



KVS – PGT

Physics

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Volume - 2



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# 1

## CHAPTER

# Gravitation

## Kepler's Laws of Planetary Motion

### Overview

Kepler's laws form the **empirical backbone** of gravitation. They describe **how** planets move without invoking force. Examiners expect you to read these laws **physically**, not as historical curiosities: speed variation along an orbit, area-time constancy as a hint of angular momentum conservation, and the third law as the gateway from geometry to dynamics. Most PYQ errors arise from (i) treating orbits as circular by default, (ii) misinterpreting "equal areas," and (iii) applying the third law outside its domain.

**By the end of this part, you should be able to:**

- State Kepler's three laws precisely
- Interpret each law physically
- Identify what each law **implies** and what it **does not**
- Anticipate common PYQ traps in assertions, graphs, and comparisons

### 1. Why Kepler's laws matter (conceptual, not historical)

- Kepler's laws were distilled from observations of planetary motion and are **kinematic** in nature. They do not mention force, mass, or gravity. Their significance lies in the fact that:
  - They apply universally to planets around the Sun (and satellites around a central body)
  - They encode deep physical symmetries later explained by Newton

#### Exam insight:

- Kepler's laws describe motion; Newton's law explains its cause.

### 2. Kepler's First Law - Law of Orbits (statement)

#### Statement:

- Every planet moves around the Sun in an **elliptical orbit**, with the Sun located at **one focus** of the ellipse.

#### Key precision points:

- Orbit is **elliptical**, not circular (circle is a special ellipse)
- Sun is at **one focus**, not at the centre

### 3. Ellipse - physical meaning (not geometry overload)

#### An ellipse is defined by:

- Two fixed points called **foci**
- The sum of distances from any point on the ellipse to the two foci is constant

#### In planetary motion:

- One focus is occupied by the Sun
- The other focus is empty

#### Exam trap:

- Placing the Sun at the centre of the ellipse.

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#### 4. Consequences of the first law - speed variation

**Because the Sun is not at the centre:**

- Planet-Sun distance changes continuously
- Gravitational interaction strength changes
- **Speed cannot remain constant**

**Thus:**

- Planet moves **fastest** at perihelion (closest point)
- Planet moves **slowest** at aphelion (farthest point)

**High-yield statement:**

- Kepler's first law already implies non-uniform speed.

#### 5. Kepler's Second Law - Law of Areas (statement)

**Statement:**

- The line joining a planet and the Sun sweeps out **equal areas in equal intervals of time.**

**Key precision:**

- Area is swept by the **radius vector**
- Time intervals must be equal

#### 6. Areal velocity - conceptual core

**The areal velocity is defined as:**

- Areal velocity =  $\frac{\text{Area swept}}{\text{Time}}$
- Kepler's second law states:
- Areal velocity is **constant**
- This is the **strongest** of Kepler's laws conceptually.

#### 7. Physical meaning of the second law

**If area swept per unit time is constant:**

- When planet is closer to Sun (smaller radius), it must move faster
- When planet is farther, it must move slower

**Thus:**

- Speed varies inversely with distance (qualitatively)

**Exam insight:**

- This law explains speed variation quantitatively **without invoking force.**

#### 8. Hidden conservation principle (preview)

**Equal areas in equal times implies:**

- No external torque about the Sun
- Angular momentum about the Sun is conserved
- This connection is **not** part of Kepler's law itself but is a key bridge to Newtonian mechanics.

#### 9. Common misconceptions about the second law

**Incorrect interpretations to avoid:**

- Planet sweeps equal **angles** in equal times (false)
- Planet covers equal **arc lengths** in equal times (false)
- Speed is constant along orbit (false)
- Only **area**, not angle or distance, is constant per unit time.

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## 10. Kepler's Third Law - Law of Periods (statement)

### Statement:

- The square of the time period  $T$  of revolution of a planet is proportional to the cube of the semi-major axis  $a$  of its orbit:

$$T^2 \propto a^3$$

### Key precision:

- $a$  is the **semi-major axis**, not instantaneous radius
- Applies to planets orbiting the **same central body**

## 11. Physical meaning of the third law

### This law implies:

- Planets farther from the Sun take much longer to complete one revolution
- Orbital size controls orbital time
- For circular orbits (special case):
- Semi-major axis equals orbital radius

### Exam trap:

- Using instantaneous distance instead of semi-major axis.

