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UPPSC – Polytechnic Lecturer

Mechanical Engineering

Uttar Pradesh Public Service Commission (UPPSC)

Volume - 5

Engineering Mechanics & Machine Design

SIERRA INNOVATIONS

INDEX

1
CHAPTER

Introduction

THEORY

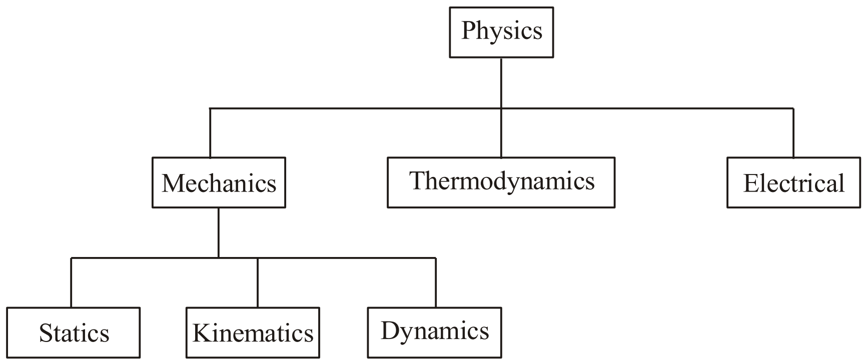
Introduction

What is Mechanics?

Mechanics is an area of science concerned with the behavior of physical bodies when subjected to forces or displacements, and the subsequent effects of the bodies on their environment.

What is Engineering Mechanics?

Engineering mechanics is the application of mechanics to solve problems involving common engineering elements. The goal of this Engineering Mechanics course is to expose students to problems in mechanics as applied to plausibly real-world scenarios.



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2
CHAPTER

System of Forces

THEORY

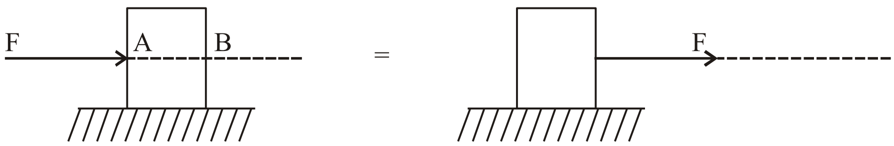
2.1 DEFINITIONS

Force - It is something that have either displacement or deformation effect on the body

Line of action of force - It is the direction of effect on the body. It is also the direction of force

2.2 LAW OF TRANSMISSIBILITY OF FORCE

Along the line of action if force is shifted, effect of force will remain same



2.3 SYSTEM OF FORCES

When multiple forces act on the body in different direction, at different point and of different magnitude, it is called system of forces

It can be of two types:

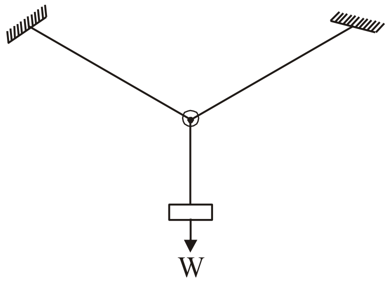
(a) Coplanar Forces (2D Forces)

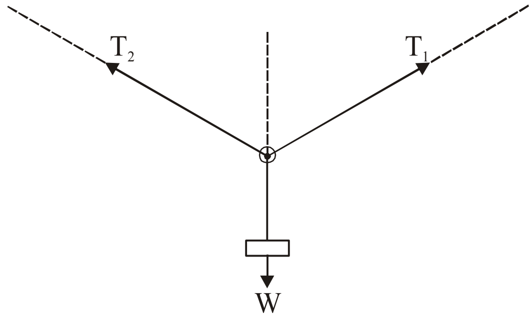
(b) Non Coplanar forces (3D Forces)

System of Forces

- Coplaner forces [Single Plane-2D]
  - → Concurrent forces
  - → Non Concurrent forces
  - → Parallel forces
  - → Collinear forces
- Non-Coplaner forces [3D forces]
  - → Concurrent forces
  - → Non Concurrent forces
  - → Parallel forces
  - → Collinear forces

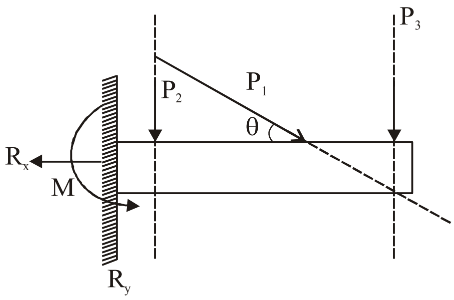
Concurrent Forces : If the line of action of all the force meet at a single point, then it is called concurrent for system.



