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UPPSC – Polytechnic Lecturer

Electronics Engineering

Uttar Pradesh Public Service Commission (UPPSC)

Volume - 3

Communication Systems

SIERRA INNOVATIONS

INDEX

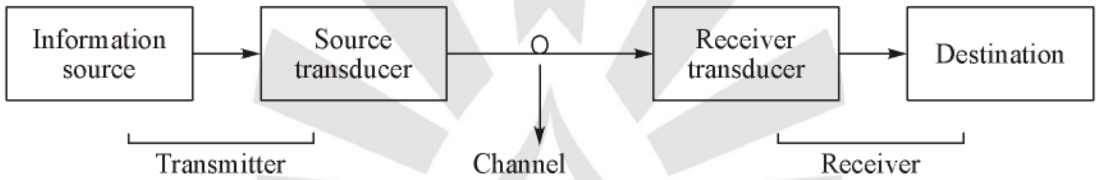
1 CHAPTER

Amplitude Modulation System

THEORY

The communication is the process of transmission of information from transmitter (source) to destination (receiver).

1. BASIC BLOCK DIAGRAM OF COMMUNICATION SYSTEM



Note: *(i) Frequency range of audio signal = 20 Hz to 20 KHz.*

(ii) Frequency range of video signal = 3 Hz to 30 KHz.

(iii) Frequency range for satellite communication = 1 GHz to 30 GHz.

Information Source : It is the source of information.

Source Transducer : It converts physical signal into electrical signal such as microphone.

Channel : It is the medium through which signal is transferred from source to destination.

Receiving Transducer : It converts electrical signal into physical signal, such as Loudspeaker.

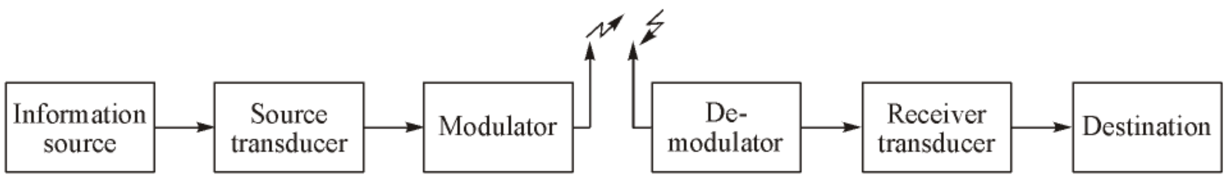
Note: *(i) Wired communication is preferred for short distance communication.*

(In wired communication, transmitter and receiver are connected physically).

(ii) Wireless communication is preferred for long distance communication.

(In wireless communication, transmitter and receiver are not connected physically but they are connected through air.

2. BLOCK DIAGRAM OF WIRELESS COMMUNICATION SYSTEM



Wireless communication is preferred for long distance but without modulator, long distance communication is not possible.

3. NEED FOR MODULATION

(i) Height of antenna is reduced if wave is transmitted at high frequency.

Minimum height of antenna required for faithful detection and transmission of wave

$$(h_{min}) = \frac{\lambda}{4}$$

where, λ = wavelength of transmitted signal $= \frac{C}{f}$

so, $$h_{min} = \frac{C}{4f}$$

where, C = velocity of light = 3×10^8 m/sec.

f = frequency in Hz.

So, for f = 15 KHz (low frequency signal)

$$h_{min} = \frac{C}{4f} = \frac{3\times10^8}{4\times15\times10^3} = 5\,\text{Km} \text{ (not practically possible)}$$

At small frequency, height of antenna is very large.

For, f = 1 MHz (high frequency signal)

$$h_{min} = \frac{3\times10^8}{4\times10^6} = 75\,\text{m} \text{ (practically possible)}$$

If same wave (small frequency signal) is transmitted with high frequency then height of antenna can be reduced. This process is called modulation.

(ii) Multiplexing

Simultaneous transmission of multiple message (more than one message) over a channel is known as multiplexing.

Channel is a medium of transmission and it may be a wired or wireless (free space).

If multiple message is transmitted without modulation over a single channel, they will interfere with one another.

So, multiple message can be transmitted over a same channel without interference using multiplexing techniques.

Multiplexing Technique

(a) Frequency division multiplexing (It uses analog modulation system).

(b) Time division multiplexing (It uses pulse modulation system).

(iii) The interference of noise and other signals can be reduced by changing the frequency of transmission.

(iv) Integration of different communication system is possible.

(v) Narrow Banding :

As modulation translates a signal from lower frequency domain to higher frequency domain, the ratio between lowest to highest frequency of modulated signal becomes close to 1.

Note: *Entire radio frequency range*

S.N.	Class	Range	wavelength (λ)	
1.	Very low frequency (VLF)	10 KHz - 30 KHz	$3 \times 10^4 - 10^4$ m	Long wave
2.	Low frequency	30 KHz - 300 KHz	$10^4 - 10^3$ m	
3.	Medium frequency	300 KHz - 3 MHz	$10^3 - 10^2$ m	
4.	High frequency	3 MHz - 30 MHz	$10^2 - 10$ m	Short waves
5.	Very high frequency	30 MHz - 300 MHz	10 – 1 m	
6.	Ultra high frequency (microwave)	Above 300 MHz	Below 1 m	

Frequency range for AM broadcasting = 550 KHz to 1600 KHz.

