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MP – Vyapam

Electrical Engineering

Madhya Pradesh Employees Selection Board (MP ESB)

Volume - 4

Network Theory & Power Electronics

SIERRA INNOVATIONS

INDEX

1 CHAPTER Basics of Circuits & Circuital laws

THEORY

1.1. *INTRODUCTION*

The phenomenon of transferring charge from one point to another is termed as electric current. An electric current may be defined as the time rate of electric charge q across-sectional boundary. Random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge.

Conductivity is the ability of the path to transfer electrons. The resistivity is the resistance offered to passage of electron. Resistance is the inverse of conductance.

Charge : Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. The smallest charge that exist is the charge carried by e^- is equal to -1.6×10^{-19} The MKS unit of charge is Coulomb (C). When an electron is removed from a substance, that substance becomes positively charged. A substance with excess of electrons is negatively charged.

Current : The time rate of flow of electric charge across a cross-sectional boundary is termed as current. Mathematically, current is given by

$$i = \frac{dq}{dt}$$

If the charge q is given in coulombs, and the time t in seconds, the current is measured in amperes and denoted by the letter 'A'. or columb/second. The charge transfer between t_0 and t is obtain by $q(t) = \int_{t_0}^{t} i(t)dt$

Voltage : Voltage is defined as work done in moving a unit positive charge once around the closed path. Mathematically, voltage is given by

$$V = \frac{dW}{dq}$$

The unit of potential is Volt (V).

Power : The instantaneous power delivered to a circuit element is the product of the instantaneous value of voltage and current of the element. Mathematically power is given by the relation.

$$p(t) = v(t)\ i(t)$$

The unit of power is Watt (W).

In terms of energy power is defined as, "The time rate of change of energy is called power". Mathematically, power is given by

$$P = \frac{dW}{dt}$$

Energy : The energy delivered to a circuit element over the time interval (t_0, t) is given by

$$E = \int_{t_0}^{t} p(t)dt = \int_{t_0}^{t} v(t)i(t)dt$$

The unit of energy is Watt-sec. or Joules.

1.2. *CIRCUIT COMPONENTS*

(i) Resistance : It is a linear, two terminal component which opposes the flow of current through it. Resistance dissipates energy as heat or some other way. It is measured in Ohms (Ω).

Most materials used to carry current in the form of wires. The resistance R of a conductor varies directly with the length *l* and is inversely proportional to the cross sectional area A.

i.e. $$R = \rho\frac{\ell}{A}$$

where ρ is the constant and is called resistivity. The unit of ρ is ohm-meter.

The ambient temperature of a body is the temperature surrounding it. When the ambient temperature of a resistor is varied, a change in resistance is noted which is related by the following relation

$$R_t = R_0\left[1+\alpha\left(T_t - T_0\right)\right]$$

where, R_0 is the resistance at temperature T_0 = 0ºC and R_t is the resistance at temperature T_tºC.

Resistances can be connected either in series or in parallel.

Series connection of resistors : If n resistors are connected in series then these can be replaced by a single resistance R_{eq} such that

$$R_{eq} = R_1 + R_2 + ... + R_n$$

In series combination of resistance, current remains same.

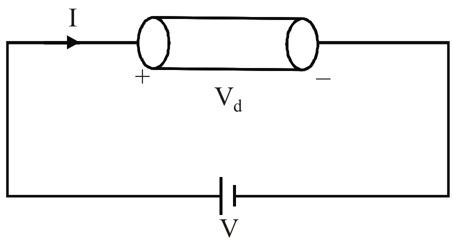
Parallel combination of resistors : In n resistances are connected in parallel then these can be replaced by a single resistance R_{eq} such that.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

In parallel combination of resistances, voltage across parallel branches remains constant.

Ohm's Law : Ohm's law states that the current (I) flowing through a conductor is directly proportional to the voltage (V) across the ends of a conductor, provided physical conditions of a conductor such as pressure, mechanical strain, temperature etc. are kept constant.

$$I \propto V$$
$$I = G \times V$$



Ohm's law also state that at constant temperature potential difference across an element is directly proportional to current flowing through the element.

i.e. $V_d \alpha I$

or $V_d = IR$

(ii) Inductance : It is a property of two terminal circuit element by virtue of which it is capable of storing energy in an electromagnetic field with the help of current passing through it. The device is known as inductor.

Inductance can be two types :

(a) Self Inductance : The property of coil due to which it opposes any increase or decrease of current or flux through it is called Self-inductance.

Self-inductance of a coil is given by the relation

$$L = \frac{N\phi}{I} = \frac{N^2 \mu A}{\ell}$$

where, N = number of turns in coil

A = area of cross-section

l = length of coil

I = Current in ampere

ϕ = flux in Weber

L = Coefficient of self-inductance in Henry.

As per lenz's law, this self induced e.m.f. opposes the cause (i.e. current in the coil). This property which opposes any change in the current is called self inductance. Current through inductor is given by

$$i_L(t) = \frac{1}{L} \int_{-\infty}^{t} V_L(t) dt$$

Energy stored by an inductor

$$W_L = \frac{1}{2} L i^2$$

(b) Mutual Inductance (M) : Mutual inductance is the property of two coil because of which each opposes any change in the current flowing through the other by developing an induced emf.