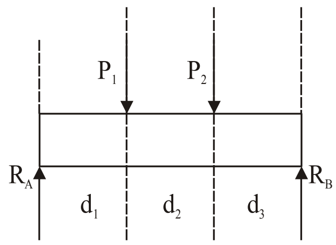


T_1, T_2 & W are Meeting at 'O'

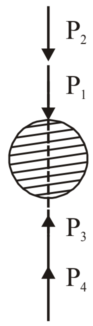
Non concurrent forces : If the line of action of all forces do not meet at a single point, then it is called non-concurrent force system.



Parallel Forces : if the line of action of all forces do not meet anywhere, then it is called parallel force system.



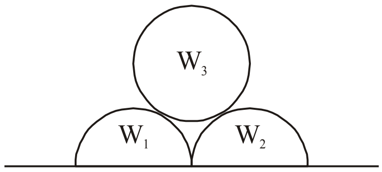
Collinear Forces : If all the forces are having same line of action, then is is called collinear force system.



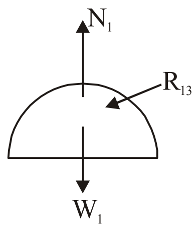
2.4 FREE BODY DIAGRAM

It is the diagram representing all the external forces acting on a body or the system of bodies.

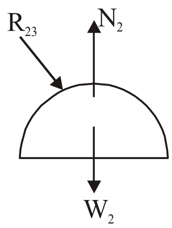
Example 1



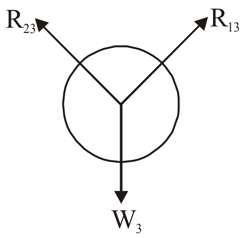
F.B.D. For W_1



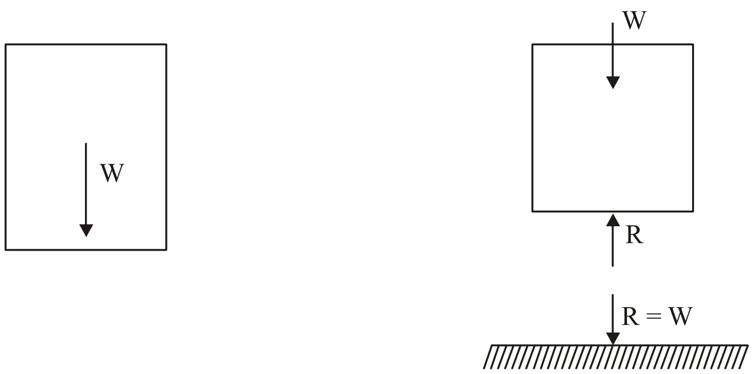
F.B.D. For W_2



F.B.D. For W_3

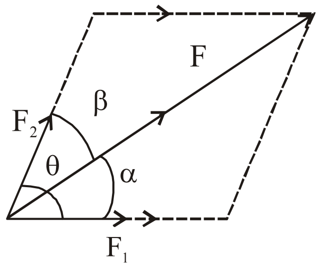


Example 2



2.5 LAW OF FORCES

2.5.1 *Law of Parallelogram*



$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

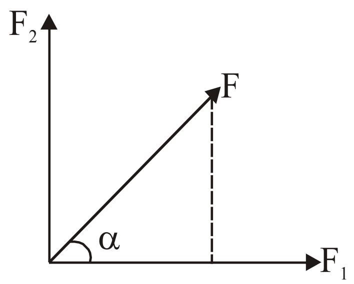
Case–I

$\theta = 0$ $\Rightarrow$ Collinear force

$$F = F_1 + F_2$$

Case–II

$\theta = 90^o$ $\Rightarrow$ Perpendicular force



$$F = \sqrt{F_1^2 + F_2^2}$$

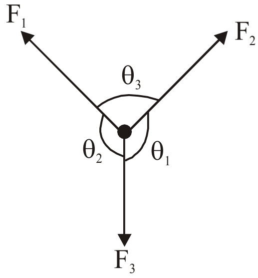
$$\tan\alpha = \frac{F_2}{F_1}$$

Case–III

$\theta = 180^o$ $\Rightarrow$ Collinear force

