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UPPSC – Polytechnic Lecturer

Electrical Engineering

Uttar Pradesh Public Service Commission (UPPSC)

Volume - 8

Signals System

SIERRA INNOVATIONS

INDEX

1 CHAPTER

Signal and Its Properties

THEORY

1.1 SIGNALS

Any set of data that contains some information and varies with respect to some independent variables, is called signal.

It may vary with respect to one variable or more than one variables. If it is varies with respect to one variable it is called one dimensional signal and generally this independent variable is time (t). On the other hand if it varies with respect to more than one variable it is called multi-dimensional signal.

Note : In our syllabus we discuss only one-variable signal and this variable is time.

1.2 CHARACTERISTIC OF A SIGNAL

There are following parameters are required to express a signal :

(i) Amplitude :

It signify the strength of the signal, and its unit depends on the type of signal.

(ii) Frequency :

It represents the rate of oscillation of the signal. Its unit is either radian per second or hertz.

(iii) Initial phase :

Initial phase represents whether the given signal is delayed version or advanced version of a standard signal. its unit is radian or degree.

Consider a signal $x(t) = A \cos(\omega_0 t + \phi)$ here

A : Amplitude of signal

ω_0 : Frequency of signal

ϕ : Initial phase of signal

DO YOU KNOW ?

Why always we discuss about sinusoidal signals only ?

As we have seen that by using fourier series and transform any signal can be represent as the sum of sinusoidal signals it means basic building block of all the signals are the sinusoidal signals only.

1.3 CLASSIFICATION OF SIGNALS

Signals can be classify into following categories :

(i) Continuous time and discrete time signals
(ii) Analog and digital signals
(iii) Energy and power signals
(iv) Periodic and Non-periodic signals
(v) Deterministics and Random signals
(vi) Even and odd signals
(vii) Causal and Non causal signal
(viii) Right sided and left sided signals.

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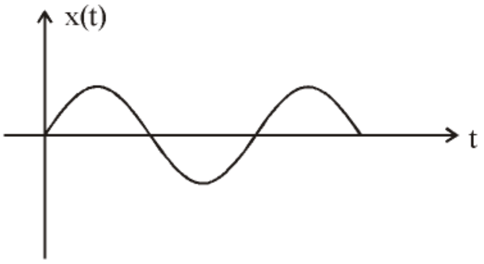
1.3.1 Continuous Time and Discrete Time Signals :

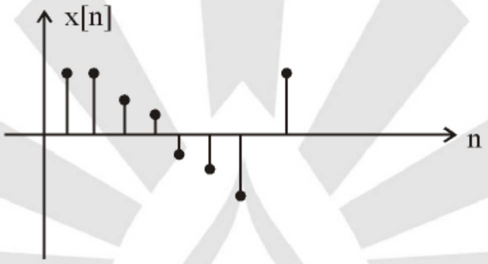
(A) Continuous Time Signals :

A continuous signal is one which is defined for all values of time in its domain :



(B) Discrete Time Signal :

Discrete time signals are those signals which are defined only at discrete value of time:



(C) Conversion of continuous signal into Discrete signal :

A discrete signal is a sampled version of a continuous signal

Consider a continuous signal.

$$x(t) = A\cos(\omega_0 t)$$

Let x(t) is sampled by a sampler having time period is T_s,

Than, $$x[n] = A\cos(\omega_0 n T_s)$$

$$x[n] = A\cos(\Omega_0 n)$$

Where, $$\Omega_0 = \omega_0 T_s$$

Ω_0 is called discrete freqency in radion per second and ω_0 is called continuous frequency.

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Example: Find the sampled version of $x(t) = 4\cos(4t)$ and hence determine the x[9], if the sampling period is $\frac{\pi}{16}$ seconds.

Solutions: $$x(t) = 4\cos(4t)$$

So, $$x[n] = 4\cos\left(4n\frac{\pi}{16}\right)$$

$$x[n] = 4\cos\left(\frac{\pi}{9}n\right)$$

Therefore, $$x[9] = 4\cos\left(\frac{9\pi}{4}\right)$$

$$= 4\cos\left(2\pi + \frac{\pi}{4}\right) = 4\cos\left(\frac{\pi}{4}\right) = \frac{4}{\sqrt{2}}$$

1.3.2 Analog and Digital Signals :

(A) Analog Signals :

If a signal can take all possible values over a finite range of amplitude axis, it is called analog signal.

(B) Digital Signals :

If a signal can take only some finite set of value over a finite range of amplitude axis, it is called digital signal.

> **DO YOU KNOW ?**
> Binary is a special case of digital signal in which signal can take only two values

(C) Difference between analog and continuous signals :

An analog signal must be continuous over time axis as well as amplitude axis where as a continuous signal is continuous over a only time axis. It may or may not be continuous over amplitude axis.

(D) Difference between discrete and digital signals :

If only time axis is discretize, it is called discrete signal where as if both the axis is discretize it is called digital.

Note: *ADC (Analog to digital converter) is a device used to convert an analog signal into digital signal.*

1.3.3 Energy and Power Signals :

A signal is said to be an energy signal if energy of the signal is finite and power of the signal is zero. i.e. for an energy signal

$$E_g = \text{finite and } P_g = 0$$

(A) Power signal :

A signal is said to be power signal if its energy is infinite and power of the signal is finite.

For a power signal $P_g = \text{Finite}$

and $E_g = \text{Infinite}$

Note:*All the practicals signals are only energy signal because there is no any practical signal whose energy is infinite.*

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1.3.4 Periodic and Non-periodic signals

If a signal repeats same waveform after certain time interval then it is called periodic otherwise it is called non-periodic signal.

1.3.5 Deterministics and Random signals

If it is possible to determine the future value of signal at any instant of time from the knowledge of previous values, then it is known as deterministic signal.

Deterministic signal is described by unique mathematical expression.

On the otherhand if a signal is described by probability function only. It is called non-deterministic or random signal.

1.3.6 Even and Odd signal

A signal x(t) is said to be an even signal if,

$$x(-t) = x(t)$$

On the otherhand a signal is said to be an odd signal if,

$$x(-t) = -x(t)$$

If any signal is neither even nor odd signal then it is called neither even nor odd (NENO) signal.

1.3.7 Causal and Non causal signal

A signal x(t) is said to be causal if it is zero for t less than zero

i.e. for x(t) to be causal

$$x(t) = 0 \text{ for } t < 0$$

A signal is said to be anticausal if,

$$x(t) = 0 \text{ for } t > 0$$

If signal is neither causal nor anti causal then it is non-causal signal.