## 12. Domain of validity - examiner favourite filter

### Kepler's third law in the form $T^2/a^3 = \text{constant}$ holds:

- Only for bodies orbiting the **same central mass**
- Not universally across different central bodies

### For satellites of different planets:

- The constant changes

### Assertion-Reason trap:

- Applying Earth-satellite constant to Sun-planet systems.

## 13. What Kepler's laws do NOT tell us

### Kepler's laws do not specify:

- Nature of the force
- Mass of the planet
- Mass of the Sun
- Cause of motion
- They are **descriptive**, not explanatory.

## 14. Typical PYQ traps (Kepler's laws)

- Treating orbits as circular by default
- Placing Sun at centre instead of focus
- Confusing equal areas with equal angles
- Assuming constant speed
- Using third law across different central bodies
- Mixing instantaneous radius with semi-major axis

## 15. How to write high-scoring descriptive answers

### Example:

- "State Kepler's laws of planetary motion and explain the physical significance of the second law."

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**Exam-safe structure:**

- State all three laws concisely
- Explain second law using areal velocity
- Connect to speed variation
- Avoid Newtonian force language
- Conclude with physical implication

**Avoid:**

- Historical narratives
- Formula-heavy explanations

**16. Synthesis - lock-in checklist****Before proceeding, ensure you can assert confidently:**

- Planetary orbits are elliptical with Sun at one focus
- Planetary speed is not uniform
- Equal areas in equal times  $\Rightarrow$  constant areal velocity
- Third law links orbital size and time period
- Kepler's laws describe motion, not cause
- This understanding is essential before introducing the **universal law of gravitation**, which explains *why* Kepler's laws hold.

<b>Universal Law of Gravitation (Newtonian Synthesis)</b>
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**Overview**

This part provides the **causal explanation** behind Kepler's empirical laws. Newton's universal law of gravitation elevates gravitation from an Earth-bound phenomenon ("gravity") to a **universal interaction** governing motion from apples to planets. Examiners focus on whether you understand **why the force is inverse-square**, how **vector direction and mutuality** operate, and how this single law **explains Kepler's laws**. High-frequency errors include confusing gravity with gravitation, mishandling distance dependence, and ignoring action-reaction symmetry.

**By the end of this part, you should be able to:**

- State and interpret the **universal law of gravitation**
- Explain the **inverse-square dependence** physically
- Use correct **vector form and direction**
- Clarify the role and meaning of the **gravitational constant  $G$**
- Distinguish **gravitation** from **gravity**
- Anticipate examiner traps in assertions and comparisons

**1. From Kepler to Newton - what needed explanation****Kepler established:**

- Elliptical orbits
- Variable speed
- A precise period-size relation

**What remained unanswered:**

- What force produces such motion?
- Why does the same pattern hold for all planets?
- Why does distance matter the way it does?
- Newton's gravitation answers all three with **one force law**.

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## 2. Statement of the universal law of gravitation

### Statement:

- Every two particles in the universe attract each other with a force that:
- Is **directly proportional** to the product of their masses
- Is **inversely proportional** to the square of the distance between their centres
- Acts along the line joining their centres
- Mathematically (magnitude):

$$F = G \frac{m_1 m_2}{r^2}$$

## 3. Vector form - direction is not optional

In vector notation, the gravitational force on  $m_2$  due to  $m_1$  is:

$$\vec{F}_{21} = -G \frac{m_1 m_2}{r^2} \hat{r}$$

### Where:

- $\hat{r}$  is the unit vector from  $m_1$  to  $m_2$
- Negative sign indicates **attraction**

### Exam trap:

- Writing only magnitude when direction is explicitly asked.

## 4. Mutual nature of gravitation - action-reaction pair

Gravitational interaction is mutual:

- Force on  $m_1$  due to  $m_2$  equals force on  $m_2$  due to  $m_1$  in magnitude
- Directions are opposite

$$\vec{F}_{12} = -\vec{F}_{21}$$

- This satisfies **Newton's Third Law**.

### High-yield clarification:

- Heavier mass does not exert a "bigger force" on the lighter one; both experience equal and opposite forces.