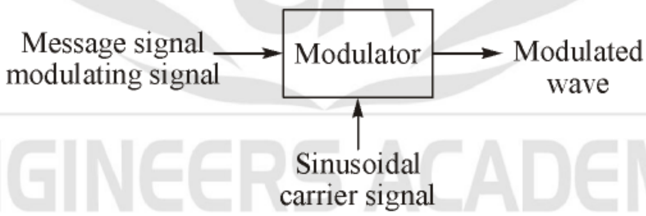
Frequency range for FM broadcasting = 88 MHz.

If baseband signal is an AM broadcast system is radiated directly with the frequency range 50 Hz to 10 KHz and ratio of highest to lowest wavelength is 200. If antenna is designed for 50 Hz, it will be too long for 10 KHz and vice versa. We may require a wide band antenna which can operate for band edge ratio of 1 : 200, which is practically impossible.

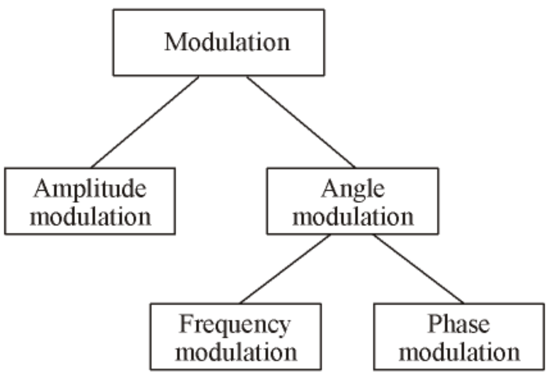
Modulation is a frequency translation technique which converts a wideband signal to narrow band signal and For narrow band, ratio between lowest to highest is 1 and same antenna will be suitable for entire band.

4. MODULATION

It is defined as the process by which some characteristic of signal called carrier is varied in accordance with instaneous value of another signal called modulating signal.



Type of Modulation :



A carrier wave is given by

$$C(t) = \underset{[\text{Amplitude}]}{A_c} \cos\left(\underset{[\text{Angular frequency}]}{2\pi f_c t} + \underset{[\text{Phase angle}]}{\theta}\right)$$

Any of the three parameters can be varied in accordance with modulating signal (also known as baseband signal / message signal). Accordingly, the modulation process is termed as amplitude modulation, frequency modulation or phase modulation.

5. AMPLITUDE MODULATION (AM):

This is the simplest form of modulation where the amplitude of the carrier wave is modulated by analog signal known as the modulating signal.

It is also known as continuous wave modulation.

Let Modulating signal / base band signal /

$$\text{message signal} = m(t)$$

and $$\text{carrier signal} = C(t) = A_c \cos(2\pi f_c t)$$

Note: *Modulating signal frequency (f_m) is less than carrier signal frequency (f_c).*

Then modulated signal $$S(t) = [A_c + m(t)]\cos(2\pi f_c t)$$

$$S(t) = [A_c + m(t)]\cos(2\pi f_c t)$$

$$\boxed{S(t) = A_c \cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)}$$

by taking the fourier transform,

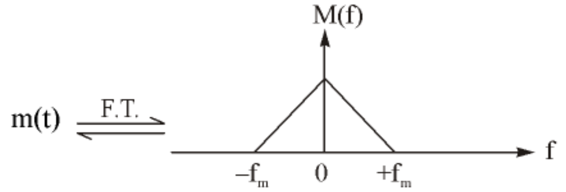
$$F[S(t)] = F[A_c \cos(2\pi f_c t)] + F[m(t)\cos(2\pi f_c t)]$$

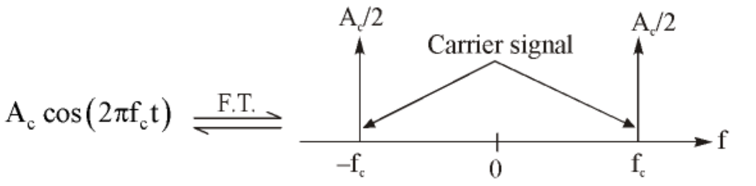
since $$F[m(t)] = M(f)$$

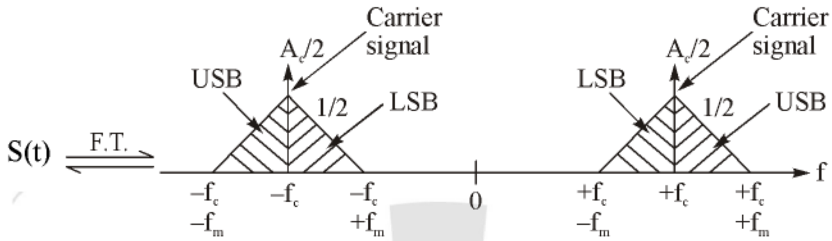
so, $$F[S(t)] = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$

$$\boxed{F[S(t)] = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}[M(f - f_c) + M(f + f_c)]}$$

(i) Graphical Representation of AM Wave







Bandwidth of AM wave $= (f_c + f_m) - (f_c - f_m) = 2f_m$

= 2 × maximum frequency of message / modulating signal

Amplitude modulated wave consists of

A. Carrier frequency component at ($\pm f_c$).

B. Upper side band (USB) = $\pm f_m = \pm(f_c + f_m - f_c)$

C. Lower side band (LSB) = $\pm f_m = \pm(f_c - f_m - f_c)$

On the basis of number frequency component within message / modulating signal, there are two types of modulation.

