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UKPSC – JE

Electrical Engineers

Uttarakhand Public Service Commission (UKPSC)

Volume - 2

(Technical)

Control Systems & Basic Electronics & Digital Electronics

SIERRA INNOVATIONS

INDEX

1 CHAPTER

Introduction to Control System

THEORY

1.1 INTRODUCTION

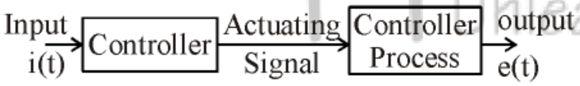
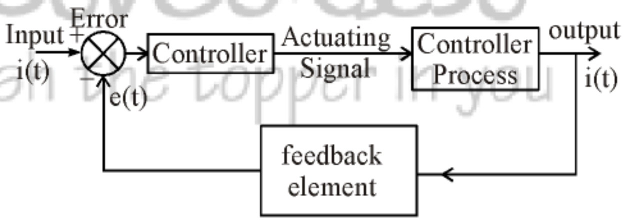
A control system is a combination of elements arranged in such manner that the working of each element produces the best output for a given input. Control systems are used in many applications such as for control of position, velocity, acceleration, temperature, pressure, voltage, current etc.

1.2 TYPES OF CONTROL SYSTEMS

(i) Open Loop

(ii) Closed Loop

The table below gives a comparison between an open loop and a closed loop.

Open Loop	Closed Loop
1. An open loop control system is a system which does not have feedback arrangement (i.e. output has no effect on the control action). 	1. In a close loop control system the output has an effect on control action through a feedback. 
2. The accuracy of this system depends on the calibration of the input.	2. Due to feedback the performance of closed loop system is accurate.
3. The open loop system is simple to construct and is cheap.	3. The closed loop system is complicated to construct and is costly.
4. Open loop systems are generally stable.	4. In closed loop system the stability depends on the system components.
5. The operation of this system degenerates due to the presence of non-linearity in its elements.	5. The performance is better as this system adjusts to the effect of non-linearity present in its elements.
Example: Traffic light controller, electric washing machine.	***Example:*** Automatic control systems, human beings.

Feedback

Feedback is that characteristic of closed loop control system which distinguishes them from open loop system.

There are two type of feedback

(a) Positive feedback

(b) Negative feedback

Negative feedback system has following properties

(i) Increased accuracy

(ii) Reduced sensitivity

(iii) Reduce effect of non linearity and distortion

(iv) Increased bandwidth

(v) Tendency towards instability

Key Points :

- *Human being is a best example of a complex closed loop control system, in which,*

  (i) Human eyes acts as observer.

  (ii) Human brain acts as controller.

  (iii) Nervous system acts as connecting media.

Use of Laplace Transform in Control System

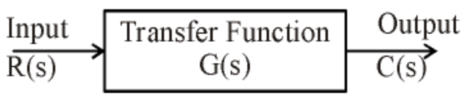
The control action of a dynamic control system is expressed in the form of differential equation in time domain. In order to calculate the solution from a differential equation in time domain it has to be transformed into an algebraic form. Laplace transform technique transforms a time domain differential equation into a frequency domain algebraic equation.

Key Points :

- *Final value theorem is applicable if and only if* $\lim_{t\to\infty} f(t)$ *exists (which means that f(t) settles down to a definite value at* $t \to \infty$).
- *Final value theorem is not applicable for sine, cosine and ramp signal.*

1.3 TRANSFER FUNCTION

The transfer function of a linear time invariant system is the ratio of laplace transform of the output variable C(s) to the laplace transform of the input variable R(s) with all initial conditions zero. The block diagram is of a simple transfer function is shown below



and the transfer function is given by the relation

i.e. $$G(s) = \frac{C(s)}{R(s)}$$

Poles and Zeros of a Transfer Function

The transfer function is represented by the ratio of two polynomials in terms of s.

i.e.
$$G(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots a_n}{b_0 s^m + b_1 s^{m-1} + \ldots b_m}$$

where, a_0, b_0, a_1, b_1,.....are constants and depends on the system.