Example: Which of the following signal is causal, anti causal or non causal

(i) $e^{t}\,u(-t)$ (ii) $e^{-t}\,u(t)$ (iii) $\sin t\,u(t-2)$

(iv) $s\,(t-2)$ (v) $e^{t}\,u(t+2)$ (vi) $e^{t}\,u(t-2)$

(vii) $e^{-t}\,u(-t-2)$

Solution: (i) Anti causal (ii) Causal (iii) Causal

(iv) Causal (v) Non-causal (vi) Causal

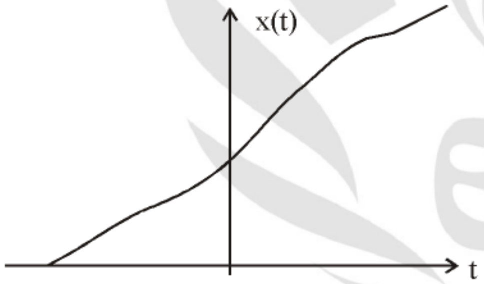
(vii) Anticausal

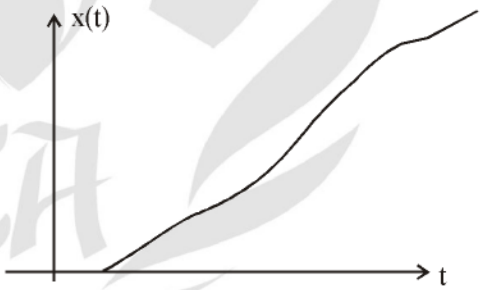
1.3.8 Right sided and left sided signals

(A) Right sided signals :

A signal extending to $+\infty$ is called right handed signal

Example:





(B) Left sided signals :

A signal extending to $-\infty$ is called left sided signal

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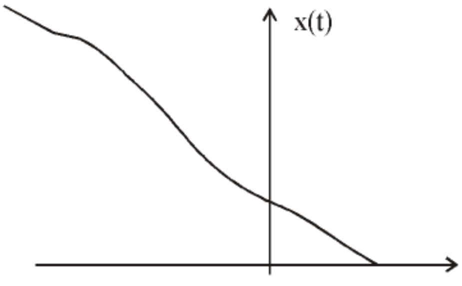
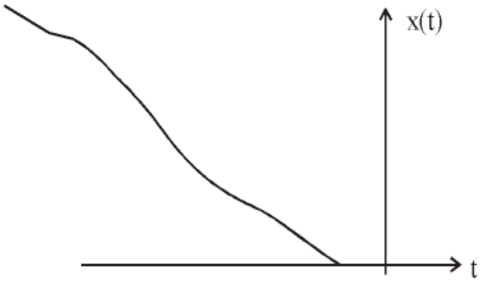
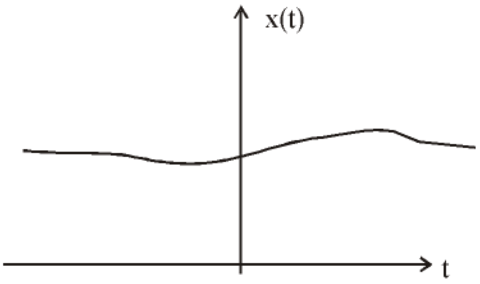
Example:





A signal extended from $-\infty$ to $+\infty$ is called etternal signal.



1.4 AREA OF A SIGNAL

Area of a signal in continuous time scale is given as

$$A=\int_{-\infty}^{\infty} x(t).dt$$

And in case of discrete time area is given as,

$$A = \sum_{n=-\infty}^{\infty} x(n)$$

1.4.1 Absolute Value of a Signal :

Absolute value of a signal in continuous time scale is given as

$$|A| = \int_{-\infty}^{\infty} |x(t)|\,dt$$

and incase of discrete time it is given as–

$$|A| = \sum_{n=-\infty}^{\infty} |x[n]|$$

1.5 BASIC OPERATIONS ON SIGNAL

Broadly operations on signal can be classify into two categories :

(i) Operation on amplitude

(ii) Operation on time scale

1.5.1 Operation on Amplitude :

Operation on amplitude can be classify into three categories as :

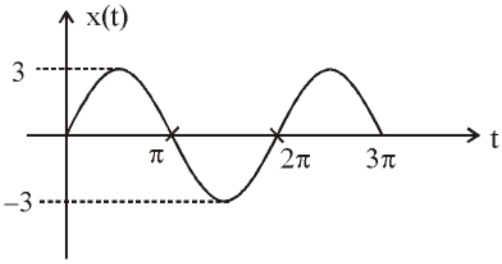
(A) Amplitude shifting or clamping

(B) Amplitude scaling

(C) Amplitude inversion

(A) Amplitude Shifting or Clamping :

Consider a signal x(t) = 3 sint, as shown in the figure.



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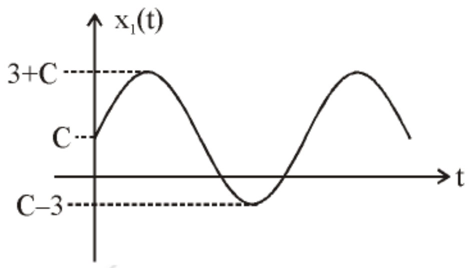
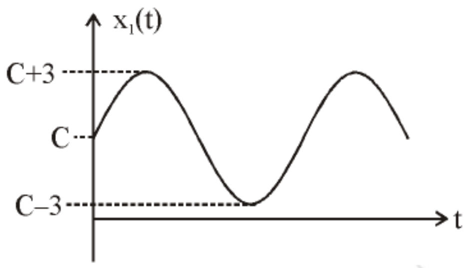
Then $x_1(t) = x(t) \pm C$ is called amplitude shifted version of x(t) i.e.

$$x_1(t) = 3 \sin t \pm C$$

For +, it is called up-shifting i.e. $x_1(t) = 3 \sin t + C$ is the up-shifting version of x(t), as shown in figure.

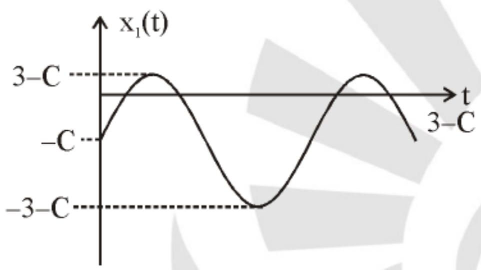


For C < 3

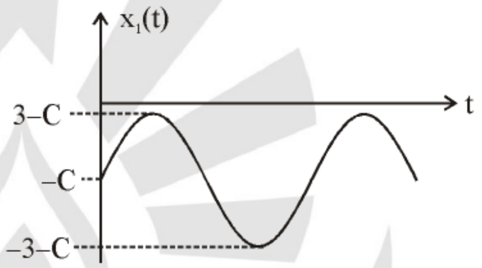


For C > 3

For –, it is called down shifting i.e. $x_1(t) = 3\sin t - C$ is the down-shifting version of x(t) as shown in figure.