(a) Single tone modulation (STM) :

If message / modulating signal has single frequency component within it. This kind of modulation is called as single tone modulation.

(b) Multi-tone modulation (MTM)–

If message signal has multiple frequency component. This kind of modulation is called as multitone modulation.

(ii) Single Tone Amplitude Modulation :

Let us assume message signal $m(t) = A_m \cos(2\pi f_m t)$

Carrier siognal $C(t) = A_c \cos(2\pi f_c t)$

Then modulated signal will be $S(t) = [A_c + m(t)]\cos(2\pi f_c t)$

$$S(t) = [A_c + A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$= A_c\left[1 + \frac{A_m}{A_c}\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

Here, $\frac{A_m}{A_c} = \mu$ = modulation index

$$= A_c\left[1+\mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$= A_c\cos(2\pi f_c t) + A_c\mu\cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$= A_c\cos(2\pi f_c t) + \mu A_c$$

$$\left[\frac{\cos(2\pi(f_c+f_m)t)+\cos(2\pi(f_c-f_m)t)}{2}\right]$$

$$\left[\text{By using } \cos A\cos B = \frac{\cos(A+B)+\cos(A-B)}{2}\right]$$

$$= A_c\cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos\left[2\pi(f_c+f_m)t\right]$$

$$+\frac{\mu A_c}{2}\cos\left[2\pi(f_c-f_m)t\right]$$

$$S(t) = \underbrace{A_c\cos(2\pi f_c t)}_{(1)} + \underbrace{\frac{\mu A_c}{2}\cos\left[2\pi(f_c+f_m)t\right]}_{(2)}$$

$$\underbrace{+\frac{\mu A_c}{2}\cos\left[2\pi(f_c-f_m)t\right]}_{(3)}$$

Term (1) = Carrier signal

Term (2) = Upper side band

Term (3) = Lower side band

- Modulated wave S(t) consists of carrier signal, upper sideband and lower side band.
- Modulation index (μ) is always less than 1.
- Percentage of modulation / depth of modulation = $\mu \times 100$

  If $\mu < 1 \rightarrow$ under modulation.

  If $\mu = 1 \rightarrow$ critical modulation.

  If $\mu > 1 \rightarrow$ over modulation.

(iii) Graphical Representation of Single Tone Amplitude Modulation:

$$S(t) = A_c\cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos(2\pi(f_c+f_m)t)$$

$$+\frac{\mu A_c}{2}\cos(2\pi(f_c-f_m)t)$$

By taking transform,

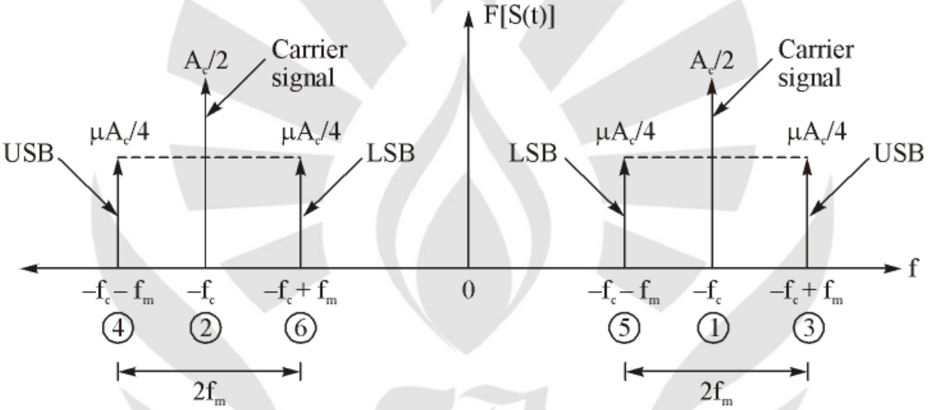
$$F\left[S(t)\right] = F\left[A_c\cos\left(2\pi f_c t\right)\right] + \frac{\mu A_c}{2}F\left[\cos\left(2\pi\left(f_c + f_m\right)t\right)\right]$$

$$+\frac{\mu A_c}{2}F\left[\cos\left[2\pi\left(f_c - f_m\right)t\right]\right]$$

$$F\left[S(t)\right] = \frac{A_c}{2}\left[\underset{(1)}{\delta\left(f - f_c\right)} + \underset{(2)}{\delta\left(f + f_c\right)}\right] + \frac{\mu A_c}{4}$$

$$\left[\underset{(3)}{\delta\left(f - \left(f_c + f_m\right)\right)} + \underset{(4)}{\delta\left(f + \left(f_c + f_m\right)\right)}\right]$$

$$+\frac{\mu A_c}{4}\left[\underset{(5)}{\delta\left(f\left(f_c - f_m\right)\right)} + \underset{(6)}{\delta\left(f + \left(f_c - f_m\right)\right)}\right]$$



Frequency domain representation of single tone modulation.