## 5. Why the force depends on product of masses

The proportionality to  $m_1 m_2$  implies:

- Larger masses interact more strongly
- Interaction strength reflects **inertial content** of both bodies

### This symmetry:

- Makes gravitation universal
- Ensures consistency with Newton's laws of motion

## 6. Inverse-square dependence - deep physical reasoning

- The  $1/r^2$  dependence is not arbitrary. Its physical logic:
- Interaction spreads uniformly in three-dimensional space
- Surface area of a sphere of radius  $r$  grows as  $4\pi r^2$
- Strength per unit area therefore falls as  $1/r^2$

### Examiner insight:

- Any long-range central force consistent with space geometry tends toward an inverse-square form.

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## 7. Consequences of inverse-square law

Because of  $1/r^2$ :

- Force weakens rapidly with distance
- Near-field effects dominate (e.g., Earth's gravity vs Moon's)
- Orbits are stable and closed (ellipses)

Changing the power of  $r$  would:

- Destroy Keplerian orbits
- Break observed planetary motion

## 8. Gravitational constant $G$ - physical meaning

The constant  $G$ :

- Sets the **strength of gravitational interaction**
- Is universal and independent of the nature of masses
- Has a very small value, explaining why gravity is weak

Dimensions:

$$[G] = M^{-1}L^3T^{-2}$$

Exam trap:

- Treating  $G$  as dimensionless or Earth-specific.

## 9. Why gravity feels weak in daily life

Compared to other fundamental forces:

- Gravity is extremely weak at microscopic scales
- Dominates only when masses are very large (planets, stars)

Thus:

- Electromagnetic forces dominate atomic systems
- Gravity governs astronomical systems

## 10. Gravitation vs gravity - do not confuse

- **Gravitation:** universal mutual attraction between any two masses
- **Gravity:** gravitational attraction **specifically due to Earth**

Thus:

- Gravity is a **special case** of gravitation
- $g$  near Earth arises from Newton's law applied to Earth-particle system
- **Examiner's favourite distinction.**

## 11. Explaining Kepler's laws (qualitative bridge)

Newton showed that:

- An inverse-square central force produces **elliptical orbits**
- Zero torque about the Sun leads to **constant areal velocity**
- Force magnitude leads to  $T^2 \propto a^3$

Thus:

- Kepler's laws emerge naturally from gravitation
- You are not required to derive them here-only to recognize the logical link.

## 12. Range and nature of gravitational force

- Gravity is **long-range**
- No shielding exists
- Always attractive (in Newtonian framework)

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**Exam trap:**

- Assuming repulsive gravitational effects at large distances.

**13. Validity and limitations of Newtonian gravitation**

**Valid when:**

- Speeds are much less than speed of light
- Gravitational fields are not extremely strong

**Limitations:**

- Does not explain relativistic effects (addressed by General Relativity)
- Fails near black holes or for precise orbital precession
- At PGT/NET level, Newtonian gravitation is **fully sufficient**.

**14. Typical PYQ traps (universal gravitation)**

- Forgetting inverse-square dependence
- Confusing gravitation with gravity
- Assuming unequal forces on unequal masses
- Ignoring vector direction/sign
- Treating  $G$  as Earth-dependent
- Applying near-Earth formulas universally

**15. How to write high-scoring descriptive answers**

**Example:**

- **“State the universal law of gravitation and explain the significance of the inverse-square dependence.”**

**Exam-safe structure:**

- State law clearly with formula
- Explain proportionality to masses
- Explain inverse-square reasoning physically
- Mention universality
- Conclude with implication for planetary motion

**Avoid:**

- Only writing the formula
- No explanation of distance dependence

**16. Synthesis - lock-in checklist**

**Before proceeding, ensure you can assert confidently:**

- Every mass attracts every other mass
- Gravitational force  $\propto m_1 m_2$  and  $\propto 1/r^2$
- Force acts along line joining centres
- Gravitational interaction is mutual
- $G$  is universal and dimensionful
- Gravity is a special case of gravitation
- This clarity is essential before specializing the law to **Earth-based gravity** and deriving **acceleration due to gravity  $g$** .

## Acceleration Due to Gravity (G): Meaning & Earth-Based Derivation

### Overview

This part specializes Newton's **universal gravitation** to the Earth-particle system and extracts one of the most tested quantities in mechanics: the **acceleration due to gravity**,  $g$ . Examiners expect you to (i) distinguish **force** from **acceleration**, (ii) explain **why free fall is mass-independent**, and (iii) understand **what assumptions** underlie the familiar constant  $g_{\text{near Earth's surface}}$ . Most errors arise from conflating gravity with gravitation, mixing mass and weight, or forgetting the **radial nature** of  $g$ .