$$F = F_1 - F_2 \quad [F_1 > F_2]$$
$$F = F_2 - F_1 \quad [F_2 > F_1]$$

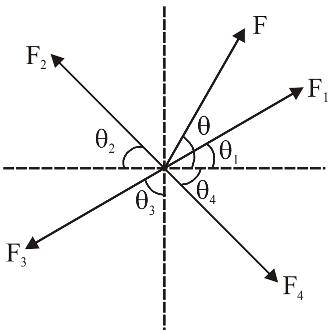
2.5.2 *Lami's Theorem (Sine Rule)*



For three concurrent forces to be in equilibrium

$$\Rightarrow \qquad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

2.5.3 *Component of Forces*



In this method forces are resolved along two perpendicular axes.

$$F_x = F_1\cos\theta_1 - F_2\cos\theta_2 - F_3\sin\theta_3 + F_4\cos\theta_4$$

$$F_y = F_1\sin\theta_1 + F_2\sin\theta_2 - F_3\sin\theta_3 - F_4\sin\theta_4$$

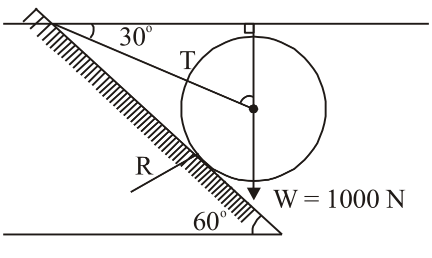
For resultant force

$$F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

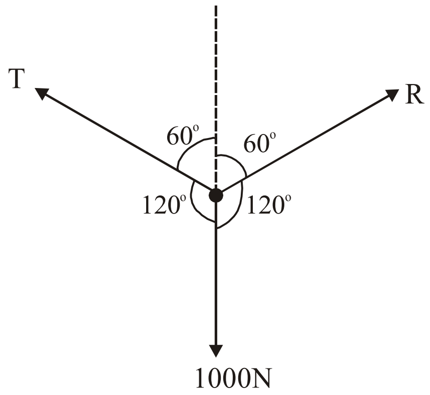
$$\tan\theta = \frac{F_y}{F_x}$$

Example 3

Determine reaction by inclined plane on the sphere



Solution:

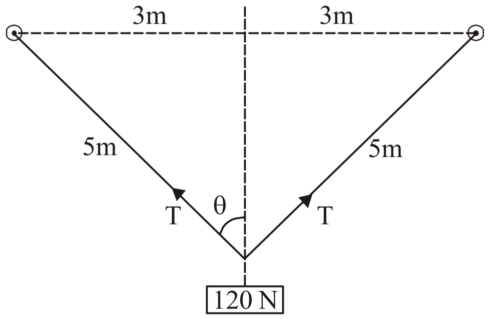


$$\frac{T}{\sin 120^o} = \frac{R}{\sin 120^o} = \frac{1000}{\sin 120^0}$$

$$T = R = 1000N$$

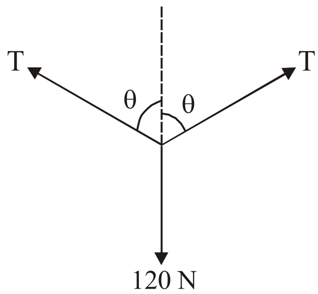
Example 4

A string of length 10m supporting a weight of 120 N as shown in fig.



Determine Tension in string

Solution:



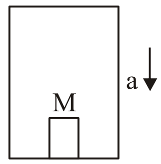
$$2T \cos\theta = 120$$

$$T = \frac{120 \times 5}{2 \times 4}$$

$$T = 75 \text{ N}$$

Example 5

With what acceleration 'a' should the box of figure descend so that the block of mass m exerts a force Mg/4 on the floor of the box?