For C < 3



For C > 3

(B) *Amplitude Scalling :*

Multiplication of a signal with any constant K is called amplitude scaling i.e. $x_1(t) = K\,x(t)$ is the amplitude scaled version of x(t)

Now if $|K| > 1$ then it is called amplification.

Now if $|K| < 1$ then it is called attenuation.

(C) *Amplitude Inversion :*

It is special case of scaling in which $K = -1$. It is the folded version of x(t) about horizontal axis.

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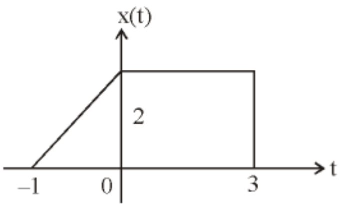
Example :

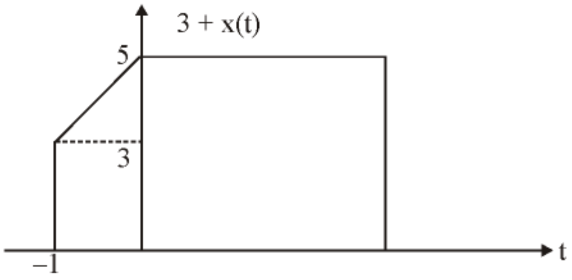


x(t) is shown in the figure sketch the following :

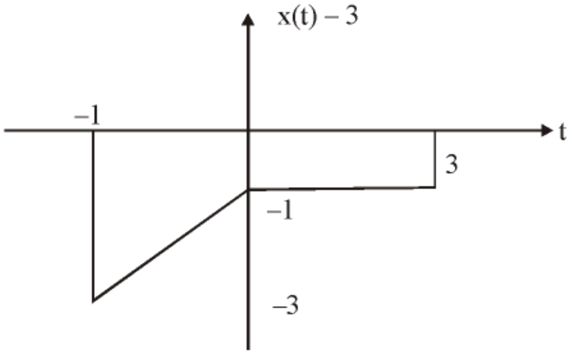
(i) $3 + x(t)$ (ii) $x(t) - 3$ (iii) $2x(t)$

(iv) $\frac{1}{2}x(t)$ (v) $-x(t) + 2$

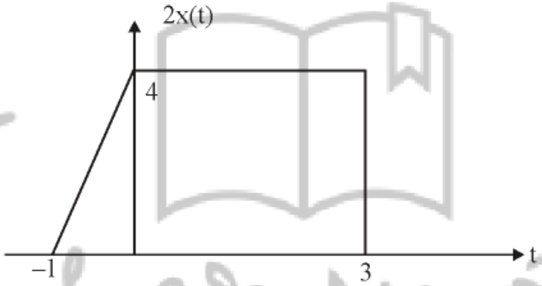
Solution: (i) $3 + x(t)$: It is the amplitude shifting by 3 units



(ii) x(t) – 3 : It is the amplitude shifting by –3 units.



(iii) 2x(t) : it is the amplitude scaling of x(t) by 2 units i.e. it is the amplification of x(t).



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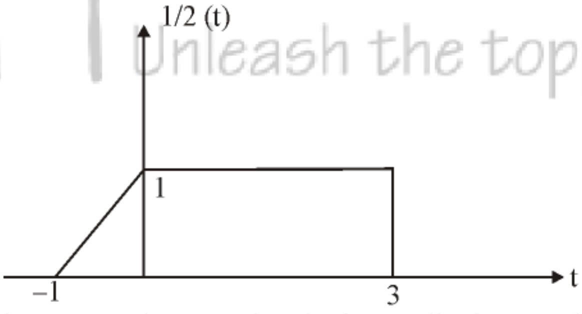
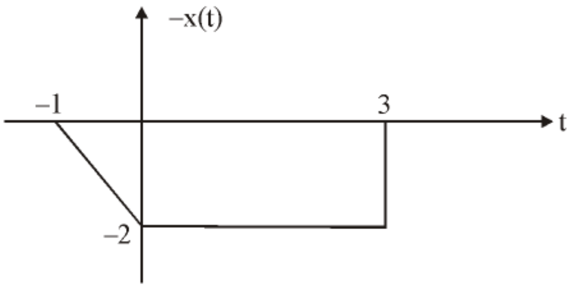
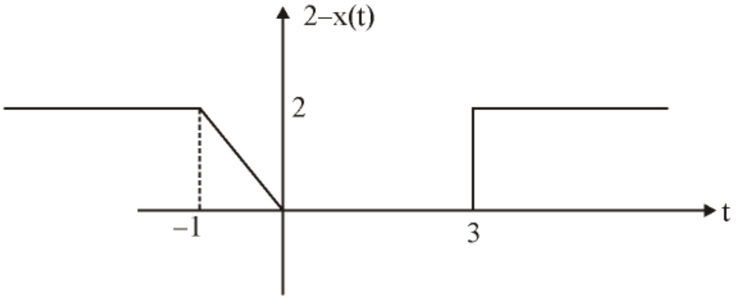
(iv) $\frac{1}{2}x(t)$: It is attenuation of x(t)



(v) –x(t) + 2 : It is a composite operation, in the amplitude operation firstly obtain –x(t) then shift by +2 unit.



Now shift $-x(t)$ by + 2 unit.



1.5.2 Operation on Time Scale :

Operations on time scale can also be classify into three categories :

(A) Time shifting

(B) Time scaling

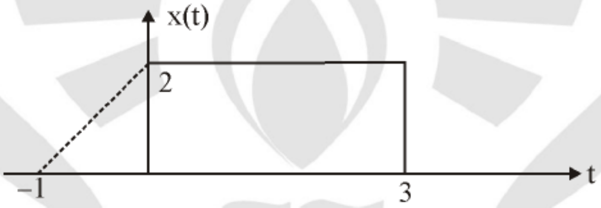
(C) Time inversion

(A) Time Shifting :

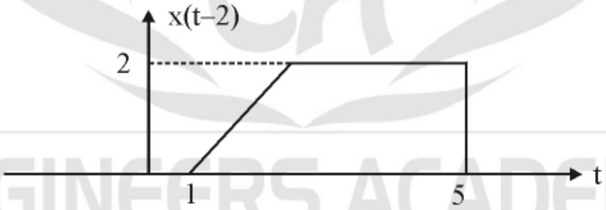
$x\ (t \pm T_0)$ is called time shifted version of x(t). It may results delay or advance.