**By the end of this part, you should be able to:**

- Define acceleration due to gravity precisely
- Derive  $g = \frac{GM}{R^2}$  from Newton's law
- Explain **mass-independence of free fall**
- Relate **weight** to gravitational force
- Clarify **gravitational mass vs inertial mass** (conceptual)
- Identify common PYQ traps and avoid them

### 1. What is acceleration due to gravity?

- **Acceleration due to gravity ( $g$ )** at a point is the acceleration acquired by a body when it falls freely under the **gravitational attraction of Earth alone**, with all other forces neglected.

**Key points:**

- $g$  is an **acceleration**, not a force
- Direction is **towards the centre of Earth**
- Near Earth's surface,  $g$  is approximately constant

**Exam trap:**

- Calling  $g$  a force or confusing it with weight.

### 2. Gravity vs gravitation - again, with precision

- **Gravitation:** universal attraction between any two masses
- **Gravity:** gravitational attraction due to Earth on nearby bodies

**Thus:**

- $g$  arises from **gravitation applied to Earth-particle system**
- It is a **local manifestation** of a universal law

### 3. Derivation of acceleration due to gravity

- Consider a particle of mass  $m$  on Earth's surface.

**From universal gravitation:**

- $F = G \frac{Mm}{R^2}$

**where:**

- $M$  = mass of Earth
- $R$  = radius of Earth
- From Newton's second law:

$$F = mg$$

- Equating:

$$mg = G \frac{Mm}{R^2}$$

- 
- Cancelling  $m$ :

$$g = \frac{GM}{R^2}$$

#### 4. Meaning of the derivation - why it matters

**This derivation shows:**

- $g$  depends on **Earth's mass and radius**
- $g$  does **not** depend on the mass of the falling body
- Near Earth, gravity behaves like a **constant acceleration field**
- **High-yield conclusion:**
- All bodies fall with the same acceleration in free fall.

#### 5. Direction of acceleration due to gravity

- $g$  is a **vector**
- Direction is **radially inward**, towards Earth's centre
- At every point on Earth's surface,  $g$  points locally "downward"

**Exam trap:**

- Treating  $g$  as vertically downward everywhere without radial interpretation.

#### 6. Free fall - conceptual clarity

**A body is in free fall when:**

- Only gravitational force acts
- Air resistance and other forces are neglected

**In free fall:**

- Acceleration =  $g$
- Motion equations of kinematics apply with  $a = g$

**Important:**

- Free fall does **not** mean "falling downward only"; upward-thrown bodies are also in free fall after release.

#### 7. Mass-independence of free fall - core insight

**From the derivation:**

$$g = \frac{GM}{R^2}$$

- No dependence on  $m$ .

**Thus:**

- Heavy and light bodies fall with the same acceleration
- Difference in fall time arises only due to air resistance

**Examiner favourite:**

- This is a direct consequence of equality of gravitational and inertial mass.

#### 8. Weight - gravitational force on a body

- **Weight ( $W$ )** is defined as:

$$W = mg$$

**Key clarifications:**

- Weight is a **force**
- Depends on local value of  $g$
- Changes with altitude, depth, or location

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**Exam trap:**

- Saying “weight is constant everywhere on Earth.”

**9. Mass vs weight - never confuse**

Mass	Weight
Scalar	Vector
Constant	Varies with $g$
Measure of inertia	Measure of gravitational force
SI unit: kg	SI unit: N

- Examiners frequently test this contrast.

**10. Gravitational mass vs inertial mass (conceptual)**

- **Gravitational mass:** determines strength of gravitational interaction
- **Inertial mass:** determines resistance to acceleration

**Experimentally:**

- $m_g = m_i$

**This equality explains:**

- Mass-independent free fall
- Consistency of Newton’s laws
- At higher levels, this idea leads to Einstein’s equivalence principle.