The above polynomial can be factorized into n and m terms of numerator and denominator respectively.

$$G(s) = \frac{k(s-s_1)(s-s_2).....(s-s_n)}{(s-s_a)(s-s_b)......(s-s_m)}$$

$$k = \frac{a_0}{b_0} \text{ is known as gain factor}$$

Following points about the transfer function should be noted

- Location of poles and zeros in s plane determines the stability of the system.
- Poles and zeros may be multiple of single. Multiple poles and zeros are avoided.
- In any practical system, number of poles are always greater than number of zeros.
- Numerator when equated to zero will give the zeros for the transfer function. At zero's frequency the response of the system is zero.
- Denominator when equated to zero will give the poles for the transfer function. At pole's frequency the response of the system is infinite.

Advantages of Transfer Function

(i) Transfer function is used to analyse and design of linear, time invariant differential equation system.

(ii) Transfer function is used to determine the time response, stability and accuracy of the system.

(iii) Any physical system can be represented by its transfer function to study its properties.

1.4 PICTORIAL REPRESENTATION OF CONTROL SYSTEM

Block Diagram

It is a pictorial representation of a complex control system in which the transfer function of each element is represented by a block diagram in a proper sequence connected by lines which shows the flow of signals in the direction of arrows.

Terms Related with Block Diagram

(i) Take off Point

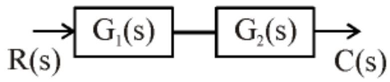
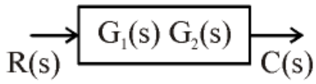
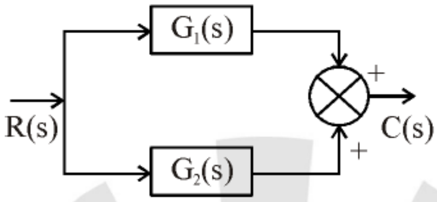
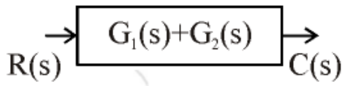
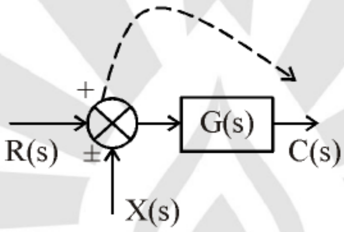
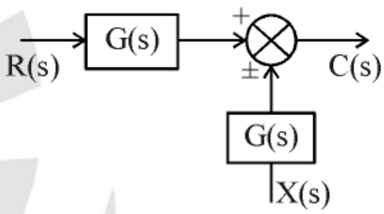
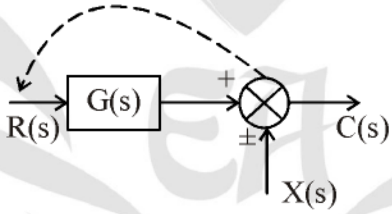
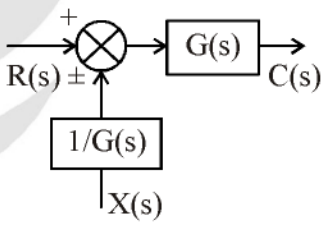
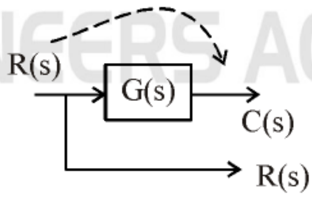
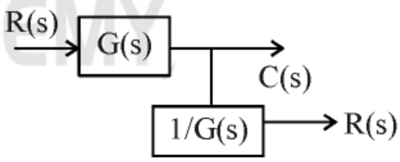
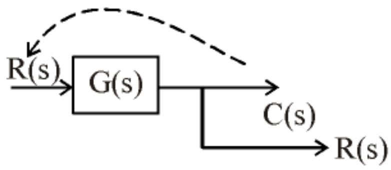
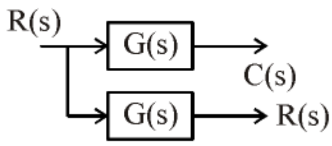
It is represented as a dot which signifies application of one input source to two or more systems.