$x(t - T_0)$ is called delayed version of x(t) and $x(t + T_0)$ is called advanced version of x(t)

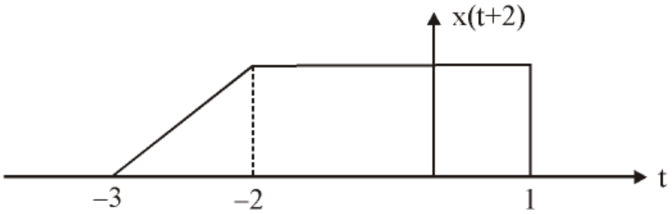
Consider x(t) as shown in the figure.



Then, x(t–2) means shifts right the signal by 2 units.



x(t+2) means shifts left the signal by 2 units.



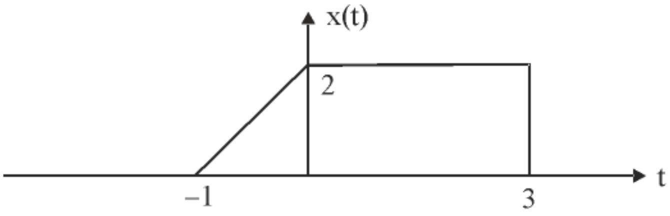
(B) Time Scaling :

x(at) is the time scaled version of x(t).

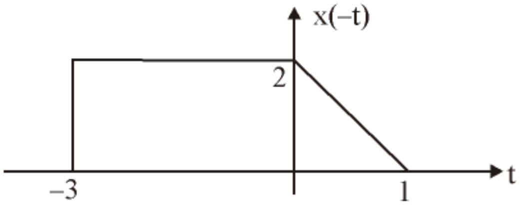
Now if $|a| > 1$ then x(at) is the compressed version of x(t) and if $|a| < 1$ then x(at) is the expanded version of x(t).

(C) Time Inversion :

Consider x(t) as shown in figure



Then x(–t),



1.6 COMPOSITE TIME OPERATION

***Procedure to sketch* x(at + b)**

Approach 1 : Let y(t) = x (at + b)

There are following steps should be follow sequentially.

Step 1 : Shift the x(t) by b units.

Step 2 : Scaled the above shifted signal by a unit.

Approach 2 : Let y(t) = x (a (t + b/a))

There are following steps should be follow sequentially.

Step 1 : Scaled the x(t) by a unit.

Step 2 : Shift the above scaled signal by b/a unit.

Note: *(i) To sketch x(–at), it does not matter whether in first step we sketch x(–t) then x(–at) or in first step x(at) then x(–at).*

(ii) x(–at + b) is the time inversion of x(at + b). Students are suggested to verify it by any numerical.

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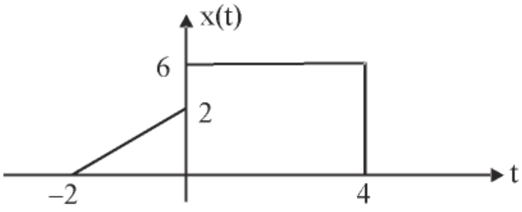
Example : The waveform of x(t) is shown in the figure

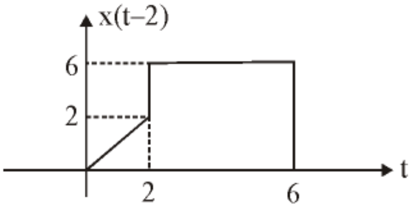


Sketch the following

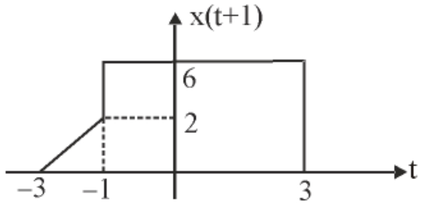
(i) x(t–2) (ii) x(t + 1) (iii) x(2t)

(iv) x(–3t) (v) x(–t+1) (vi) x(–t–1)

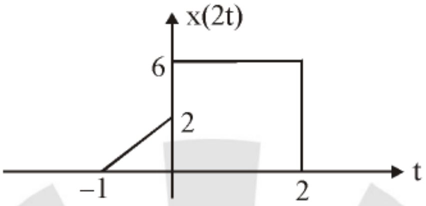
Solution: (i) x(t–2) is the time delayed version of x(t) by 2 units



(ii) x(t+1) is the time advanced version of x(t) by 1 unit

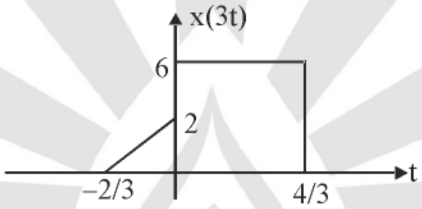


(iii) x(2t) is the compressed version of x(t), by 2 unit

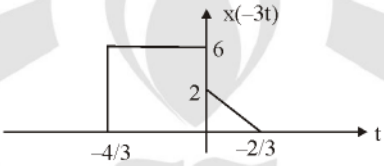


(iv) x(–3t) is the compressed as well as invert of x(t)

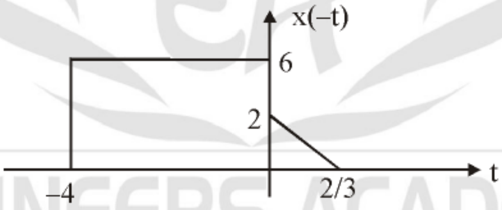
Approach 1 : In first step obtain x(3t)



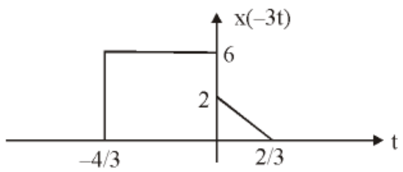
and in second step invert the graph with respect to vertical axis.