Solution:

The block is at rest with respect to the box which is accelerated with respect to the ground. Hence, the acceleration of the block with respect to the ground is 'a' downward. The force on the block are

(i) Mg downward (by the earth) and

(ii) N upward (by the floor).

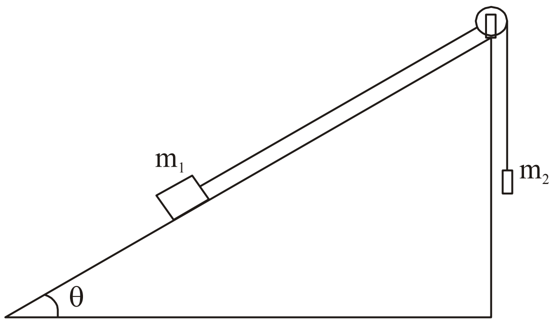
The equation of motion of the block is, therefore,

$$Mg - N = Ma.$$

If N = Mg/4, the above equation gives a = 3g/4. The block and hence the box should descend with an acceleration 3g/4.

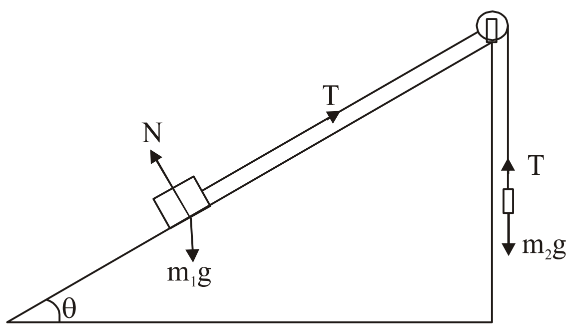
Example 6

Two bodies of masses m_1 and m_2 are connected by a light string going over a smooth light pulley at the end of an incline. The mass m_1 lies on the incline and m_2 hangs vertically. The system is at rest. Find the angle of the incline and the force exerted by the incline on the body of mass m_1.



Solution:

Figure shows the situation with the forces on m_1 and m_2 shown. Take the body of mass m_2 as the system. The forces acting on it are



(i) m_2g vertically downward (by the earth),

(ii) T vertically upward (by the string).

As the system is at rest, these forces should add to zero.

This gives $T = m_2g.$ ...(i)

Next, consider the body of mass m_1 as the system. The forces acting on this system are

(i) m_1g vertically downward (by the earth),

(ii) T along the string up the incline (by the string)

(iii) N normal to the incline (by the incline).

As the string and the pulley are all light and smooth, the tension in the string is uniform everywhere. Hence, same T is used for the equations of m_1 and m_2. As the system is in equilibrium, these forces should add to zero.

Taking components parallel to the incline,

$$T = m_1 g \cos\left(\frac{\pi}{2} - \theta\right) = m_1 g \sin\theta. \quad \text{...(ii)}$$

Taking components along the normal to the incline,

$$N = m_1 g \cos\theta \quad \text{...(iii)}$$

Eliminating T from (i) and (ii),

$$m_2 g = m_1 g \sin\theta$$

or, $\sin\theta = m_2/m_1$

giving $\theta = \sin^{-1}(m_2/m_1).$

From (iii) $N = m_1 g\sqrt{1-(m_2/m_1)^2}$

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3

CHAPTER

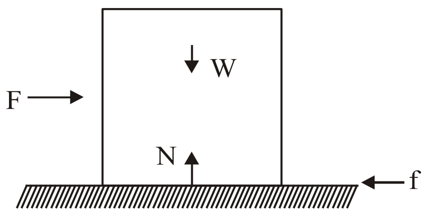
Friction & Application

THEORY

3.1 FRICTION

It is the resistance force between contact surfaces due to roughness of surfaces

It always acts opposite to the relative motion of contact surfaces



F → applied force

W → weight of body

N → Normal Reaction

f → Friction or frictional force.

3.2 LAW OF FRICTION

Maximum frictional force is directly proportional to the normal force between the surfaces

$$f = \mu N$$

μ → coefficient of friction.

$$(f_s)_{max} = \mu_s N$$

$(f_s)_{max}$ → Limiting friction.

When the applied force is less than Maximum friction force . The body is in rest and the friction forces is equal to applied force.