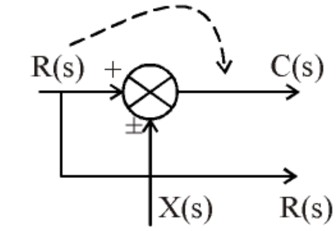
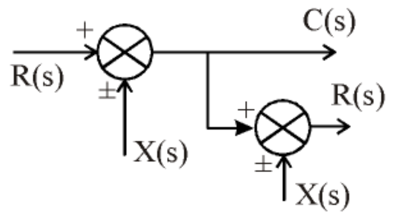
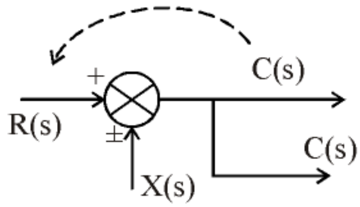
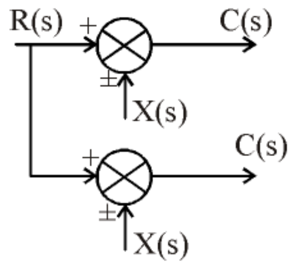
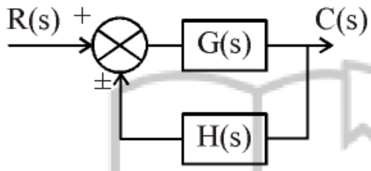
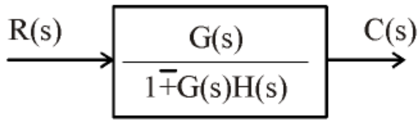
(ii) Summing Point

It represents summation of two or more input signal entering in a system.

Block Diagram Reduction

In order to obtain the overall transfer function, complete block diagram configuration can be simplified by a procedure called block diagram reduction technique.

Rule	Original Diagram	Equivalent Diagram
1. Combining block in cascade		
2. Combining block in parallel		
3. Moving a summing point before from block to after a block		
4. Moving a summing point ahead of a block		
5. Moving a take off point after a block		
6. Moving a take off point ahead of a block.		

<table>
<tr><th>Rule</th><th>Original Diagram</th><th>Equivalent Diagram</th></tr>
<tr><td>7. Shifting of a take off point from position before a summing point to after it.</td><td>
</td><td>
</td></tr>
<tr><td>8. Shifting of a take off point after a summing point to before it.</td><td>
</td><td>
</td></tr>
<tr><td>9. Elimination of a feedback loop.</td><td>
</td><td>
</td></tr>
</table>

1.5 SIGNAL FLOW GRAPH

Signal flow graph further shortens the representation of a control system by way of eliminating the summing symbol, take off points and blocks. This elimination is achieved by replacing the variables by point called "Nodes" and the transfer function is termed as "Transmittance" which is replaced by a line called "branch".

(i) Node

Node is a representation of variables in a signal flow graph.

(ii) Branch

A signal in the graph travels along the branch from one node to another in the direction indicated by arrow and multiplied by the transmittance.

(iii) Loop

It is a close path without repetition of any node.

Rules for Drawing Signal Flow Graphs

1. The signal travels along a branch in the direction of an arrow.
2. The input signal is multiplied by the transmittance to obtain the output signal.
3. Input signal at a node is the sum of all signals entering at that node.
4. A node transmits signals in all branches leaving that node.
5. Take off point after a summing point are represented by a single node.
6. Take off point precedes a summing point are represented by two separate nodes with a transmittance of unity between them.

In a signal flow graph the overall transmittance can be determined by "Mason's gain formula" given by the following equation.

$$T = \sum_{k=1}^{m} \frac{P_k \Delta_k}{\Delta}$$

Where,

k = Total number of forward paths.

P_k = Forward path transmittance of the path where no node is encountered more than once.

Δ_k = Path factor associated with k^{th} path involves all closed loops in the graph which are isolated from the forward path under consideration.

Δ = Graph determinant

= 1– [sum of all individual loop transmittance] + [all possible pairs of non-touching loops] – [sum of loop transmittance products of all possible triplets of non-touching loops] + [.......] – [.........]