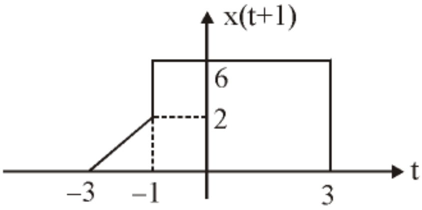
Approach 2 : In first step obtain x(–t)



and in second step scaled the above graph by 3 unit and we get x(–3t)



(v) x(–t + 1) In first step obtain x(t+1) by shifting left to 1 unit :



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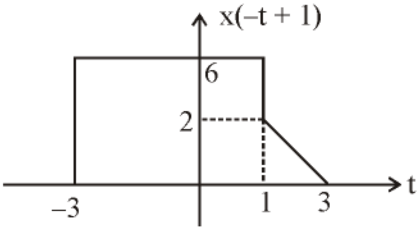
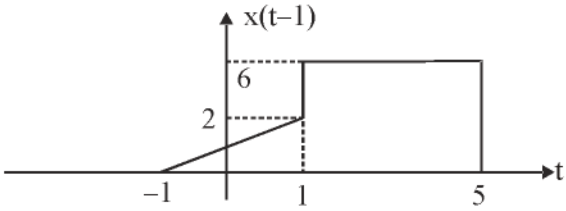
and in second step invert the above graph and we get x(–t +1)

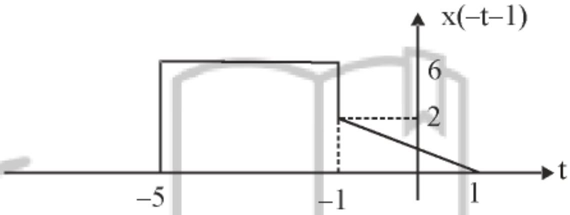


(vi) x (–t – 1)

In first step obtain x(t–1) by shifting right to 1 unit.



in second step invert the above graph and we get x(–t –1)



Example: If x(n) = $\{-1.5, \underset{\uparrow}{2}, -3, 1, 5\}$ then sketch the following

(i) x(n–2) (ii) x(n+1) (iii) x(n/3)

(iv) x(2n) (v) x(–n+1)

Solution: (i) x(n–2) means right shifting of x(n) by 2 units ;

Since. x(n) is shown in figure

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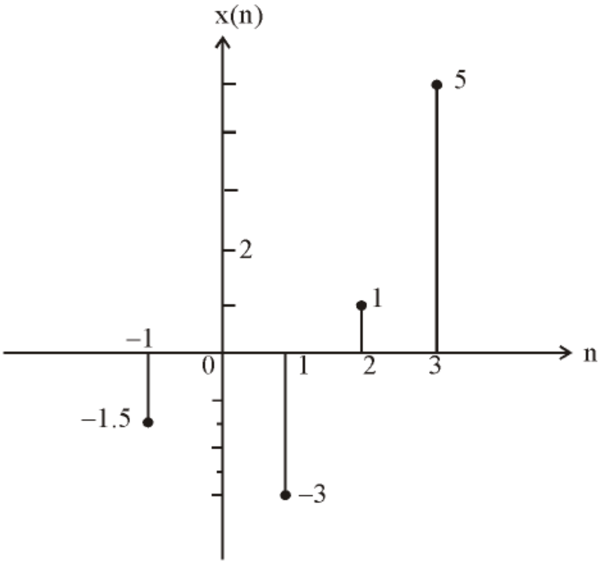
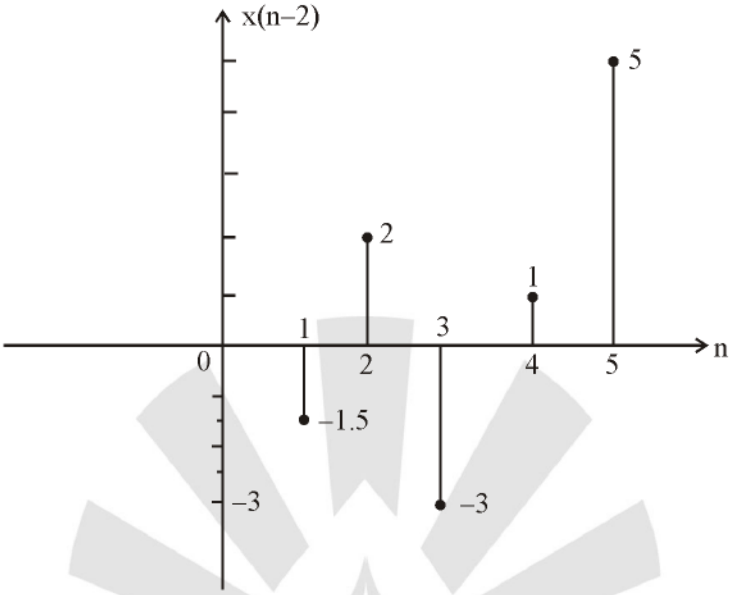


Therefore x(n–2)



(ii) x(n+1) means shift left by 1 unit,

(iii) x(n/3) : it is the expansion, expansion of discrete time signal is called as the interpolation, If 0 is added between the given points it is called zero interpolation.

If 1 is added it is called as step interpolation.

x(n/3) : Zero interpolation is shown in figure.

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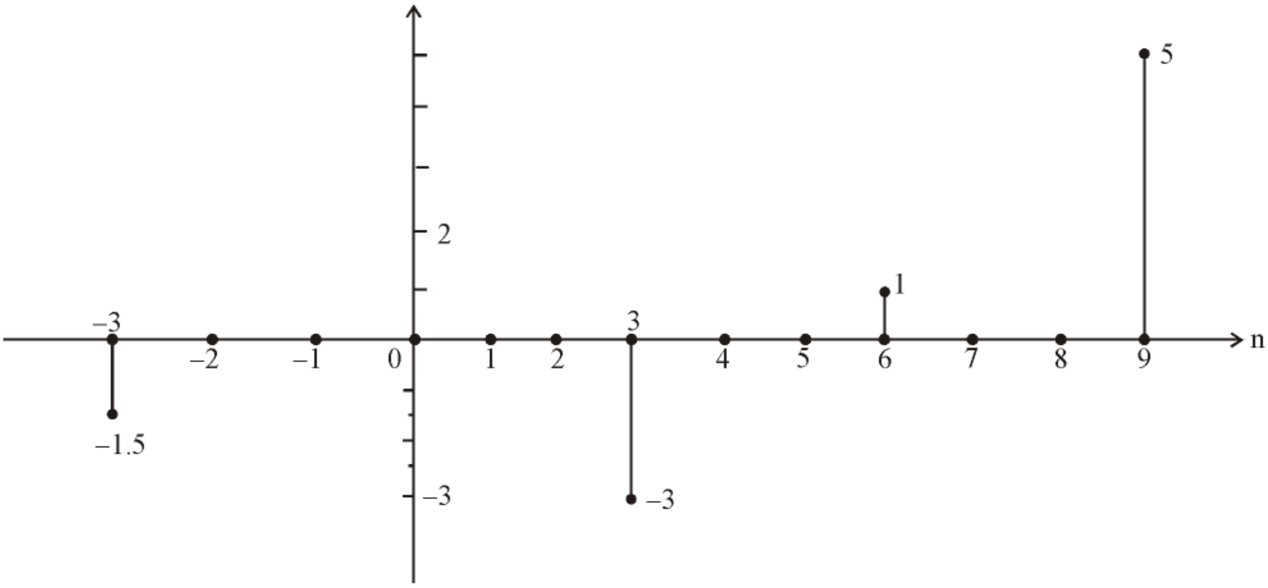
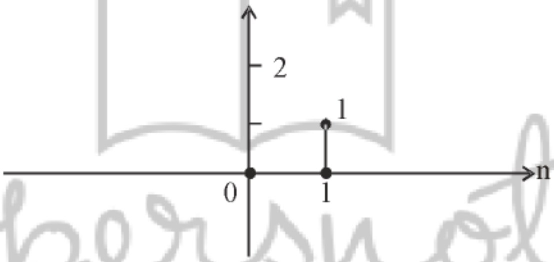