When applied force is equal to maximum friction force, the body is about to move, this condition is called impending motion.

When applied force is more than maximum friction force, the body is in motion and friction on the body is kinetic friction.

$$(f_k) = \mu_k N$$

3.3 Application of Friction

(1) Wedges

(2) Ladder

(3) Screw Jack

(4) Brakes & Clutches

Example 1

The coefficient of static friction between a block of mass m and an incline is $\mu_s = 0.3$. (a) What can be the maximum angle θ of the incline with the horizontal so that the block does not slip on the plane? (b) If the incline makes an angle θ/2 with the horizontal, find the frictional force on the block.

Solution:

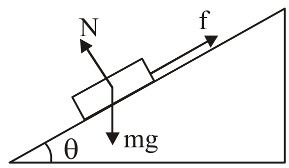
The situation is shown in figure.

(a) The forces on the block are

(i) the weight mg downward by the earth,

(ii) the normal contact force N by the incline, and

(iii) the friction 'f' parallel the incline up the plane, by the incline.



As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is a case of limiting equilibrium and so $f = \mu_s N$.

Taking components perpendicular to the incline,

$$N - mg\cos\theta = 0$$

or, $$N = mg\cos\theta \quad ....(i)$$

Taking components parallel to the incline,

$$f - mg\sin\theta = 0$$

or, $$f = mg\sin\theta$$

or, $$\mu_s N = mg\sin\theta \quad ...(ii)$$

Dividing (ii) by (i) $\mu_s = \tan\theta$

or, $$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.3).$$

(b) If the angle of incline is reduced to θ/2, the equilibrium is not limiting, and hence the force of static friction f is less than $\mu_s N$. To know the value of f, we proceed as in part (a) and get the equations

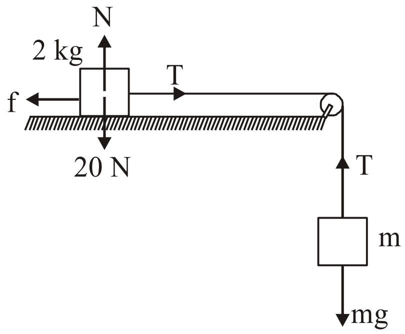
$$N = mg\cos(\theta/2)$$

and $$f = mg\sin(\theta/2).$$

Thus, the force of friction is mg sin (θ/2).

Example 2

The coefficient of static friction between the block of 2 kg and the table shown in figure is $\mu_s = 0.2$. What should be the maximum value of m so that the blocks do not move ? Take g = 10 m/s^2. The string and the pulley are light and smooth.



Solution:

Consider the equilibrium of the block of mass m. The forces on this block are

(a) mg downward by the earth and

(b) T upward by the string.

Hence, $$T - mg = 0 \text{ or, } T = mg. \quad ...(i)$$

Now consider the equilibrium of the 2 kg block. The forces on this block are

(a) T towards right by the string.

(b) f towards left (friction) by the table,

(c) 20 N downward (weight) by the earth and

(d) N upward (normal force) by the table.

For vertical equilibrium of this block,

$$N = 20 \text{ N}. \quad ...(ii)$$

As m is the largest mass which can be used without moving the system, the friction is limiting.

Thus, $$f = \mu_s N. \quad ...(iii)$$

For horizontal equilibrium of the 2 kg block,

$$f = T. \quad ...(iv)$$

Using equations (i), (iii), and (iv)

$$\mu_s N = mg$$

or, $$0.2 \times 20 \text{ N} = mg$$

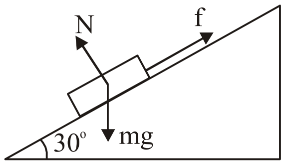
or, $$m = \frac{0.2 \times 20}{10} \text{ kg} = 0.4 \text{ kg}.$$

Example 3

A block slides down an incline of angle 30^o with an acceleration g/4. Find the kinetic friction coefficient.

Solution:

Let the mass of the block be m. The forces on the block are



(a) mg downward by the earth (gravity),

(b) N normal force by the incline and

(c) f up the plane, (friction) by the incline.

Taking components parallel to the incline and writing Newton's second law,

$$mg \sin 30^o - f = mg/4$$

$$f = mg/4.$$

There is no acceleration perpendicular to the incline.