Key Points :

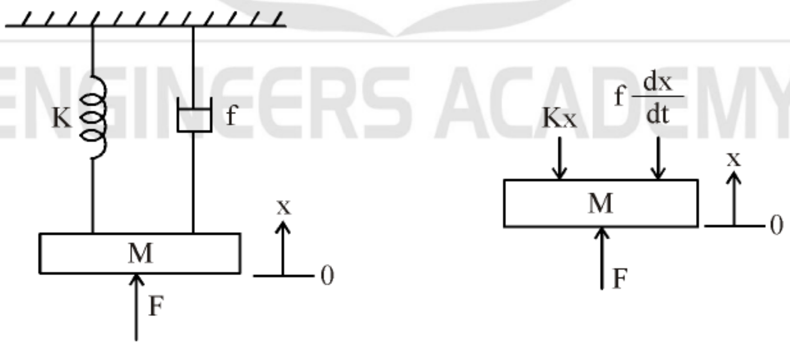
- *While drawing signal flow graph from given block diagram representation, take off point after summing point is represented by a single node. Where as summing point preceding a take off point are represented by two different nodes.*
- *The path factor Δ_k for the k^{th} path is equal to the value of the graph determinant of a signal flow graph which exists after removing the k^{th} path from the graph.*

1.6 MODELLING OF CONTROL SYSTEM

In this section, we deal with some physical systems, analyze their performance and then determines their transfer function by drawing block diagram. This process is called as modelling of control system.

1.6.1 Translation Mechanical System

Let us consider a translational spring-mass-damper system such as shown in figure below in which force is applied in the upward direction. Then this force is distributed on all the three elements of the mass-spring damper system.



The force F is applied on mass M (unit is Kg) results into displacement X. If the spring deflection constant is K (Newton/metre) and friction coefficient is f (Newton/rad/sec), then equation of motion for the system is obtained by applying second law of translation motion.

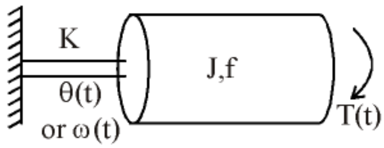
$$F = M\frac{d^2x}{dt} + f\frac{dx}{dt} + Kx \quad \text{...(i)}$$

Now, the transfer function of the system with initial conditions zero is given by

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + K}$$

1.6.2 Rotational Mechanical System

Here we apply torque T(t) (in N-m), which is analogous to force in translation system. In place of mass we take moment of inertia J(in kg-m^2). Coefficient of viscous friction remains same as before i.e. f (in N-m/rad/sec) K is known as torsional constant here. In the similar way we write the equation for rotational mechanical system.



$$T(t) = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta$$

where θ = angular displacement in radians

and transfer function is given by

$$\frac{\theta(s)}{T(s)} = \frac{1}{(Js^2 + fs + K)}$$

If we take the output quantity in terms of angular velocity of shaft ω(t) in rad/sec, then

$$\omega(t) = \frac{d\theta}{dt} \text{ or } \omega(s) = s\theta(s)$$

$$\therefore \quad \frac{\omega(s)}{T(s)} = \frac{s}{Js^2 + fs + K}$$

Normally for rotational system, K = 0 then transfer function is given by

$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + fs}$$

or

$$\frac{\omega(s)}{T(s)} = \frac{1}{Js + f}$$

1.6.3 Force-Voltage Analogy

- Force f(t) is analogous to voltage v(t).
- Displacement x(t) is analogous to charge q(t).
- Mass M is analogous to L.
- Coefficient of viscous friction f is analogous to R.
- Spring constant K is anlogous to $\frac{1}{C}$.

1.6.4 Force Current Analogy

- Force f(t) is analogous to current i(t).
- Displacement x(t) is analogous to flux ψ.
- Mass M is analogous to capacitance C.
- Coefficient of viscous friction f is analogous to $\frac{1}{R}$.
- Spring constant K is analogous to $\frac{1}{L}$.