Coefficient of Coupling : Coefficient of coupling (K) between two coils is the measure of coupling of two coils.

$$M = 1$$

Mathematically, $K = \frac{M}{\sqrt{L_1 L_2}}$, value of K < 1.

Coefficient of coupling $K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$

Dot Convention for Coupled Circuits

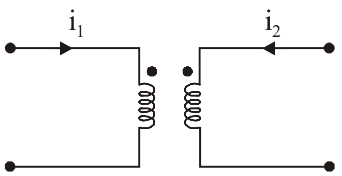


Figure (a)

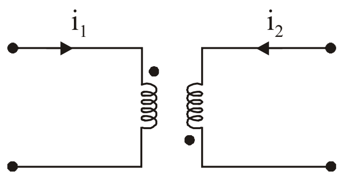


Figure (b)

A dot placed at any end of the two coils indicates that, these coils are mutually coupled.

Now the mutual inductance M between two coils

(i) Will be positive if the current enters through both the dots or leaves both the dots, (as shown in the above figure (a).

(ii) Will be negative when current enters through a dot in one coil and leaves the dot in another coil as shown in figure (b) above.

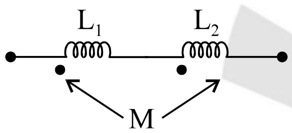
Series combination of inductors : If a number of inductors are connected in series, then they can be replaced by a single inductor of inductance L_{eq} such that

$$L_{eq} = L_1 + L_2 + L_3 + ... + L_n$$

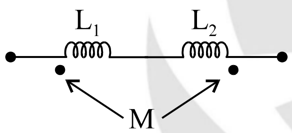
Parallel combination of inductors : If a number of inductors are arranged in a parallel to one another then the equivalent inductance is given by:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ... + \frac{1}{L_n}$$

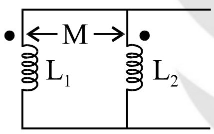
Combination of Inductors with Mutual Inductance

(i) 

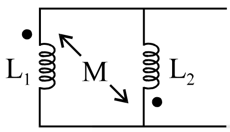
$$L_{eq} = L_1 + L_2 + 2M$$

(ii) 

$$L_{eq} = L_1 + L_2 - 2M$$

(iii) 

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

(iv) 

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

(iii) Capacitance : An electrical element which stores energy due to voltage across it and is independent of the current through it is called capacitor. The unit of capacitance is Farad and denoted by 'C'.

The voltage across the capacitance is given by

$$V = \frac{Q}{C} = \frac{1}{C}\int_{-\infty}^{t} i\,dt \quad (\therefore \text{ where Q is the charge on capacitance in coulombs.})$$

The current through the capacitor is given by

$$i = C\frac{dV}{dt}$$

For a parallel plate capacitor whose plates are having an area of A (in square meter) each the separation between the plates is d (in meters). Then capacitance is given by

$$C = \frac{\varepsilon A}{d} \text{ Farad}$$

Various types of capacitors are as follows

(a) Ceramic capacitor : They are manufactured by depositing directly on each side of ceramic dielectric, coating of silver that serves as capacitor plates. These capacitors have very high capacitance per unit volume.

(b) Paper capacitor : They are manufactured by winding long narrow sheets of alternate layers of aluminium foil and was impregnated paper, into impact rolls.

(c) Electrolytic capacitors : They can only be used in circuits that maintains the polarity of the voltage in one direction. If the voltage polarity is reversed, the capacitor acts as a short circuit. These capacitors have a very high values of capacitance ranging from 1 to 2μF to the order to several thousand microfarads.

1.3 *CIRCUIT TERMINOLOGY*

(i) Circuit element : Any individual circuit component like inductor, resistor, capacitor etc. with two terminals.

(ii) Branch : A branch is a group of circuit elements with two terminals. Thus a branch consists of number of elements.

(iii) Node : A node is the meeting point of three or more branches at a common point. It is also referred as junction.

(iv) Mesh and Loop : A set of branches forming a closed path, with the omission of any branch makes the path open. Mesh must not have any other circuit inside it. Loop may have other loops or meshes inside it.

(v) Active Network : A network containing energy sources together with other circuit elements.

(vi) Passive Network : A network containing circuit elements without any energy sources.

(vii) Lumped Network : Network whose elements are physically separable are called as lumped elements, such as resistors, inductors, capacitors etc.

(viii) Distributed Network : A network in which resistors, capacitors, inductors etc. are not electrically separable and isolated as individual elements. A transmission line has distributed resistor, inductor, and capacitor which are not isolated from the network.

(ix) Bilateral Network : A bilateral network is defined as those whose elements can transmit well in either direction. For example, elements made of high conductivity material like iron core conductors are bilateral elements.

(x) Unilateral Network : A unilateral network is one whose elements follow different laws relating to voltage and current for different direction of current and voltage polarities. For example, vacuum tubes, crystal and metal rectifiers are unilateral elements.