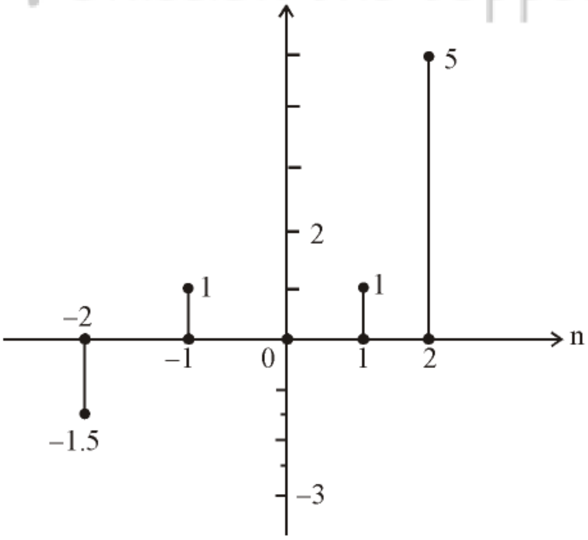
(iv) x(2n) : It is the compression of discrete time signal :

Compression of discrete time signal is called decimation. (loss of information)

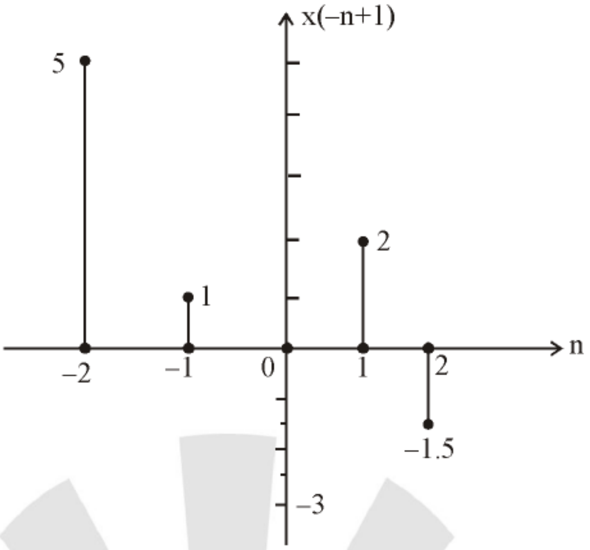


Toppersnotes® Unleash the topper in you

(v) x(−n+1) : In first step shift left by 1 unit



Now in second step invert the above signal for x(−n+1)

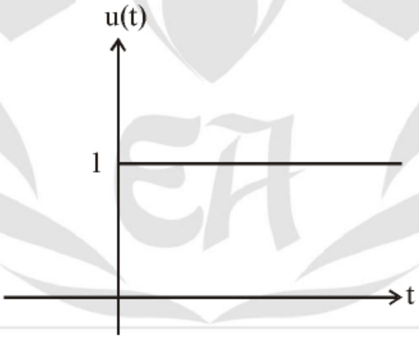


1.7 BASIC CONTINUOUS SIGNALS

1.7.1 Step Signal :

$$Au(t) = \begin{cases} A & \text{for} \quad t > 0 \\ 0 & \text{for} \quad t < 0 \end{cases}$$

For A = 1 it is called unit step signal i.e. $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$



Similarly, $u(-t) = \begin{cases} 1 & \text{for} \quad -t > 0 \quad \text{or} \quad t < 0 \\ 0 & \text{for} \quad -t < 0 \quad \text{or} \quad t > 0 \end{cases}$

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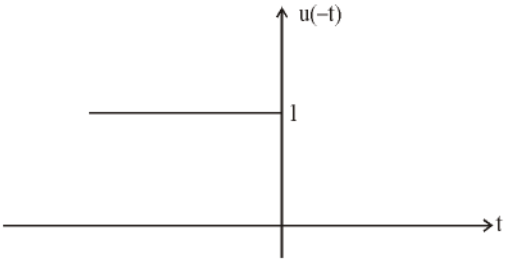
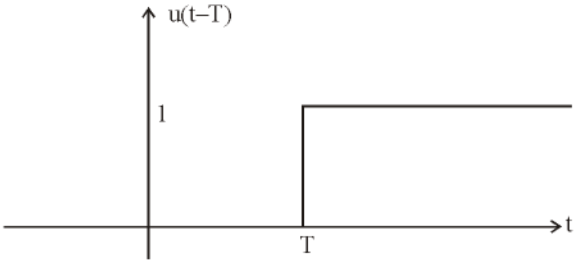
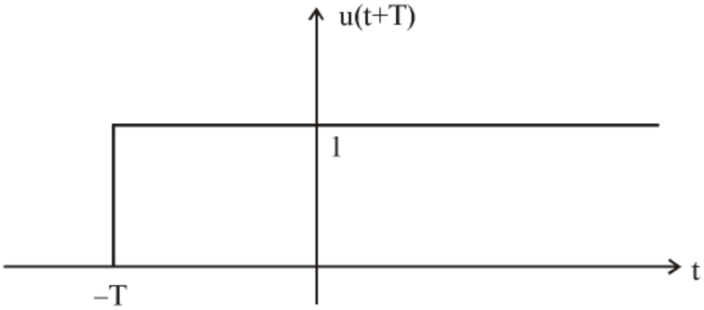


Note: $1 = u(t) + u(-t)$

Delayed unit step : $u(t-T) = \begin{cases} 1 & \text{for} \quad t - T > 0 \quad \text{or} \quad t > T \\ 0 & \text{for} \quad t - T < 0 \quad \text{or} \quad t < T \end{cases}$