Analogy with Various Systems

Electrical	**Thermal**	**Mechanical**	**Liquid system (Hydraulic)**	**Pneumatic (air)**
Charge	Heat (joules)	Length (meter)	Volume of water flow $(meter)^3$	Volume of air flow $(meter)^3$
Voltage	Temperature (°C)	Force (Newton)	Head (meter)	Difference of pressure ($Newton/m^2$)
Current	Heat flow rate (joule/sec)	Velocity (meter/sec)	Liquid flow rate (m^3/sec)	Air flow rate ($meter^3/sec$)
Resistance	Resistance (°C-sec/joules)	Resistance Newton/sec/meter	Resistance (sec/m^2)	Resistance ($Newton\text{-}sec/m^5$)
Capacitance	Capacitance (joule/°C)	Capacitance (meter/Newton)	Capacitance ($meter^2$)	Capacitance ($m^5/Newton$)
Inductance	Inductance ($°C\text{-}sec^2/joules$)	Mass (kg.)	Inductance ($sec^2/meter^2$)	Inductance ($Newton\text{-}sec^2/m^5$)

Analogy between Electrical and Mechanical System

Force - voltage	**Force - current**	**Mechanical translatory system**	**Mechanical rotational system**
Voltage v(t)	Current	Force f(t)	Torque T(t)
Charge q(t)	flux	Displacement x(t)	Angular dispalcement θ(t)
Current i(t)	voltage	Velocity v(t) = x(t)	Angular velocity ω(t) = θ(t)
Inductance L	C	Mass M	Moment of Inertia J
Resistance R	$\frac{1}{R}$	Friction coefficient f	Friction coefficient f
Reciprocal of capacitance $\frac{1}{C}$	$\frac{1}{L}$	Spring constant K	Torsional constant K

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2 CHAPTER Time Response Analysis

THEORY

2.1 INTRODUCTION

Time response means how a system behaves in accordance with time when a specified, input test signal is applied.

The time response of a control system is divided in two parts namely

(i) Transient response

(ii) Steady state response

The transient part of time response reveals the nature of response. (i.e. oscillatory or overdamped) and also gives an indication about its speed.

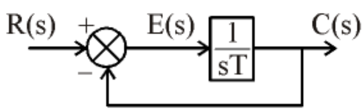
The steady state part of time response reveals the accuracy of a control system. Steady state error is observed if the actual output does not exactly match with the input.

Difference Between Transient Response and Steady State Response

Transient response	Steady state response
1. The time of response upto 3T or 4T is called transient response goes to zero as time becomes large.	1. Response after 4T is called steady state (T = time constant) i.e. the response which remains after the transient has died out, indicate the final accuracy of the system.
2. It does not depends on input signal.	2. It depends on the input signal applied.
3. It depends on poles and zeros of a system and reveals the nature of response and also give an indication about its speed.	3. The steady state part reveals the accuracy of a control system.

2.2 TIME RESPONSE OF A FIRST ORDER CONTROL SYSTEM

For a linear system, whenever highest power of s in the denominator of its transfer function equal to 1, it is called first order control system. The simple block diagram of a first order closed loop system is shown in figure below



Thus, a first order control system is expressed by a transfer function given by

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

(i) When the input is a unit step function

For a unit step function

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

Therefore the output of the system is expressed as

$$C(s) = \frac{1}{s}\cdot\frac{1}{sT+1} = \frac{1}{s} - \frac{1}{s+1/T}$$

or $$c(t) = 1 - e^{-t/T}$$

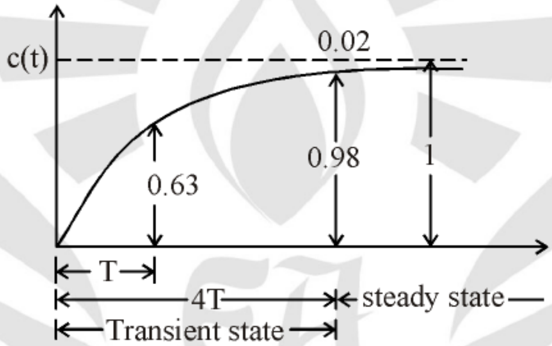
The error is given by

$$e(t) = r(t) - c(t) = 1 - (1 - e^{-t/T}) = e^{-t/T}$$

and steady state error is given by

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} e^{-t/T} = 0$$

The graphical representation of the time response is shown below. Figure shows that the response is exponential with steady state value of 1.