(xi) Linear and Non Linear Networks : A linear network is one for which the principle of superposition holds. A circuit element is linear if the relation between current and voltage involves a constant coefficient.

e.g. $$V = Ri, V = L\frac{di}{dt}, \ V = \frac{1}{C}\int i \, dt$$

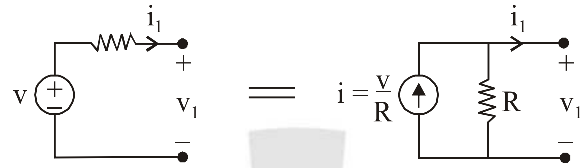
Therefore, it is concluded that a linear network must be bilateral, but a bilateral network is not necessarily linear. For example, an iron-core conductor is bilateral but it is non-linear.

(xii) Current Source : A generator which maintain its output current independent of the voltage across its terminals. It is indicated by a circle enclosing an arrow for reference current direction. An ideal current source has infinite internal resistance.

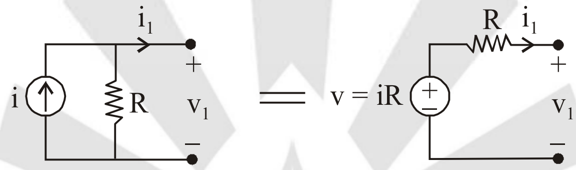
(xiii) Voltage Source : A generator which maintain its value of potential independent of the output current. An AC source is indicated by a circle enclosing a wave line. An ideal voltage source has zero internal resistance.

1.4 *SOURCE TRANSFORMATION*

A voltage source is transformed into its equivalent current source as

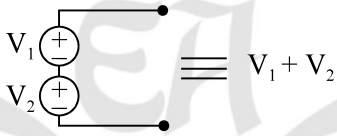


A current source is transformed into its equivalent voltage source as

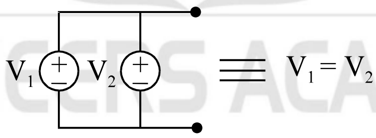


✍ ***Key points :***

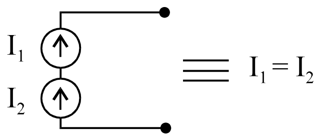
- An ideal voltage source has zero internal resistance.
- Ideally the internal resistance of a voltmeter should be infinite.
- An ideal current source has infinite internal resistance.
- Ideally the internal resistance of an ammeter should be zero.
- If two voltage sources are connected in series, they are summed up.



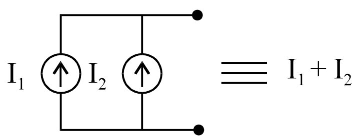
- If two voltage sources are connected in parallel, they are treated as equal.



- If two current sources are connected in series, they are treated as equal.



- If two current sources are connected in parallel, they are summed up.



- **Circuit Response of single elements**

Element	Voltage across element	Current in element
Resistance, R	$v(t) = R\,i(t)$	$i(t) = \frac{v(t)}{R}$
Inductance, L	$v(t) = L\frac{di}{dt}$	$i(t) = \frac{1}{L}\int_0^t v\,dt$
Capacitance, C	$v(t) = \frac{1}{C}\int_0^t i\,dt$	$i(t) = C\frac{dv}{dt}$

1.5 *KIRCHOFF'S LAWS*

Kirchoff's basic circuit laws provides two methods for the solution of networks. They are as follows

(i) Kirchoff's Current Law (KCL) : It is based on conservation of charge. It states that, "The algebraic sum of currents at any node of a circuit is zero".

The direction of current coming towards node is taken as positive and outgoing current as negative. The reverse sense of directions can also be taken.

Minimum number of nodal equations to calculate the voltage and current of every port of network is = n – 1 (where, n = number of nodes)

Sign Convention : *Take positive sign for current entering a node and negative sign for leaving a node.*

(ii) Kirchoff's Voltage Law (KVL) : It is based on conservation of energy. It states that, "The algebraic sum of voltages in any closed path of network that is transverse in any single direction is zero".

Sign Convention : *Current i causes a positive drop of voltage when flowing from positive to negative potential and a negative drop when flowing from negative to positive potential.*

Minimum number of loop equations = b – n + 1

where b = number of branches.

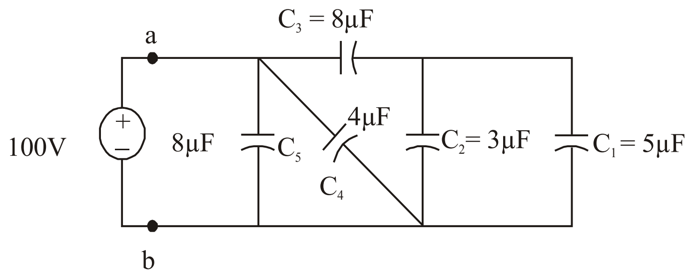
n = number of loops.

Solved Examples

Example : For the circuit shown in figure below

(a) Calculate the equivalent capacitance across the terminals a-b.

(b) Also calculate the charging time to charge these capacitances by a steady direct current of constant magnitude of 10A.