**11. Why  $g$  is approximately constant near Earth****Near Earth’s surface:**

- Height  $h \ll R$
- Distance from Earth’s centre  $\approx$  constant

**Thus:**

- $g = \frac{GM}{(R+h)^2} \approx \frac{GM}{R^2}$

- This approximation underlies:
- Kinematic equations with constant  $g$
- Everyday mechanics problems

**12. Common PYQ traps (acceleration due to gravity)**

- Confusing  $g$  with force
- Writing  $g$  depends on mass of body
- Forgetting radial direction of  $g$
- Treating weight as mass
- Applying constant  $g$  far from Earth
- Ignoring free-fall condition

**13. How to write high-scoring descriptive answers****Example:**

- “Derive an expression for acceleration due to gravity on Earth.”

**Exam-safe structure:**

- State universal law of gravitation
- Write force on a mass  $m$  at Earth’s surface
- Apply Newton’s second law

- Cancel mass and obtain  $g$
- Interpret mass-independence

**Avoid:**

- Jumping straight to the result
- No physical interpretation

#### 14. Synthesis - lock-in checklist

**Before proceeding, ensure you can assert confidently:**

- $g$  is acceleration due to Earth's gravity
- $g = \frac{GM}{R^2}$  near Earth's surface
- Direction of  $g$  is towards Earth's centre
- Free fall acceleration is mass-independent
- Weight is force  $mg$ , not mass
- Constant- $g$  approximation has limits
- This clarity is essential before studying **how  $g$  changes with altitude and depth**, a very high-PYQ zone.

### Variation of Acceleration Due to Gravity with Altitude and Depth

#### Overview

This part targets one of the **most repeatedly tested segments** of Gravitation. Examiners probe whether you can distinguish **external** (altitude) from **internal** (depth) behavior of Earth's gravity, apply the **right approximation**, and interpret **graphs and proportionalities** correctly. The biggest errors occur when candidates (i) use surface formulas blindly, (ii) confuse altitude with depth effects, or (iii) miss the **change in functional dependence** inside Earth.

**By the end of this part, you should be able to:**

- Derive how  $g$  varies with **altitude**
- Apply **small-height approximations** correctly
- Derive how  $g$  varies with **depth inside Earth**
- Explain why  $g = 0$  at Earth's centre
- Compare altitude vs depth behavior clearly
- Read and reason with  **$g$ -distance graphs**
- Avoid the most common PYQ traps

#### Part A - Variation of $g$ with Altitude (Outside Earth)

##### 1. Why altitude affects gravity

**Gravity depends on distance from Earth's centre:**

- $g(r) = \frac{GM}{r^2}$

**At Earth's surface:**

- $g_0 = \frac{GM}{R^2}$

**At height  $h$  above the surface:**

- $r = R + h$
- Thus, gravity must decrease with altitude.

---

## 2. Exact expression of $g$ at altitude $h$

Using Newton's law:

$$g_h = \frac{GM}{(R+h)^2}$$

- Comparing with surface value:

$$g_h = g_0 \left( \frac{R}{R+h} \right)^2$$

- This is the **exact relation** (no approximation yet).

## 3. Small-altitude approximation (very high PYQ frequency)

For heights much smaller than Earth's radius:

- $h \ll R$

Using binomial approximation:

$$\left(1 + \frac{h}{R}\right)^{-2} \approx 1 - \frac{2h}{R}$$

- Hence:

$$g_h \approx g_0 \left(1 - \frac{2h}{R}\right)$$

## 4. Physical interpretation (altitude)

- Gravity decreases **slowly** with height
- For everyday heights, change in  $g$  is negligible
- Significant reduction occurs only at very large altitudes

**Exam trap:**

- Assuming  $g$  drops sharply just a few kilometres above Earth.

## 5. Graphical behavior (altitude)

- $g$  vs  $h$ : decreasing **non-linearly**
- Steeper near Earth, flatter at large distances
- Approaches zero as  $h \rightarrow \infty$

## Part B - Variation of $g$ with Depth (Inside Earth)

### 6. Why internal gravity behaves differently

Inside Earth:

- Only the mass **enclosed within radius  $r$**  contributes to gravity
- Outer spherical shells exert **no net force** (shell theorem)
- This changes the dependence completely.

### 7. Assumption for depth derivation (must be stated)

We assume Earth:

- Is a **uniform sphere**
- Has constant density  $\rho$
- This assumption is **explicitly expected** in derivations.