Key Points :

- *System having small time constant are fast i.e. for large time constant system is sluggish.*

$$\textit{Speed of system} \propto \frac{1}{\text{Time constant}}$$

- *Speed of the system is governed by the largest time constant of the system.*

(ii) When input is unit ramp function

For a unit ramp function $r(t) = t$

$\therefore$ $$R(s) = \frac{1}{s^2}$$

then the output of the system is expressed as

$$C(s) = \frac{1}{s^2}\cdot\frac{1}{sT+1} = \frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+(1/T)}$$

Taking inverse laplace transform on both sides, we get

$$c(t) = t - T + Te^{-t/T}$$

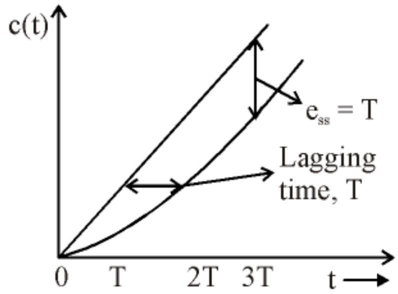
The error is given by

$$e(t) = r(t) - c(t) = t - (t - T + Te^{-t/T}) = T - Te^{-t/T}$$

and the steady state error is given by

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty}(T - te^{-t/T}) = T$$

It is a positional error, as output lags the input by time T, lower the time constant less the positional error and lesser time lag. The graphical representation of such a system is shown below.



(iii) When input is unit impulse function

For a unit impulse response

$$r(t) = \delta(t)$$

$$\therefore \quad R(s) = 1$$

The output of the system is expressed as

$$C(s) = 1.\frac{1}{sT+1} = \frac{1}{T}\left[\frac{1}{s+1/T}\right]$$

Taking inverse laplace transform on both the sides, we get,

$$c(t) = \frac{1}{T}e^{-t/T}$$

The error is given by

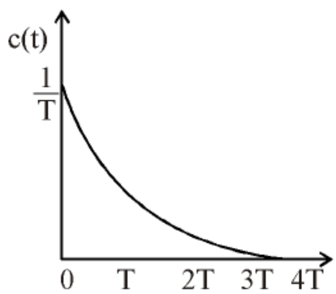
$$e(t) = r(t) - c(t)$$

and its laplace transform is given by

$$E(s) = R(s) - C(s) = 1 - \frac{1}{sT+1} = \frac{sT}{sT+1}$$

and the steady state error is given by

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \times \frac{sT}{sT+1} = 0$$



The time response of such a system is drawn above

Relation Between First Order System Responses

Input to the system	Response	Relation
Unit impulse 1	$e(t)=\frac{1}{T}e^{-t/T}$	Integration
Unit step $\frac{1}{s}$	$c(t)=1-e^{-t/T}$	Integration
Unit ramp $\frac{1}{s^2}$	$c(t)=t-T+Te^{-t/T}$	Integration

From the above table it is clear that unit step is the first derivative of unit ramp function and unit impulse is the first derivative of unit step function.

2.3 TIME RESPONSE OF A SECOND ORDER CONTROL SYSTEM FOR UNIT STEP INPUT

Here the highest power of s in the denominator of its transfer function equal to 2. The transfer function of a second order control system is given by the expression.

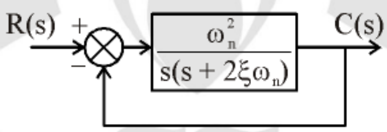
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Where, ω_n = Natural frequency of oscillation

ξ = Damping ratio

$\xi\omega_n$ = Damping factor or actual damping or damping coefficient.

Figure below shows a standard from of a second order control system in closed loop form.



Then, the output response of a second order system with unit step input is given by

$$c(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)$$

where $\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

We know error is given by

$$e(t) = r(t) - c(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)$$

and steady state error is given by

$$e_{ss} = \lim_{t\to\infty}\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$e_{ss} = 0$$

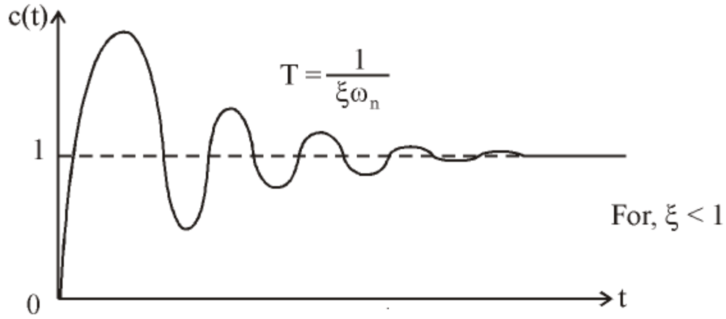
From above analysis it is clear that time response of second order control system is influenced by damping ratio ξ. Let us consider the following cases for different values of ξ.