Solution **:** Equivalent capacitance between terminals a-b is given by-

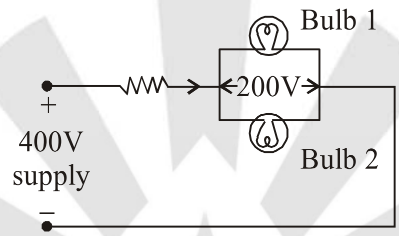
$$C_{eq} = \left[\frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} \,||\, C_4\right] || \, C_5 = \left[\frac{(5\mu + 3\mu) \times 8\mu}{5\mu + 3\mu + 8\mu} \,||\, 4\mu\right] || \, 8\mu = 4\mu F$$

Then, net charge is given by

$$Q_{net} = C_{eq} \times V = 4 \times 10^{-6} \times 100 = 400\mu F$$

Therefore, charging time is $\quad t = \dfrac{Q_{net}}{I} = \dfrac{400 \times 10^{-6}}{10} = 40\,\mu\,\text{sec}$

Example : Two bulbs of rating 100 W and 220 V are required to be connected across a 400 Volts supply. Find the value of the resistance to be inserted in the line so that the voltage across the bulbs does not exceed 220 V



Solution : Total power drawn from the circuit is 2 × 100 = 200 W

Hence supply current I is given by $\quad I = \dfrac{200W}{220V} = 0.91A$

Let R be the resistance connected in series such that the voltage across the bulbs is 220 V

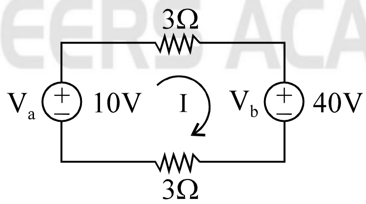
Now $\quad$ Supply voltage = drop in R + voltage across bulbs

$$400 = V_R + 220$$

or $\quad V_R = IR = 180$

or $\quad R = \dfrac{180}{0.91} = 197.8\Omega$

Example : The two constant voltage sources V_a and V_b act in the same circuit as shown in figure below. Calculate the power delivered by each voltage source ?



Solution : Applying KVL to the circuit

$$3 \times I + 3 \times I - 10 + 40 = 0$$

or $\quad I = -5A$

Power delivered by $\quad V_a = V_a \times I = 10(-5) = -50W$

Power delivered by $\quad V_b = V_b \times I = 40(5) = 200\text{ W}$

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2 CHAPTER Network Theorems

THEORY

2.1 *INTRODUCTION*

Ohm's law and Kirchoff's law are the fundamental tools for network analysis, while network theorems are very powerful tools for solving complicated network problems. We will discuss the following important network theorems

2.2 *SUPERPOSITION THEOREM*

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch or voltage at any node is the algebraic sum of the currents or node voltages that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Following points should be noted about superposition theorem :

(i) This theorem is used to calculate voltage and current.

(ii) It is applicable only when more than one active source is present in the network.

(iii) All elements should be linear.

(iv) Elements may be time variant or invariant.

(v) Superposition is the combined property of homogeneity and additively.

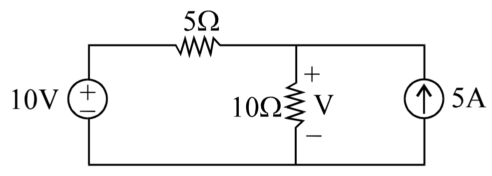
Steps for solving a network using Superposition theorem

(i) Take only independent source at a time and remove all the other sources (voltage source is removed by a short circuit and current source by an open circuit). Calculate branch current or node voltage with that single source.

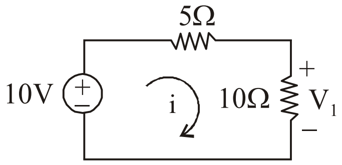
(ii) Now repeat above step and calculate branch current or node voltage for each source.

(iii) To calculate net current in the branch or net voltage at the node, find the algebraic sum of currents or node voltages obtained for each source.

Example : Calculate voltage V in the circuit shown below using superposition theorem ?

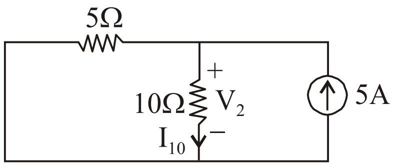


***Solution* :** Considering only 10V source and remove 5A source (by an open circuit). Modified circuit is shown below



$$V_1 = 10 \times i = 10 \times \frac{10}{5+10} = 6.67V$$

Again considering 5A source only and removing 10V source (by a short circuit). Modified circuit shown above.



$$i_{10} = 5 \times \frac{5}{5+10} = 1.67A$$

and $V_2 = 1.67 \times 10 = 16.7V$

By superposition theorem $V = V_1 + V_2 = 6.67 + 16.7 = 23.37$ V

2.3 *RECIPROCITY THEOREM*

In any linear network containing bilateral linear impedances and single source, the ratio of a voltage V introduced in one mesh to the circuit I in any second mesh is the same as the ratio obtained if the positions of V and I are interchanged.

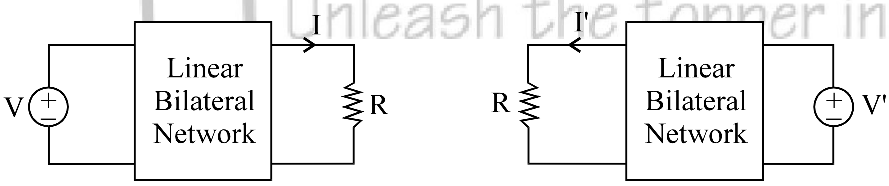
Following points should be noted :

(i) Any reciprocal network should not have any dependent source.