### 8. Mass enclosed at depth $d$

Let:

- Depth below surface =  $d$

- Distance from centre =  $r = R - d$

**Mass enclosed:**

- $M_r = M \left( \frac{r^3}{R^3} \right)$

### 9. Expression for $g$ at depth $d$

**Gravitational acceleration at radius  $r$ :**

$$g_r = \frac{GM_r}{r^2}$$

**Substituting:**

$$g_r = \frac{GM}{R^3} r$$

- Since  $g_0 = \frac{GM}{R^2}$ :

$$g_d = g_0 \left( 1 - \frac{d}{R} \right)$$

### 10. Key result - linear decrease inside Earth

- $g$  decreases **linearly** with depth
- At centre ( $d = R$ ):
- $g = 0$
- High-yield conclusion:**
- Gravity vanishes at Earth's centre.

### 11. Physical reason why $g = 0$ at centre

**At the centre:**

- Gravitational pulls from all directions cancel
- Net force is zero
- This is a **symmetry argument**, not a numerical accident.

### 12. Graphical behavior (depth)

- $g$  vs depth: straight-line decrease
- Maximum at surface
- Zero at centre
- Examiners frequently test **graph identification**.

## Part C - Altitude Vs Depth (Comparative Logic)

### 13. Side-by-side comparison (must be clear)

Aspect	Altitude (outside Earth)	Depth (inside Earth)
Governing mass	Entire Earth	Only enclosed mass
Dependence	Inverse-square	Linear
Nature of curve	Non-linear	Linear
$g$ at centre	Not applicable	Zero
PYQ frequency	High	Very high

### 14. Common misconceptions to avoid

- Using  $g_h = g_0 \left( 1 - \frac{h}{R} \right)$
- Assuming same formula for altitude and depth
- Forgetting uniform-density assumption
- Treating altitude and depth effects symmetrically

## 15. Typical PYQ traps (variation of $g$ )

- Forgetting  $2h/R$  factor in altitude case
- Using inverse-square law inside Earth
- Ignoring shell theorem logic
- Assuming  $g$  constant underground
- Wrong sign in depth expression
- Confusing height  $h$  with depth  $d$

## 16. How to write high-scoring descriptive answers

**Example:**

- “Derive expressions for variation of acceleration due to gravity with altitude and depth.”

**Exam-safe structure:**

- State base expression  $g = \frac{GM}{r^2}$
- Derive altitude case clearly
- Apply approximation for small  $h$
- State uniform Earth assumption
- Derive depth expression
- Compare and conclude

**Avoid:**

- Mixing both derivations together
- Skipping assumptions

## 17. Synthesis - lock-in checklist

**Before proceeding, ensure you can state confidently:**

- $g$  decreases with altitude as inverse-square
- Near Earth,  $g_h \approx g_0(1 - \frac{2h}{R})$
- Inside Earth,  $g$  decreases linearly with depth
- $g = 0$  at Earth's centre
- Altitude and depth variations follow **different physics**
- Graph interpretation is essential
- This mastery is essential before moving to **gravitational potential energy**, where energy descriptions replace force descriptions.

## Gravitational Potential Energy (GPE)

### Overview

This part transitions gravitation from a **force-based** description to an **energy-based** framework. Examiners consistently test whether candidates understand (i) **why gravitational potential energy is negative**, (ii) the **choice of reference level**, and (iii) the **difference between near-Earth approximation and exact Newtonian form**. The most frequent errors arise from importing  $mgh$  indiscriminately, mishandling signs, and confusing potential energy with gravitational potential

**By the end of this part, you should be able to:**

- Define gravitational potential energy rigorously
- Derive GPE from work done by gravitational force
- Explain the **negative sign** and its physical meaning

- 
- Distinguish **exact Newtonian GPE** from near-Earth approximation
  - Interpret **energy-distance graphs**
  - Explain **bound vs unbound gravitational systems**
  - Avoid high-frequency PYQ traps related to reference choice and sign

### 1. Why an energy description is needed

- Force descriptions answer: *what acceleration is produced.*
- Energy descriptions answer: *how motion changes without tracking direction at every instant.*
- Gravity is particularly suited for an energy approach because:
  - It is a **conservative force**
  - Work done depends only on initial and final positions
  - Energy methods simplify escape, orbit, and binding problems

#### Exam insight:

- Whenever motion occurs under gravity without non-conservative forces, energy is the fastest route.