(a) Underdamped System

When $\xi < 1$, then the response is called underdamped response.

For this value of damping ratio ξ, oscillations occur in the system is transient period and these oscillations decays with time depending on the value of ξ (Higher the value of ξ, faster the damping). The steady state error of this system approaches zero in a time 4T.

For underdamped system, the response of a second order system for unit step input is drawn below.



(b) Undamped System

When $\xi = 0$ then system has sustained or undamped oscillations.

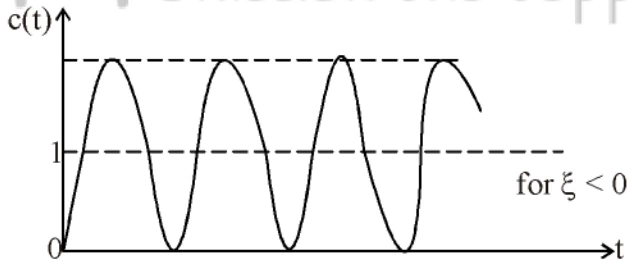
For this value of ξ, there is no damping observed in the response of the system. We know output response of a second order system is given by.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Putting, $\xi = 0$ for undamped oscillations, in the above equation,

we get, $$c(t) = 1 - \sin(\omega_n t + \tan^{-1}\infty)$$
$$= 1 - \cos\omega_n t$$

For undamped oscillations, the response of the system is plotted below.



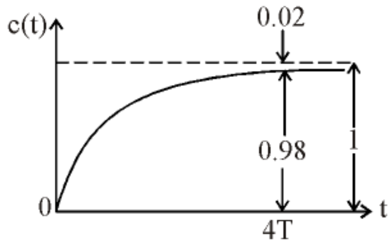
(c) Critically Damped System

When, $\xi = 1$ the system has critically damped response. (i.e. non-oscillatory).

For this value of ξ oscillations just disappeared. The speed of critically damped system is faster than the overdamped system ($\xi > 1$), but slower than the underdamped system ($\xi < 1$). The ratio between the actual damping and critical damping is known as the damping ratio and is given by the following relation.

$$\frac{\text{Actual damping}}{\text{Critical damping}} = \frac{\xi\omega_n}{\omega_n} = \xi = \text{Damping ratio}$$

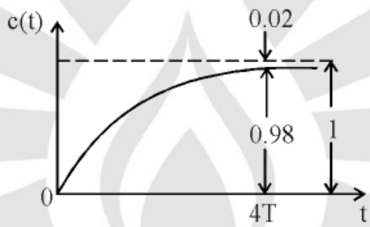
For critically damped system the response for a second order system for unit step input is plotted below. The response is increasing monotonically from zero value towards the steady state value of unity.



(d) Overdamped System

When, $\xi > 1$ the response is called overdamped response.

For an overdamped system, the response of a second order system subjected to unit step input is plotted below. The response is increasing monotonically from zero value towards the steady state value which is unity.



2.4 CHARACTERISTIC EQUATION

As we know the general expression for the transfer function of a second order control system is given by:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

The denominator of the above expression is the characteristic equation.