(ii) All the element should be time invariants.

(iii) Reciprocal networks do not have initial conditions.

Consider a network shown below

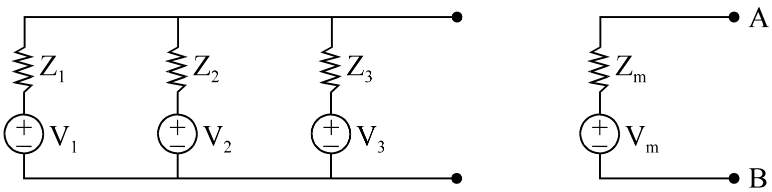


If the networks are reciprocal, then from reciprocity theorem,

$$\frac{V}{I} = \frac{V'}{I'}$$

2.4 *MILLMAN'S THEOREM*

Millman's theorem states that if several voltage sources in series with admittances are connected as shown in figure below. Then the equivalent may be represented by a voltage source V in series with an impedance Z as shown below



where $$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3} = \frac{\sum V_iY_i}{\sum Y_i}$$

and $Z_m = \frac{1}{Y_m} = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{\sum Y_i}$

Steps for solving the network using Millman's theorem :

(i) Calculate the equivalent admittance Y of the network by removing the load.

(ii) Apply Millman's theorem to find equivalent voltage source given by the relation

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3} = \frac{\sum V_iY_i}{\sum Y_i}$$

(iii) Calculate the equivalent impedance by using the relation

$$Z_m = \frac{1}{Y_m} = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{\sum Y_i}$$

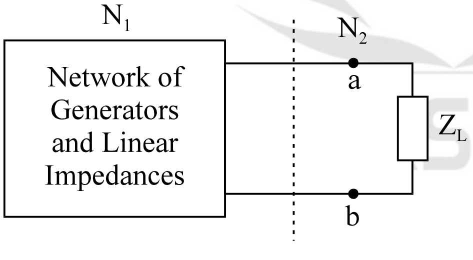
(iv) The current through the load is then given by the relation

$$I_L = \frac{V_m}{Z_m + Z_L}$$

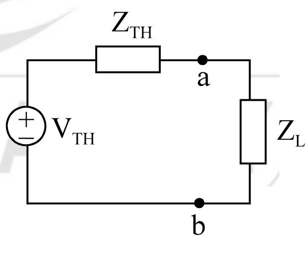
2.5 *THEVENIN'S THEOREM*

Any two-terminal network consisting of linear impedance and energy sources may be replaced by an e.m.f. acting in series with an impedance. The e.m.f. is the open circuited voltage at the terminals, and the impedance is the impedance viewed at the terminals when all the sources in the network have been replaced by impedances equal to their internal impedances.

Consider a general network N_1 consisting of generators and linear impedance shown in figure below and network N_2 which is replaced by load Z_L connected across terminals a and b.



General Network



Thevenin's Equivalent

Following important points about Thevenin's theorem should be noted :

(i) N_1 should have linear elements only.

(ii) N_1 can have both dependent and independent sources.

(iii) N_1 can have initial conditions.

(iv) N_1 and N_2 should not be mutually coupled to each other.

(v) N_2 can have linear, non-linear, time variant and time invariant.

(vi) N_2 can have any voltage source independent or dependent.

(vii) N_2 can have initial conditions.

(viii) N_2 should not be mutually coupled.

Steps for solving a network using Thevenin's theorem

(i) Remove the load impedance Z_L and find the open circuit voltage V_{TH} across the open circuited load terminals, a-b.

(ii) To find the Thevenins impedance Z_{TH} across terminal a-b, remove the current and voltage source. Voltage source is removed by making it short with its internal resistance and current source by making it open circuit.

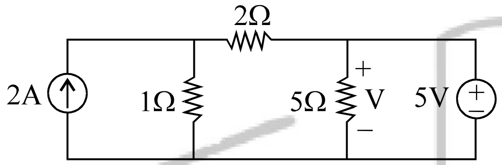
(iii) Draw the thevenin equivalent circuit by placing Z_{TH} in series with V_{TH}.

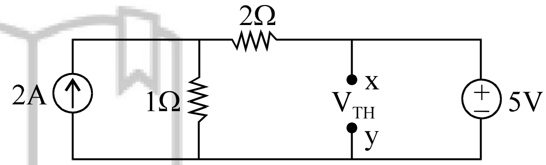
(iv) Reconnect load impedance Z_L across the load terminals.