Advanced unit step : $u(t+T)=\begin{cases}1 & \text{for} \quad t+T>0 \quad \text{or} \quad t>-T\\ 0 & \text{for} \quad t+T<0 \quad \text{or} \quad t<-T\end{cases}$

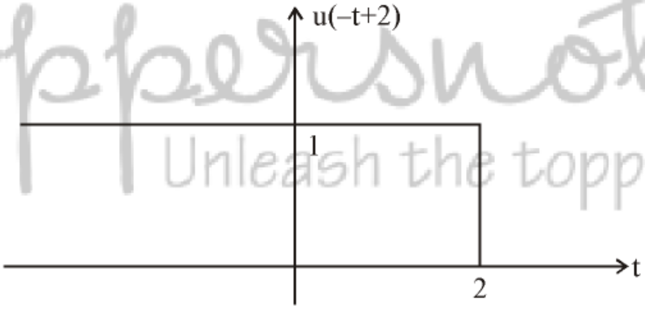


Example: Sketch the following signals

(i) u(–t+2) (ii) u(–t–2)

Solution : (i) u(–t+2)

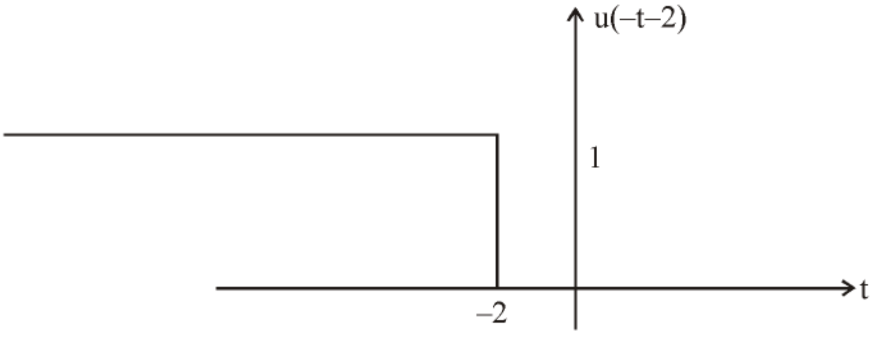
$$u(-t+2)=\begin{cases}1 & \text{for} \quad -t+2>0 \quad \text{or} \quad t<2\\ 0 & \text{for} \quad -t+2<0 \quad \text{or} \quad t>2\end{cases}$$



Toppersnotes® Unleash the topper in you

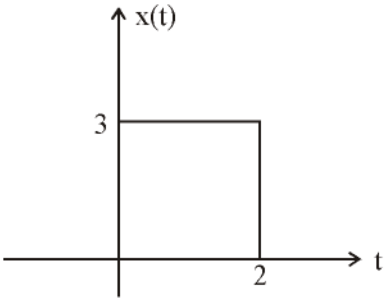
(ii) u(–t–2)

$$u(-t-2)=\begin{cases}1 & \text{for} \quad -t-2>0 \quad \text{or} \quad t<-2\\ 0 & \text{for} \quad -t-2<0 \quad \text{or} \quad t>-2\end{cases}$$

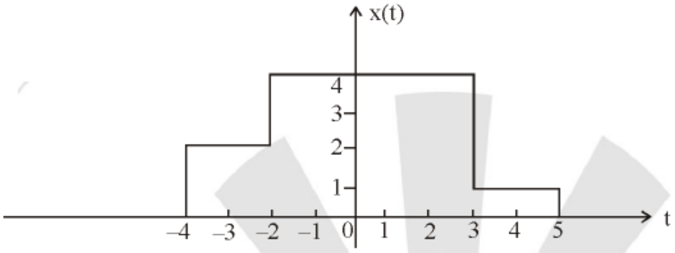


Example: Write the expression of given waveform in terms of unit step.

(i)

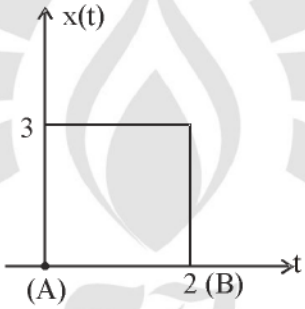


(ii)



Solution: (i) For such types of problems firstly point out the critical point (Point where wave changes its shape)

and then find amount of jump at each critical point.



At point A & B wave changes its shape so these two are the critical point.

Jump at point A = 3 unit,

Jump at point B = –3 unit,

So, x(t) = 3 u(t) – 3 u(t–2)

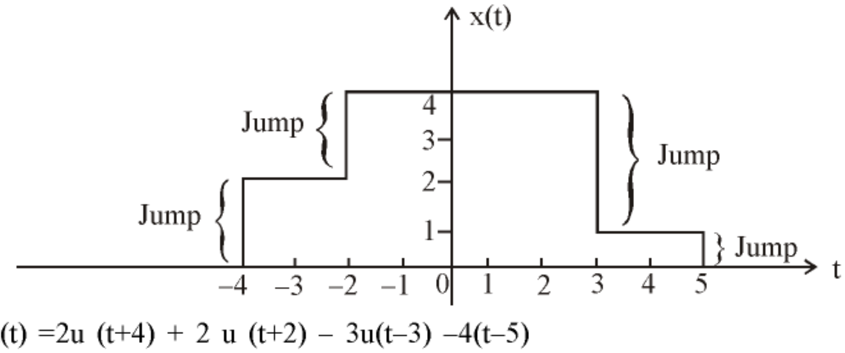
(ii) Critical points are, –4, –2, 3, 5,

Jump at point –4 = 2 unit

Jump at point –2 = 2 unit

Jump at point 3 = –3 unit

Jump at point 5 = –1 unit



So, x(t) =2u (t+4) + 2 u (t+2) – 3u(t–3) –4(t–5)

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(iii) Critical points are, –5, –3, –2, 3, 5