### 2. Conservative nature of gravitational force

#### A force is conservative if:

- Work done is **path-independent**
- A scalar potential energy function can be defined
- Gravitational force satisfies both conditions.

#### Key consequence:

- Gravitational potential energy exists and is uniquely defined once a reference is chosen.

### 3. Definition of gravitational potential energy

- **Gravitational potential energy (U)** of a system is defined as the **negative of the work done by gravitational force** in bringing a body from a chosen reference position to the given position.

#### Symbolically:

- $U = -W_{\text{grav}}$

#### This definition embeds:

- The conservative nature of gravity
- The sign convention naturally

### 4. Choosing the reference level - critical decision

- For gravitational interactions of astronomical range:
  - The natural reference is **infinite separation**
  - Potential energy is taken as zero at infinity

#### Thus:

- $U(\infty) = 0$

#### Exam trap:

- Using Earth's surface as zero potential in universal gravitation problems.

### 5. Derivation of gravitational potential energy (exact form)

- Consider a mass  $m$  at distance  $r$  from the centre of Earth (mass  $M$ ).

#### Gravitational force magnitude:

- $F(r) = \frac{GMm}{r^2}$

- 
- Work done by gravity in moving the mass from infinity to distance  $r$ :

- $$W = \int_{\infty}^r \frac{GMm}{r^2} dr$$

**Evaluating:**

- $$W = GMm \left[ \frac{1}{r} \right]_{\infty}^r = \frac{GMm}{r}$$

- Hence, gravitational potential energy:

- $$U(r) = -\frac{GMm}{r}$$

## 6. Interpretation of the negative sign - examiner's favourite

**The negative sign indicates:**

- Gravitational interaction is **attractive**
- Energy must be supplied to separate the masses
- The system is in a **bound state** when  $U < 0$

**High-yield sentence:**

- Negative gravitational potential energy signifies a bound gravitational system.

## 7. Physical meaning of zero potential at infinity

**At infinite separation:**

- Gravitational interaction vanishes
- No work is required to maintain separation

**Thus:**

- Zero potential energy represents **no gravitational binding**

**Any finite separation corresponds to:**

- Lower (negative) potential energy
- A bound configuration

## 8. Variation of GPE with distance - qualitative understanding

**From:**

- $$U(r) = -\frac{GMm}{r}$$

**As  $r$  increases:**

- $U$  increases (becomes less negative)
- Approaches zero asymptotically

**As  $r$  decreases:**

- $U$  becomes more negative
- Binding becomes stronger

## 9. Graph of gravitational potential energy

**The  $U$  vs  $r$  graph:**

- Is a **rectangular hyperbola**
- Lies entirely below the zero-energy axis
- Approaches zero at large  $r$
- Drops steeply near the central mass

**Exam trap:**

- Drawing GPE as linear with distance.

## 10. Near-Earth approximation - where $mgh$ comes from

Near Earth's surface:

- Height  $h \ll R$
- Distance from Earth's centre:  $r = R + h$

Exact GPE:

$$U = -\frac{GMm}{R+h}$$

Using binomial expansion:

$$U \approx -\frac{GMm}{R} + \frac{GMm}{R^2}h$$

- Since  $g = \frac{GM}{R^2}$ :
- $U \approx \text{constant} + mgh$

Thus:

- $mgh$  is a **local approximation**
- Valid only for small heights

## 11. Key distinction - exact vs approximate GPE

Exact GPE	Near-Earth GPE
$-\frac{GMm}{r}$	$mgh$
Universal	Local
Reference at infinity	Reference at Earth's surface
Used for satellites, escape	Used for daily mechanics

**Examiner filter:**

- Use the correct form for the correct physical situation.

## 12. Gravitational PE inside Earth (qualitative)

Inside Earth:

- Potential energy varies differently due to changing enclosed mass
- PE is **minimum at centre**
- PE increases outward to surface
- Outside Earth, PE follows  $-1/r$
- Detailed derivation is not required here, but conceptual clarity is expected.

## 13. Bound and unbound gravitational systems

A gravitational system is:

- **Bound** if total mechanical energy  $E < 0$
- **Unbound** if  $E \geq 0$

Where:

$$E = K + U$$

**Negative total energy implies:**

- Object cannot escape to infinity
- Motion is confined (orbit or fall-back)

## 14. Energy perspective on falling bodies

- When a body falls under gravity:
- Potential energy decreases