Hence

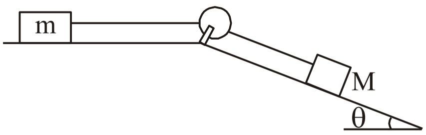
$$N = mg \cos 30^o = mg. \frac{\sqrt{3}}{2}$$

As the block is slipping on the incline, friction is $f = \mu_k N$.

So $$\mu_k = \frac{f}{N} = \frac{mg}{4 mg\sqrt{3}/2} = \frac{1}{2\sqrt{3}}.$$

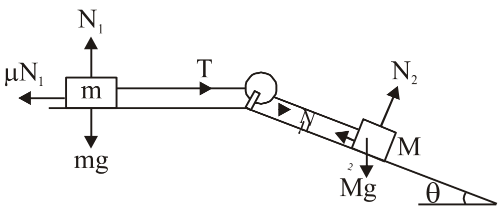
Example 4

Find the maximum value of M/m in the situation shown in figure so that the system remains at rest. Friction coefficient at both the contacts is μ. Discuss the situation when $\tan\theta < \mu$.



Solution:

Figure shows the forces acting on the two blocks. As we are looking for the maximum value of M/m,. the equilibrium is limiting. Hence, the frictional forces are equal to μ times the corresponding normal forces.



Equilibrium of the block m gives

$$T = \mu N_1 \text{ and } N_1 = mg$$

which gives

$$T = \mu\ mg. \quad ...(i)$$

Next, consider the equilibrium of the block M. Taking components parallel to the incline.

$$T + \mu N_2 = Mg \sin\theta.$$

Taking components normal to the incline

$$N_2 = Mg \cos\theta.$$

These give $T = Mg(\sin\theta - \mu \cos \theta)$. ...(ii)

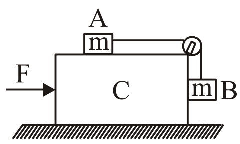
From (i) and (ii), $\mu\ mg = Mg (\sin\theta - \mu \cos \theta)$

or, $$M/m = \frac{\mu}{\sin\theta - \mu\cos\theta}$$

If $\tan\theta < \mu$, $(\sin\theta - \mu \cos\theta) < 0$ and the system will not slide for any value of M/m.

Example 5

Consider the situation shown in figure . The horizontal surface below the bigger block is smooth. the coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller blocks at rest with respect to the bigger block.

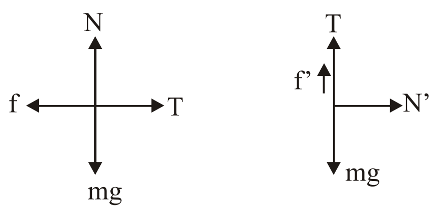


Solution:

If no force is applied, the block A will slip on C towards right and the block B will move downward. Suppose the minimum force needed to prevent slipping if F. Taking A + B+ C as the system, the only external horizontal force on the system is F. Hence, the acceleration of the system is

$$a = \frac{F}{M + 2m} \quad ...(i)$$

Now take the block A as the system. The forces on A are,



(i) tension T by the string towards right,

(ii) friction f by the block C towards left,

(iii) weight mg downward and

(iv) normal force N upward.

For vertical equilibrium N = mg

As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus,

$$f = \mu N = \mu\ mg.$$

As the block moves towards right with an acceleration a,

$$T - f = ma$$

or, $$T - \mu\ mg = ma \qquad ...(ii)$$

Now take the block B as the system. The forces are,

(i) tension T upward,

(ii) weight mg downward,

(iii) normal force N towards right, and

(iv) friction f' upward.

As the block moves towards right with an acceleration a,

$$N' = ma.$$

As the friction is limiting, $$f' = \mu N' = \mu ma\ .$$

For vertical equilibrium

$$T + f' = mg$$

or, $$T + \mu\ ma = mg. \qquad ...(iii)$$

Eliminating T from (ii) and (iii)

$$a_{min} = \frac{1-\mu}{1+\mu} g.$$

When a large force is applied the block A slips on C towards left and the block B slips on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is

$$a_{max} = \frac{1+\mu}{1-\mu} g.$$

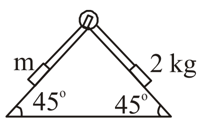
Thus, a lies between $\frac{1-\mu}{1+\mu} g$ and $\frac{1+\mu}{1-\mu} g$.