Band width (B.W.) of single tone modulated wave

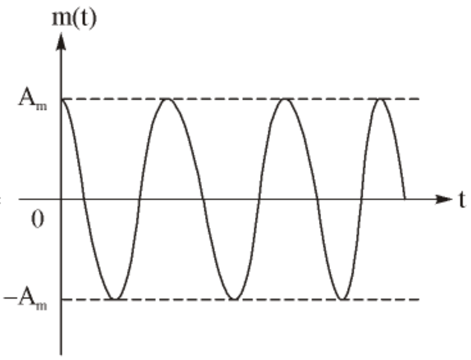
$$= \left(f_c + f_m\right) - \left(f_c - f_m\right) = 2f_m$$

B.W. $= 2f_m = 2 \times$ Maximum frequency component of modulating signal frequency

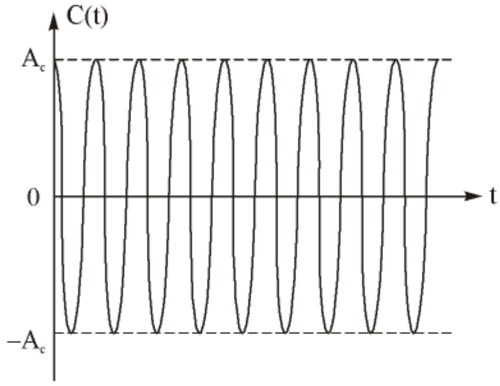
> ***Note:*** *The frequency of modulated signal is same as frequency of carrier signal but its amplitude varies in proportion to the instantaneous value of modulating signal.*

In single tone amplitude modulation

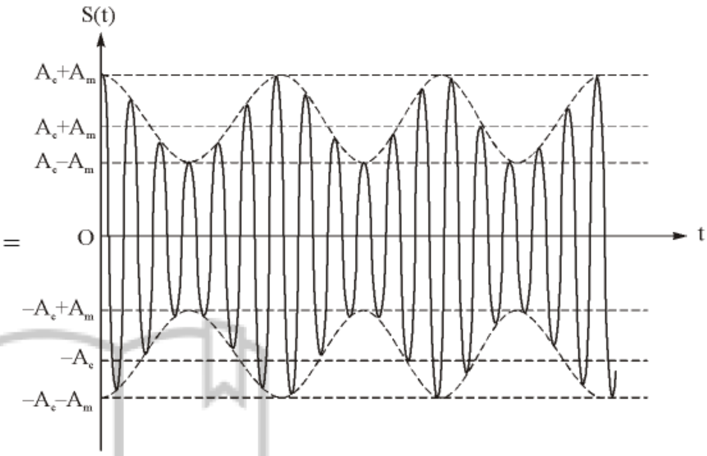
$$m(t) = A_m\cos\left(2\pi f_m t\right) =$$



$C(t) = A_c \cos(2\pi f_c t) =$



$S(t) = (A_c + m(t))\cos(2\pi f_c t) =$



Time domain graphical representation of amplitude modulated wave

(iii) Modulation Index From Waveform:

From time domain graphical representation of amplitude modulated wave,

maximum amplitude $= A_{max} = A_c + A_m$

minimum amplitude $= A_{min} = A_c - A_m$

from these two, $$A_c = \frac{A_{max} + A_{min}}{2}$$

$$A_m = \frac{A_{max} - A_{min}}{2}$$

Modulation index, $$\mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Note: *(i) For satisfactory modulation* $0 \le \mu \le 1$.

(ii) $A_{max} = A_c [1 + \mu]$ *and* $A_{min} = A_c [1 - \mu]$.

(iv) Power Carried by Amplitude Modulated Wave

Total power $(P_t) = P_c + P_{LSB} + P_{USB}$

here, P_c = power of carrier signal

P_{LSB} = power of lower side band (LSB)

P_{USB} = power of upper side band (USB)

Note: If $x(t) = A_m$ then $P = \frac{A_m^2}{R}$

If $x(t) = A_m \cos(2\pi f_0 t)$ then $P = \frac{A_{rms}^2}{R}$

$$P = \frac{\left(A_m/\sqrt{2}\right)^2}{R} = \frac{A_m^2}{2R}$$

$$P_c = \frac{A_c^2}{2R}$$

$$P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

$$P_{USB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

So,
$$P_t = P_c + P_{LSB} + P_{USB}$$
$$= \frac{A_c^2}{2R} + \frac{A_c^2\mu^2}{8R} + \frac{A_c^2\mu^2}{8R}$$
$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{4R} = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

$$P_t = P_c + \frac{P_c \mu^2}{2}$$

$$\boxed{P_t = P_c + P_{SB}}$$

$$\boxed{P_{SB} = \frac{P_c \mu^2}{2}}$$ = total power carried by both sidebands.

$$\boxed{P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{P_c \mu^2}{4}}$$

Power carried by the side band is the useful power.

As μ is increased, peak value of modulated signal is increased and carrier signal remains unaffected.

Total power (P_t) increases with increase in μ.

For $\mu = 0,\ P_t = P_c$

$$S(t) = A_c \cos(\omega_c t),$$

This is the case of no modulation.

$$P_{SB} = 0\% \text{ of } P_t$$

For $\mu = 1\ ,\ P_t = \frac{3}{2}P_c$

This is the case of 100% modulation.

$$P_t = \frac{3}{2}P_c$$

$$P_c = \frac{2}{3}P_t$$

Since, $P_t = P_c + P_{SB}$

$$P_t = \frac{2}{3}P_t + P_{SB}$$

$$P_{SB} = \frac{1}{3}P_t = 33.3\% \text{ of } P_t.$$

Only 33.33% of total power is effective power. Huge amount of transmitted power is wasted in form of carrier transmission.