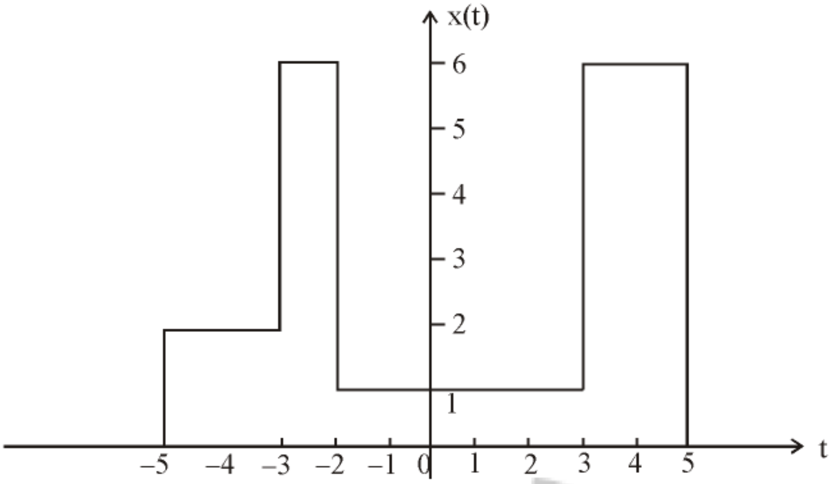
Jump x(t) at point –5 = 2 unit

Jump x(t) at point –3 = 4 unit

Jump x(t) at point –2 = –5 unit

Jump x(t) at point 3 = 5 unit

Jump x(t) at point 5 = –6 unit.



So, x(t) =2 u(t+5) + 4u(t+3) – 5u(t+2) + 5u(t–3) – 6 u(t–5)

1.7.2 Impulse Function :

$$A\delta(t) = \begin{cases} 0 & \text{for} \quad t \neq 0 \\ \infty & \text{for} \quad t = 0 \end{cases}$$

and area of impulse function = $\int_{-\infty}^{\infty} A\delta(t)dt = A$

For A = 1 it is called unit impulse function

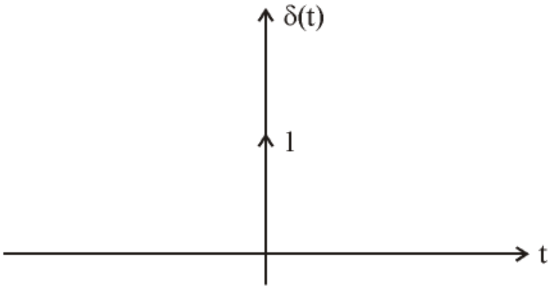
$$\delta(t) = \begin{cases} 0 & \text{for} \quad t \neq 0 \\ \infty & \text{for} \quad t = 0 \end{cases}$$

and area of impulse function = $\int_{-\infty}^{\infty} A\delta(t)dt = A$

For A = 1 it is called unit impulse function

$$\delta(t) = \begin{cases} 0 & \text{for} \quad t \neq 0 \\ \infty & \text{for} \quad t = 0 \end{cases}$$

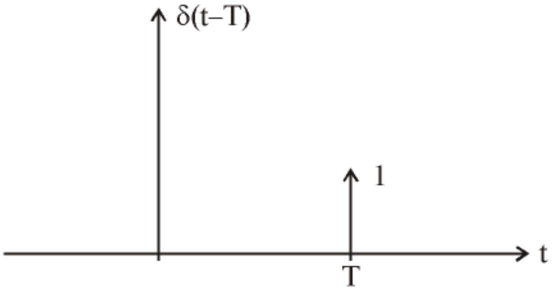
and, Area = $\int_{-\infty}^{\infty} \delta(t).dt = 1$



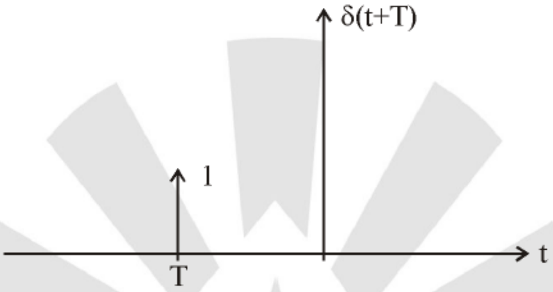
Note: *The height of arrow in the graph represents the area of impulse function, it does not represents amplitude at that point.*

Delayed and Advanced version of impulse function :

(i) Delay impulse function :



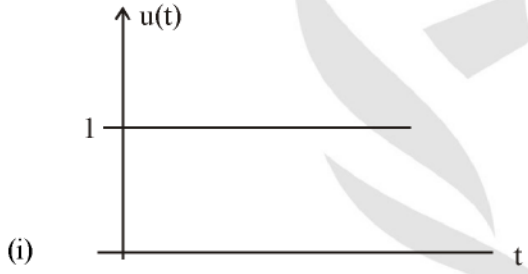
(ii) Advance impulse function :

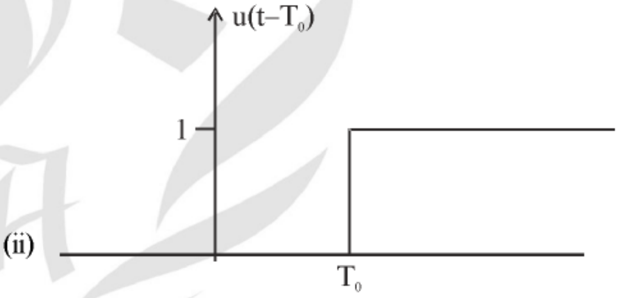


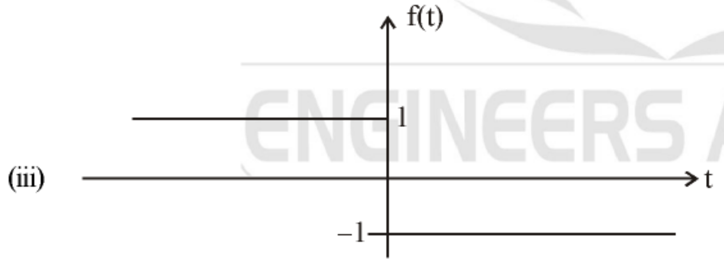
1.7.3 Relationship Between Step and Impulse Function :

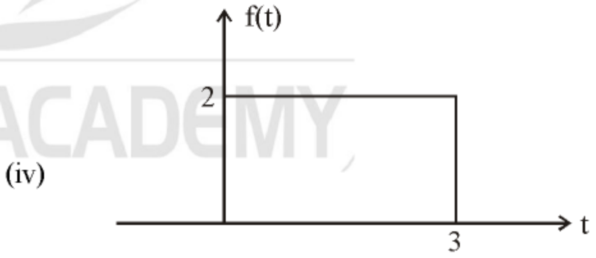
$$\delta(t) = \frac{d}{dt}u(t) \quad \text{and} \quad u(t) = \int_{-\infty}^{t} \delta(\tau).d\tau$$

Example: Differentiate following wave forms with respect t









Solution: (i) Since it is constant so differentiation is zero except at the point of jump, and at this jump there is an impulse.