From (i) the force F should be between

$\frac{1-\mu}{1+\mu}(M + 2m)\ g$ and $\frac{1+\mu}{1-\mu}(M + 2m)\ g$.

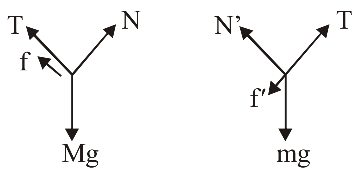
Example 6

Figure shows two blocks connected by a light string placed on the two inclined parts of a triangular structure. The coefficients of static and kinetic friction are 0.28 and 0.25 respectively at each of the surfaces. (a) Find the minimum and maximum values of m for which the system remains at rest. (b) Find the acceleration of either block if m is given the minimum value calculated in the first part and is gently pushed up the incline for a short while



Solution:

(a) Take the 2 kg block as the system. The forces on this block are shown in figure with M = 2 kg. It is assumed that m has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.



Taking components ⊥ to the incline

$$N = Mg \cos 45^o = Mg/\sqrt{2}.$$

Taking components || to the incline

$$T + f = Mg \sin 45^o = Mg/\sqrt{2}$$

or, $$T = Mg/\sqrt{2} - f.$$

As it is a case of limiting equilibrium,

$$f = \mu_s N$$

or, $$T = \frac{Mg}{\sqrt{2}} - \mu_s \frac{Mg}{\sqrt{2}} = \frac{Mg}{\sqrt{2}}(1 - \mu_s). \quad ...(i)$$

Now consider the other block as the system. The forces acting on this block are shown in figure. Taking components ⊥ to the incline,

$$N' = mg \cos 45^o = mg/\sqrt{2}.$$

Taking components || to the incline

$$T = mg \sin 45^o + f' = \frac{mg}{\sqrt{2}} + f'.$$

As it is the case of limiting equilibrium

$$f' = \mu_s N' = \mu_s \frac{mg}{\sqrt{2}}.$$

Thus, $$T = \frac{mg}{\sqrt{2}}(1 + \mu_s), \quad ...(ii)$$

From (i) and (ii)

$$m(1 + \mu_s) = M(1 - \mu_s) \quad ...(iii)$$

or, $$m = \frac{(1-\mu_s)}{(1+\mu_s)} M = \frac{1-0.28}{1+0.28} \times 2 \text{ kg}$$

$$= \frac{9}{8} \text{ kg}.$$

When maximum possible value of m is supplied, the directions of friction are reversed because m has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of m can be obtained from (iii) by putting $\mu_s = -0.28$. Thus, the maximum value of m is

$$m = \frac{1+0.28}{1-0.28} \times 2 \text{ kg}$$

$$= \frac{32}{9} \text{kg.}$$

(b) If m = 9/8 kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k \cdot \frac{Mg}{\sqrt{2}} \text{ and } f' = \frac{\mu_k mg}{\sqrt{2}},$$

where $\mu_k = 0.25$. If the acceleration is a, Newton's second law for M gives.

$$Mg \sin 45^o - T - f = Ma$$

or, $$\frac{Mg}{\sqrt{2}} - T - \frac{\mu_k Mg}{\sqrt{2}} = Ma. \quad ...(iv)$$

Applying Newton's second law

$$T - mg \sin 45^o - f' = ma$$

or, $$T - \frac{mg}{\sqrt{2}} - \frac{\mu_k mg}{\sqrt{2}} = ma. \quad ...(v)$$

Adding (iv) and (v)

$$\frac{Mg}{\sqrt{2}}(1 - \mu_k) - \frac{mg}{\sqrt{2}}(1 + \mu_k) = (M + m)\, a$$

or, $$a = \frac{M(1-\mu_k) - m(1-\mu_k)}{\sqrt{2}(M+m)} g$$

$$= \frac{2 \times 0.75 - 9/8 \times 1.25}{\sqrt{2}(2+9/8)} g$$

$$= 0.31 \text{ m/s}^2$$

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