Therefore, the current I in the circuit is calculated by

$$I = \frac{V_{TH}}{Z_{TH} + Z_L}$$

Example : Calculate the current through the 5Ω resistor in the circuit shown below

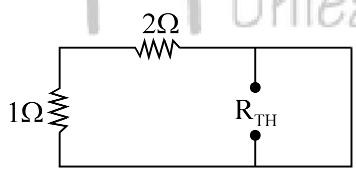


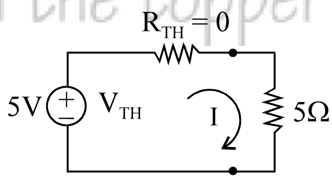


Solution **:** First remove 5Ω resistor from the circuit and calculate the open circuit voltage V_{TH} across the open circuited terminal x-y. Voltage V_{TH} across terminal x-y is

$$V_{TH} = 5V$$

Thevenin's resistance across the terminal x-y is obtained by replacing the energy sources. After removing current and voltage sources, the circuit becomes





From above circuit, the Thevenin's resistance is $R_{TH} = 0$

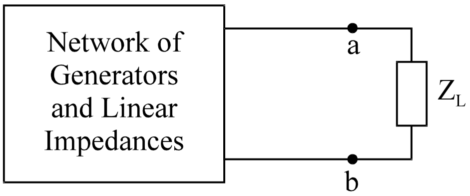
Thus the Thevenin's current is given by

$$I = \frac{V_{TH}}{R_{TH} + R_L} = \frac{5}{0+5} = 1A$$

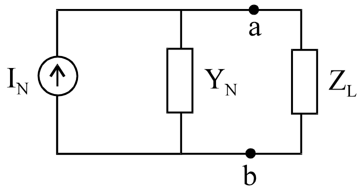
2.6 *NORTON'S THEOREM*

Any two-terminal linear network consisting of energy sources and impedances can be replaced by an equivalent circuit consisting of current source I_N in parallel with an admittance Y_N. The I_N is the short circuit current between the terminals a and b of the network and Y_N is the admittance measured between the terminals with all energy sources replaced by their internal resistances.

Consider a general network consisting of generators and linear impedance shown in figure below. Z_L is the load impedance connected across terminals a and b.



General Network



Norton's Equivalent

Steps for solving a network using Norton's theorem

(i) Short the load terminal and find the short circuit current, I_N flowing through the shorted load terminals.

(ii) To find the Norton admittance Y_N across terminal a-b, remove the current and voltage source. Voltage source is removed by making it short with its internal resistance and current source by making it open circuit.

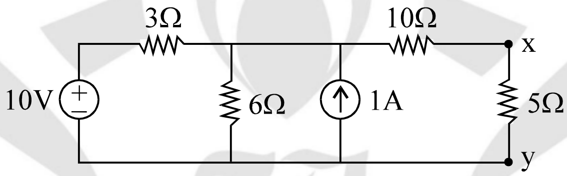
(iii) Draw the Norton equivalent circuit by placing Y_N in parallel with I_N.

(iv) Reconnect load impedance Z_L across the load terminals.

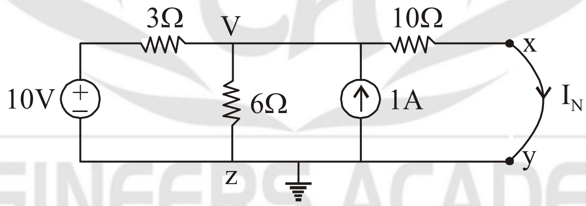
Therefore, the current I_L through the load Z_L is given by

$$I_L = \frac{Z_N}{Z_N + Z_L} \times I_N$$

Example : Calculate the current in the 5Ω resistor in the circuit shown below



***Solution* :** Remove the load resistance 5Ω from the circuit and short the terminal x-y as shown below:



Assuming the voltage V to be +ve at node w and z is connected to ground in the above circuit, nodal analysis gives

$$\frac{V-10}{3} + \frac{V}{6} - 1 + \frac{V}{10} = 0$$

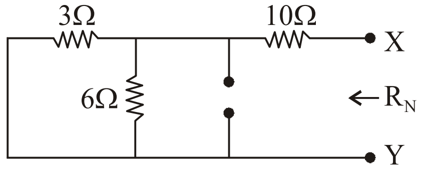
$$V = 7.22V$$

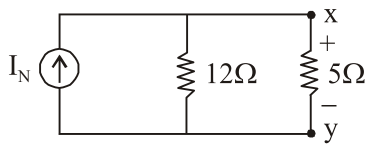
Applying KVL at the loop wxyz in above figure,

$$-V + I_N \times 10 = 0$$

$$I_N = 0.722A$$

To find the Norton equivalent resistance, replace all the sources with their equivalents, and calculate equivalent resistance across terminal x-y. The circuit reduces to





$$R_N = \frac{3 \times 6}{3+6} + 10 = 12\Omega$$

Norton equivalent circuit is shown in figure above. Therefore, current through 5Ω resistor is given by

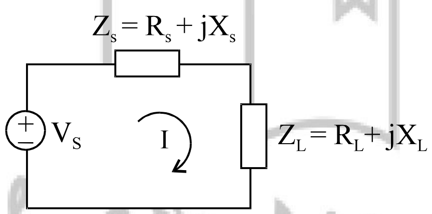
$$I_{5\Omega} = \frac{R_N}{R_N + R_L} \times I_N = \frac{12}{12+5} \times 0.772 = 0.5096A$$

2.7 *MAXIMUM POWER TRANSFER THEOREM*

This theorem states that, the maximum power is absorbed by one network from another joined to it at two terminals, when the impedance of one is the conjugate of other.

It means that for maximum power transfer to take place, the resistance of the load should be equal to the source and the reactance of the load should be equal to that of source in magnitude but opposite in sign. It means, if load is capacitive then source is inductive and vice-versa.