(v) Modulation Efficiency (η):

It specifies percentage of power distributed to side band freuency components. It is denoted by η.

$$\eta = \frac{P_{SB}}{P_t} \times 100 = \frac{\frac{\mu^2}{2}P_c}{\left(1+\frac{\mu^2}{2}\right)P_c} \times 100$$

$$\eta = \left(\frac{\mu^2}{\mu^2+2}\right) \times 100$$

If $\mu = 0 \rightarrow \eta = 0 \rightarrow P_{SB} = 0\%$ of P_t; $P_c = 100\%$ of P_t

If $\mu = 0.5 \rightarrow \eta = 0.11 \rightarrow P_{SB} = 11\%$ of P_t; $P_c = 89\%$ of P_t

If $\mu = 0.707 \rightarrow \eta = 0.2 \rightarrow P_{SB} = 20\%$ of P_t; $P_c = 80\%$ of P_t

If $\mu = 1 \rightarrow \eta = \frac{1}{3} = 0.33 \rightarrow P_{SB} = 33.3\%$ of P_t; $P_c = 66.6\%$ of P_t

Since P_c is independent of μ as increased but percentage of P_c will be decreased.

Example: If the radiated power of AM transmitter is 10 kW, what is the power in the carrier for modulation index of 0.6?

Solution: Given that $P_t = 10$ kW

$P_c = ?$

we know that $\mu = 0.6$

$$P_t = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$10\text{kW} = P_c\left[1+\frac{(0.6)^2}{2}\right]$$

$$10\times 10^3 = P_c\left[1+\frac{0.36}{2}\right]$$

$$10^4 = P_c[1.18]$$

$$P_c = \frac{10^4}{1.18} = 8.47\,\text{kW}$$

Example: If modulation index of AM wave is changed from 0 to 1, what is the percentage change in total transmitted power.

Solution:

$$P_t = P_c\left[1+\frac{\mu^2}{2}\right]$$

for $\mu = 0$,

$$P_t = P_c\left[1+\frac{0^2}{2}\right] = P_c$$

for $\mu = 1$,

$$P_t = P_c\left[1+\frac{1^2}{2}\right] = \frac{3}{2}P_c = 1.5\,P_c$$

$\therefore$ Percentage change in power $= \dfrac{1.5\,P_c - P_c}{P_c}\times 100 = 50\%$

Example: The peak amplitude of AM signal varying between 2V and 10V. Find μ_1 and η.

Solution: $A_{min} = 2$ and $A_{max} = 10$

$$A_c = \frac{10+2}{2} = 6\text{V and } A_m = \frac{10-2}{2} = 4$$

$$\mu = \frac{A_m}{A_c} = \frac{4}{6} = 0.66$$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{0.66^2}{2+0.66^2} = 0.178$$

$$P_c = \frac{A_c^2}{2R} = 18\,W \quad (\text{Assume } R = 1\Omega)$$

$$P_t = P_c\left[1+\frac{\mu^2}{2}\right] = 18\left[\frac{1+0.66^2}{2}\right]$$

$$P_t = 21.9\ W$$

Example: A carrier signal of 10 $\cos\left(2\pi 10^6 t\right)$ is amplitude modulated by message signal of $4\cos\left(4\pi\times 10^3 t\right)$ with $\mu = 0.5$. Antenna resistance is 5Ω. Find all the parameters of AM.

Solution: Carrier signal $C(t) = A_c\cos\left(2\pi f_c t\right) = 10\cos\left(2\pi 10^6 t\right)$

$A_c = 10V$ and $f_c = 10^6$ Hz = 1000 KHz

Message signal $m(t) = A_m\cos\left(2\pi f_m t\right) = 4\cos\left(4\pi\times 10^3 t\right)$

$A_m = 4V$ and $f_m = 2\times 10^3$ Hz = 2 KHz.

Bandwidth of amplitude modulated wave

$$= 2f_m = 2\times 2\times 10^3\,\text{Hz} = 4\ \text{KHz}$$

Total power $P_t = P_c\left(1+\frac{\mu^2}{2}\right)$

$$P_c = \frac{A_c^2}{2R} = \frac{10^2}{2\times 5} = 10\,W$$

$$\mu = \frac{A_m}{A_c} = \frac{4}{10} = 0.4$$

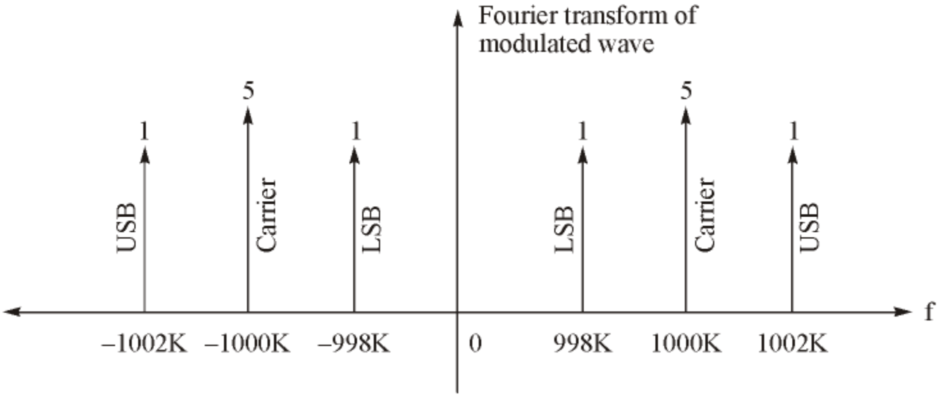
$$P_t = 10\left[1+\frac{(0.4)^2}{2}\right] = 10.8\,W$$