Consider a network consisting of a load impedance Z_L and source impedance Z_s as shown below



In the above circuit, maximum power is observed when, $X_L = -X_S$ and $R_L = R_S$ and power is given by

$$P_{L\max} = \frac{V_S^2}{4R_L}$$

Steps for solving the network relating to Maximum Power theorem

(i) Remove the load resistance and find the Thevenin's resistance R_{TH} of the source network.

(ii) From maximum power transfer theorem, this resistance R_{TH} equals load resistance i.e. $R_{TH} = R_L$ for maximum power transfer.

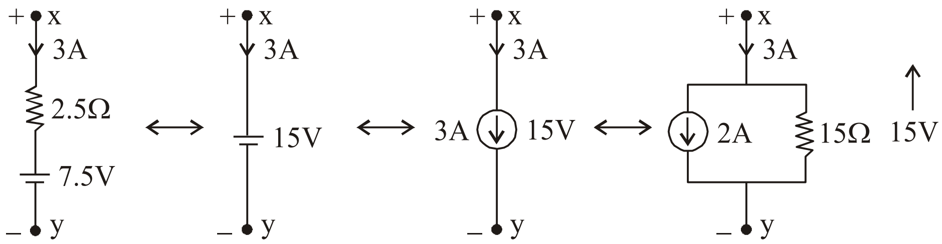
(iii) Find the Thevenins voltage V_{TH} across the open circuited load terminal.

(iv) Maximum power transfer is given by $P_{L\max} = \frac{V_{TH}^2}{4R_L}$

2.8 *SUBSTITUTION THEOREM*

The voltage across the current through any branch of a bilateral network being known, this branch can be replaced by any combination of elements that will make the same voltage across the current through the chosen branch.

Consider the following equivalent network that are substituted in place of other.



The limitation of this theorem is that the theorem cannot be used to solve the network containing two or more sources that are not in series or in parallel.

2.9 *COMPENSATION THEOREM*

In a linear time invariant network when the impedance Z of an uncoupled branch carrying a current I is changed by ΔZ, the currents in all the branches would change and can be obtained by assuming that an ideal voltage source of $V_C = I\Delta Z$ has been connected in series with $(Z + \Delta Z)$ when all other sources in the network are replaced by their internal resistances.

2.10 *TELLENGEN'S THEOREM*

For any given time, the sum of power delivered to each branch of any electric network is zero. In other words, the summation of instantaneous power of summation of complex power of sinusoidal sources in a network is zero. The network may be linear or non linear, passive or active and time invariant or varying.

Mathematically, for instantaneous power this theorem can be expressed as

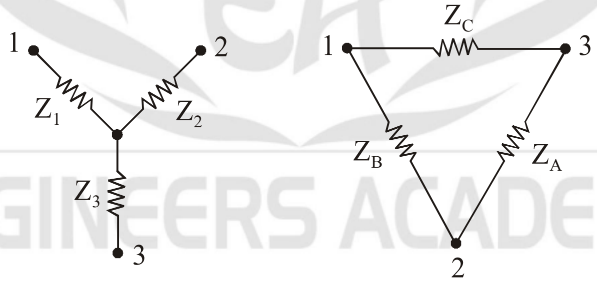
$$\sum_{n=1}^{k} V_n I_n = 0$$

where k is the number of branches in a network.

2.11 *STAR DELTA CONVERSION THEOREM*

This theorem states that at any frequency a T-section (or Star or Y network) can be interchanged to any network with a π section (mesh or delta) and vice versa, provided certain relations are maintained between the elements of the two sections.

Figure below shows a Star and delta networks



Conversion formulaes from Delta to Star Conversion

$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}, \qquad Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}, \qquad Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

Conversion formulaes from Star to Delta Conversion

$$Z_A = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}, \qquad Z_B = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}, \qquad Z_C = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

❍❍❍

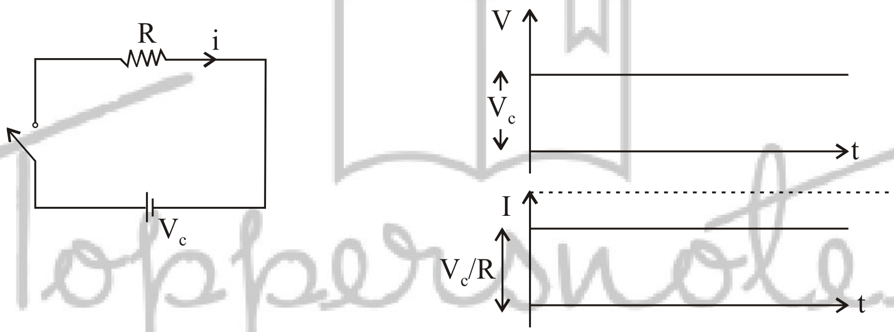
3 CHAPTER Transient & Laplace Circuits

THEORY

3.1. *INTRODUCTION*

Before formulating the initial condition in networks, we first consider effect of switching action on resistors, inductors and capacitors.

1. Resistors : In the ideal resistor, current and voltage are related by ohm's law V= RI. If a step input of voltage as shown is applied to a resistor network, the current will have the same waveform altered by scale factor (1/R).



The current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly voltage with change instantaneously on applying instantaneous changing current.