$$P_{SB} = \frac{\mu^2 A_c^2}{4R} = \frac{(0.4)^2(10)^2}{4\times 5} = \frac{4}{5} = 0.8$$

$$P_{LSB} = P_{USB} = \frac{P_{SB}}{2} = 0.4$$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(0.4)^2}{2+(0.4)^2} = 0.074$$

$$\%\eta = 7.4\ \%$$



(vi) Current Relations in AM:

$$P_t = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$I_t^2 R = I_c^2 R\left[1+\frac{\mu^2}{2}\right]$$

$$I_t^2 = I_c^2\left[1+\frac{\mu^2}{2}\right]$$

$$\boxed{I_t = I_c\sqrt{1+\frac{\mu^2}{2}}}$$

Same with voltage,

$$\boxed{V_t = V_c\sqrt{1+\frac{\mu^2}{2}}}$$

Example: The antenna current of an AM transmitter is 8A if only the carrier is sent, but it increases to 8.93 if the carrier is modulated by single sinusoidal wave. Determine the percentage of modulation and also find the antenna current if the percent of modulation changes to 0.8.

Solution: (i)

$$I_t = I_c\sqrt{1+\frac{\mu^2}{2}}$$

$$8.93 = 8\sqrt{1+\frac{\mu^2}{2}}$$

$$\left(\frac{8.93}{8}\right)^2 - 1 = \frac{\mu^2}{2}$$

$$\mu = 70.1\%$$

(ii)

$$I_c = 8A \; ; \; \mu = 0.8$$

$$I_t = I_c\sqrt{1+\frac{\mu^2}{2}} = 9.19\,A$$

(vii) Multitone Modulation:

When carrier is modulated simultaneously more than one sinusoidal signal, the modulation is called multitone modulation.

Let $$m(t) = A_{m_1}\cos\left(\omega_{m_1}t\right) + A_{m_2}\cos\left(\omega_{m_2}t\right) + ...$$

and carrier $$C(t) = A_c\cos\left(\omega_c t\right)$$

Then $$C(t) = (A_c + m(t))\cos(\omega_c t)$$

$$S(t) = \left(A_c + A_{m_1}\cos\left(\omega_{m_1}t\right) + A_{m_2}\cos\left(\omega_{m_2}t\right) + ...\right)\cos\left(\omega_c t\right)$$

$$= A_c\left(1 + \mu_1\cos\left(\omega_{m_1}t\right) + \mu_2\cos\left(\omega_{m_2}t\right) + ...\right)\cos\left(\omega_c t\right)$$

Here, $$\mu_1 = \frac{A_{m_1}}{A_c}, \mu_2 = A_{m_2}/A_c \dots$$

$$S(t) = A_c\cos\left(\omega_c t\right) + \mu_1 A_c\cos\left(\omega_{m_1}t\right)\cos\left(\omega_c t\right)$$

$$+\mu_2 A_c\cos\left(\omega_{m_2}t\right)\cos\left(\omega_c t\right) + ...$$

Assume $\omega_{m_2} > \omega_{m_1}$ and $\mu_2 > \mu_1$ because $A_{m_3} > A_{m_2} > A_{m_1}$

$$S(t) = A_c\cos\left(\omega_c t\right)$$

$$+\frac{\mu_1 A_c}{2}\left[\cos\left[\left(\omega_c + \omega_{m_1}\right)t\right] + \cos\left[\left(\omega_c - \omega_{m_1}\right)t\right]\right]$$

$$+\frac{\mu_2 A_c}{2}\left[\cos\left[\left(\omega_c + \omega_{m_2}\right)t\right] + \cos\left[\left(\omega_c - \omega_{m_2}\right)t\right]\right] + ...$$

$$S(t) = A_c\cos\left(\omega_c t\right)$$

$$+\frac{\mu_1 A_c}{2}\cos\left[\left(\omega_c + \omega_{m_1}\right)t\right] + \frac{\mu_2 A_c}{2}\cos\left[\left(\omega_c - \omega_{m_1}\right)t\right]$$

$$+\frac{\mu_2 A_c}{2}\cos\left[\left(\omega_c + \omega_{m_2}\right)t\right] + \frac{\mu_2 A_c}{2}\cos\left[\left(\omega_c - \omega_{m_2}\right)t\right] + ...$$

By taking fourier transform on both the side

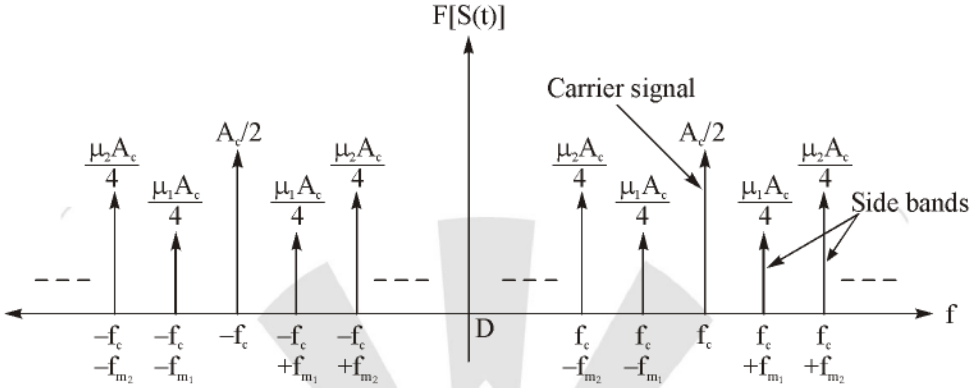
$$F\left[S(t)\right] = \frac{A_c}{2}\left[\delta\left(f - f_c\right) + \delta\left(f + f_c\right)\right]$$

$$+\frac{\mu_1 A_c}{4}\left[\delta\left[f - \left(f_c + f_{m_1}\right)\right] + \delta\left[f + \left(f_c + f_{m_1}\right)\right]\right]$$

$$+\frac{\mu_1 A_c}{4}\left[\delta\left[f - \left(f_c - f_{m_1}\right)\right] + \delta\left[f + \left(f_c - f_{m_1}\right)\right]\right]$$

$$+\frac{\mu_2 A_c}{4}\left[\delta\left[f-\left(f_c+f_{m_2}\right)\right]+\delta\left[f+\left(f_c+f_{m_2}\right)\right]\right]$$

$$+\frac{\mu_2 A_c}{4}\left[\delta\left[f-\left(f_c-f_{m_2}\right)\right]+\delta\left[f+\left(f_c-f_{m_2}\right)\right]\right]+\ldots$$



Bandwidth = 2 × heighest frequency component of message signal.

(viii) Power Content in Multitone Amplitude Modulated Wave:

Modulated wave is given by

$$S(t) = A_c\cos(\omega_c t)+\frac{\mu_1 A_c}{2}\cos\left[\left(\omega_c-\omega_{m_1}\right)t\right]$$

$$+\frac{\mu_1 A_c}{2}\cos\left[\left(\omega_c+\omega_{m_1}\right)t\right]+\frac{\mu_2 A}{2}\cos\left[\left(\omega_c-\omega_{m_2}\right)t\right]$$

$$+\frac{\mu_2 A_c}{2}\cos\left[\left(\omega_c+\omega_{m_2}\right)t\right]+\ldots\ldots$$

$$\text{Power } P_t = \frac{A_c^2}{2R}+\frac{\mu_1^2 A_c^2}{8R}+\frac{\mu_1^2 A_c^2}{8R}+\frac{\mu_2^2 A_c^2}{8R}+\frac{\mu_2^2 A_c^2}{8R}+\ldots\ldots$$

$$= \frac{A_c^2}{2R}+\frac{\mu_1^2 A_c^2}{4R}+\frac{\mu_2^2 A_c^2}{4R}+\ldots\ldots$$

$$= \frac{A_c^2}{2R}\left[1+\frac{\mu_1^2}{2}+\frac{\mu_2^2}{2}+\ldots\ldots\right] = P_c\left[1+\frac{\mu_1^2+\mu_2^2+\ldots\ldots}{2}\right]$$

Assume $\mu_t^2 = \mu_1^2+\mu_2^2+\ldots\ldots$

$$\boxed{\mu_t = \sqrt{\mu_1^2+\mu_2^2+\ldots\ldots}}$$

$$\boxed{P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]}$$

Example: A 10 kW carrier is sinusoidally modulated by two carriers corresponding to a modulation index of 30% and 40% respectively. Find out the total radiated power?

Solution:

$$\mu_t^2 = \mu_1^2 + \mu_2^2 = (0.3)^2 + (0.4)^2 = 0.25$$

$$P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right] = 10k\left[1 + \frac{0.25}{2}\right]$$

$$= 10 \times 10^3 [1.125] = 11.25\,kW$$

Example: Find out the modulation index, amplitude and frequency of sidebands.

$$V(t) = 100\left[1 + 0.2\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(2\pi \times 10^6 t\right)$$

Solution: Given, $V(t) = 100\left[1 + 0.2\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(2\pi \times 10^6 t\right)$

On comparing given equation with standard AM equation,

$$V(t) = A_c\left[1 + \mu\cos(\omega_m t)\right]\cos(\omega_c t)$$

So, modulation index $\mu = 0.2$

$$A_c = 100$$

$$f_m = 10^3 \text{ Hz.}$$

$$f_c = 10^6 \text{ Hz.}$$

$$LSB = f_c - f_m = (1000 - 1)\,kHz = 999\,kHz$$

$$USB = f_c + f_m = (1000 + 1)\,kHz = 1001\,kHz.$$

Amplitude of side band $= \frac{\mu A_c}{2} = \frac{0.2 \times 100}{2} = 10\,V$

Example: An AM transmitter radiates 10K watt with carrier unmodulated an antenna current is 10A. The antenna current increases to 11A when carrier is modulated by a single sine wave. The antenna current increases to 12A when wave is modulated by another sine wave. Find out the modulation index if carrier is modulated using only second sine wave. Also find out radiated power when carrier is modulated with both sine wave simultaneously.