2. Inductor : The current through an inductor cannot change instantaneously. If an energy source is suddenly connected to an inductor then it will not cause current to flow initially. Thus inductor acts as open circuit for the newly applied source at the instant of switching.

If a current of value I_0 flows in the inductor at the instant switching takes place, that current will continue to flow. For the initial instant, the inductor may be considered as a current source of current I_0.

3. Capacitor : The voltage across a capacitor cannot change instantaneously.

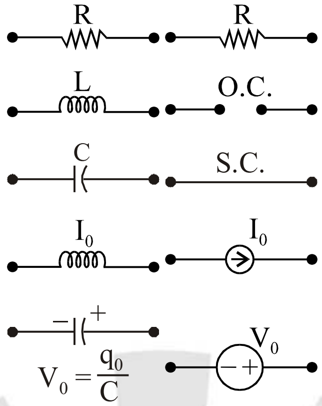
Since we have $V = \frac{q}{C}$, hence zero charge corresponds to zero voltage. Thus for a suddenly applied energy source, voltage across capacitor is initially zero i.e. equivalent to a short circuit.

Hence on connecting an uncharged capacitor to an energy source a current flows instantaneously.

But with an initial charge in the system, the capacitor is equivalent to a voltage source of value $V_0 = \frac{q_0}{C}$, where q_0 is the initial charge.

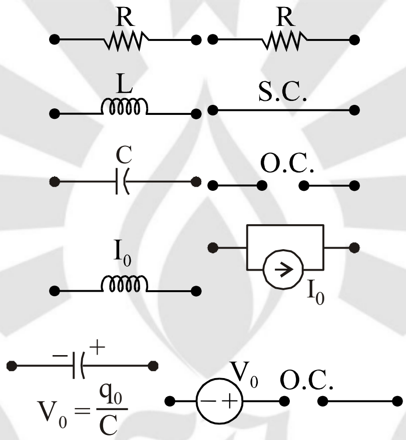
Element in terms of Initial conditions

Element (& final condition) Equivalent circuit at t = 0^+

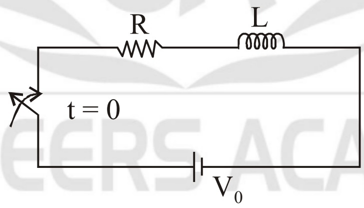


Elements in Terms of Final Conditions

Element (& initial condition) Equivalent circuit at t = ∞



Initial value of derivatives : Consider a series RL circuit connected suddenly to a constant voltage source V_0 by closing in the switch S at t = 0.



Then application of KVL gives

$$L\frac{di}{dt} + Ri = V_0 \qquad ...(i)$$

$$\frac{di}{dt} = \frac{1}{L}(V_0 - iR) \qquad ...(ii)$$

For an inductor, on switching in the supply voltage (at time t = 0^+) current i must be zero, because inductor L hold the same current for some time i.e.

$$i(0^-) = i(0^+) = 0 \text{ hence, } \left[\frac{di(0^+)}{dt} = \frac{V_0}{L}\right] \qquad ...(iii)$$

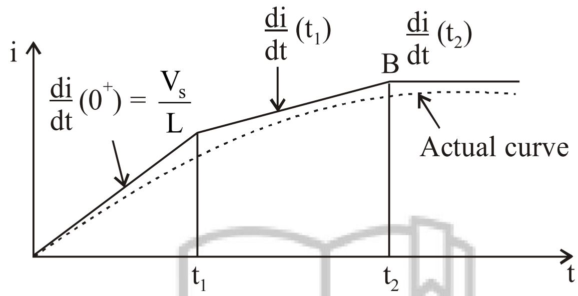
According to eq. (iii), initially the current i increases linearly at the rate $\frac{V_0}{L}$ and may be considered to continue to do so, for a small time interval t_1.

Thus after time interval t_1, the new value of current is $i_1 = \frac{V_0}{L}t_1$

At this time, in accordance with eq. (ii) the time rate of change of current $\frac{di}{dt}$ is given by

$$\frac{di}{dt}(t_1) = \frac{1}{L}\left[V_0 - i_1R\right]$$

Let the current i continue to increase at this new rate for another small time interval t_1 to t_2 as shown by region AB in figure.



This process may be continued to give a graph as shown. Thus we conclude that the first derivative represents the slope of the curve plotting the dependent variable (i) as a function of independent variable (t).

Procedure for evaluating initial conditions

The following steps are followed :

1. Solving for the initial values of the variables namely currents and voltages at $t = 0^+$. This is done by following steps

(i) Replace every inductor by an open circuit, if current $t = 0^+$ through inductor is zero or by a current generator having source current equal to that flowing at time $t = 0^+$.

(ii) Replace every capacitor by short circuit, if the voltage across capacitor is zero at $t = 0^+$ or by a voltage generator having source voltage V_0 equal to q_0/C if there is an initial charge q_0.

(iii) Leave every resistor in the network unchanged.,

2. Solving for the derivatives of variables at $t = 0^+$.

Example : In the circuit shown in figure, V = 10V, R = 10 ohm, L = 1H, C = 1μF and $V_C(0^-) = 0$.

Find $i(0^+), \frac{di(0^+)}{dt}$ and $\frac{d^2i(0^+)}{dt^2}$