Solution:

$$P_c = 10 \text{ kW and } I_c = 10 \text{ A}$$

$$I_{Modulat\ 1} = 11 \text{ A}$$

$$I_{Modulat\ 2} = 12 \text{ A}$$

$$I_{Modulat\ 1} = \left[1 + \frac{\mu_1^2}{2}\right]^{1/2} I_c$$

$$11 = \left[1 + \frac{\mu_1^2}{2}\right]^{1/2} 10$$

$$\mu_2 = 0.648$$

and $$12 = \left[1+\frac{\mu^2}{2}\right]^{1/2} \times 10$$

$$\mu = 0.938$$

But $$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$(0.938)^2 = (0.648)^2 + \mu_2^2$$

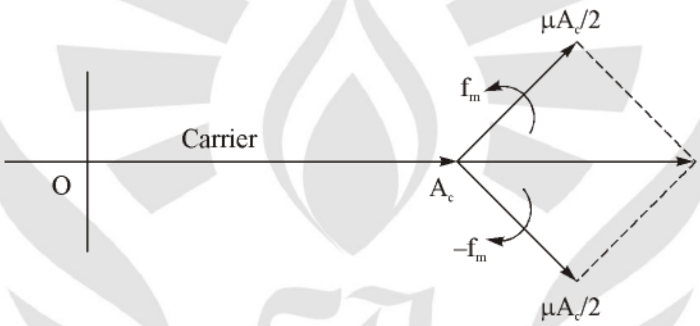
$$\mu_2 = 0.678$$

Total radiated power $$P_T = \left[1+\frac{(0.938)^2}{2}\right] \times 10 \times 10^3 \text{ W}$$

$$P_T = 14.4 \text{ kW}$$

(ix) Phasor Diagram of AM Wave:

Let us draw a 'phasor' diagram, using carrier signal as the reference.



Let $$S(t) = A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos\left((\omega_c + \omega_m)t\right) + \frac{\mu A_c}{2}\cos\left((\omega_c - \omega_m)t\right)$$

$$S(t) = A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos\left((\omega_c + \omega_m)t\right) + \frac{\mu A_c}{2}\cos\left((\omega_c - \omega_m)t\right)$$

$$S(t) = \text{Re}\left[A_c e^{j\omega_c t} + \frac{\mu A_c}{2} e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{2} e^{+j(\omega_c-\omega_m)t}\right]$$

$$S(t) = A_c + \frac{\mu A_c}{2} e^{j\omega_m t} + \frac{\mu A_c}{2} e^{-j\omega_m t}$$

With respect to carrier signal

The term $\frac{A_c\mu}{2}e^{j\omega_m t}$ can be represented as a rotating vector with a magnitude of $\frac{\mu A_c}{2}$, rotating counter clockwise at the rate of f_m Hz. Similarly $\frac{\mu A_c}{2}e^{-j\omega_m t}$ can be shown as a vector with clockwise rotational speed of f_m Hz.

(x) Generation of AM Wave:

Let x(t) be the input to an LTI system with the impulse resonse h(t) and let y(t) be the output. Then

$$y(t) = x(t) * h(t) \quad \text{[in time domain]}$$

$$Y(f) = X(f)H(f) \quad \text{[in frequency domain]}$$

That's why an LTI system can only alter a frequency component (either boost or attenuate), that is present in the input signal. In other words, an LTI system connot generate at its output frequency components that are not present in X(f). It means, to generate AM signal non-linear or time-varying systems are used.

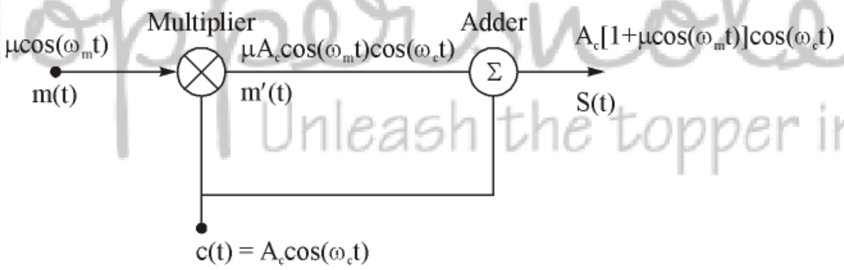
Metthod of generation of AM waves are:

A. Product modulator

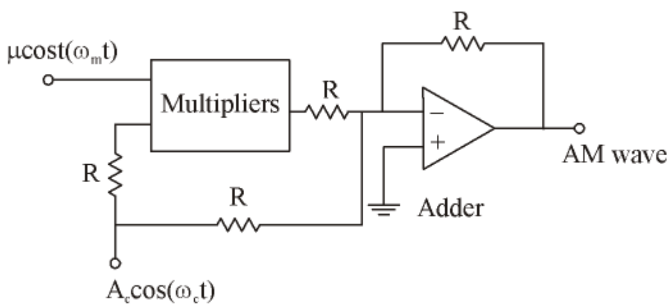
B. Square law modulator / Non-linear modulator

C. Switching modulator (time-varying method)

A. Product modulator:



The above block diagram can be implemented by using either transistor (BJT/FET) or OP-AMP.



It is used at low frequency and low power due to limitation imposed by multiplier circuit.