i.e. $s^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$

The roots of the above quadratic equation are

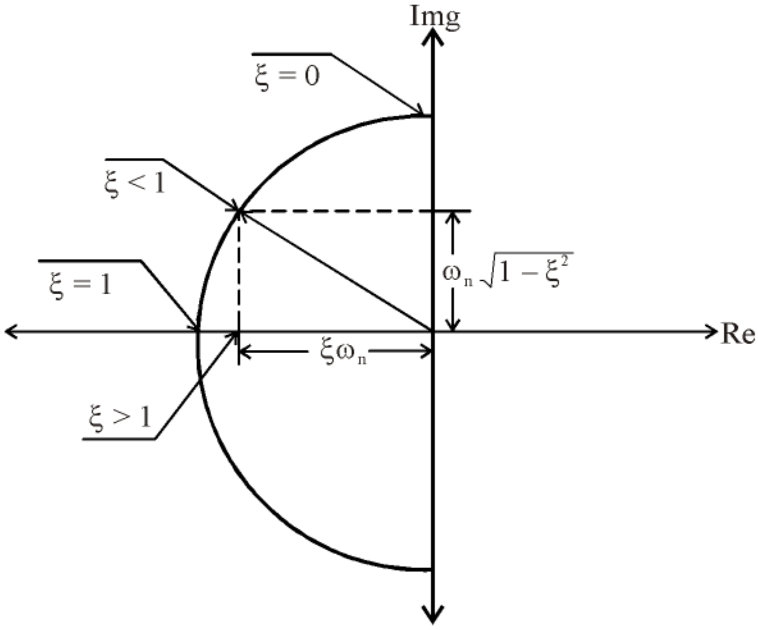
$$s_1,\ s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

The roots s_1 and s_2 gives information about the nature of the time response. The real part of the roots represent the damping and the imaginary part represents the damped frequency of oscillation of the system.

The plot of values of these roots for different ξ is called root locus. For second order system or real system root locus exists always in negative s-plane.

Imaginary part is responsible for oscillations and if there is no imaginary part then oscillations are observed. Sustained oscillations are obtained when poles are lying on imaginary axis.

Figure below shows the time response of second order control response for different values of ξ.

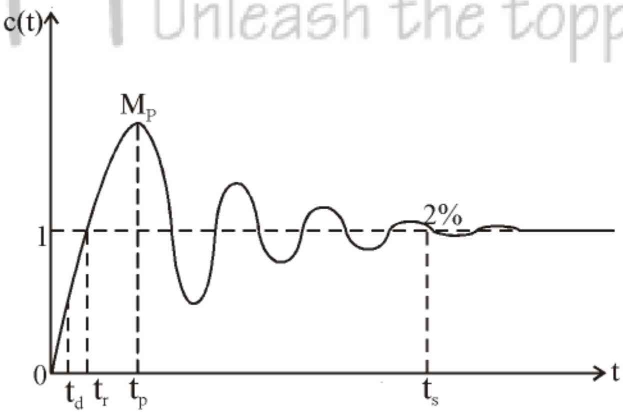


Different values of Damping ratio, ξ

$\xi = 0$	–	Roots are imaginary	–	Undamped system
$\xi < 1$	–	Roots are complex	–	Underdamped system
$\xi = 1$	–	Roots are real and same	–	Critically damped system
$\xi > 1$	–	Roots are real and different	–	Overdamped system

2.5 TRANSIENT RESPONSE SPECIFICATION OF SECOND ORDER CONTROL SYSTEM (UNDERDAMPED)

The transient response of an underdamped control system consists of oscillation which are damped out with time. Figure below shows the time response during transient part.



1. Delay Time t_d

It is the time required for the response to reach 50% the final value in first attempt.

2. Rise Time t_r

It is the time required for the response to rise from 0 to 100% of its final value. (while 10% to 90% rise for overdamped system). It is given by the following relation.

$$t_r = \frac{\pi - \phi}{\omega_n \sqrt{1-\xi^2}}$$

where $$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

3. Peak Time t_p

It is the time required for the response to reach the peak value at first instant. It is also defined as the time required to reach the maximum overshoot is called peak time. Peak time is given by the relation.

$$t_P = \frac{n\pi}{\omega_n \sqrt{1-\xi^2}}$$

Here we take, $n = 1$ for first overshoot,

$n = 2$ for first undershoot.

$n = 3$ for second overshoot and so on.

4. Maximum Overshoot M_P

The maximum positive deviation of the output w.r.t. its desired value is called as maximum overshoot. It is denoted by M_P and is given by

$$M_P(\%) = \frac{c(t_P) - c(\infty)}{c(\infty)} \times 100\%$$

For step input $c(\infty) = 1$

Maximum overshoot is given by expression.

$$M_P(\%) = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100\%$$

5. Settling Time t_s

It is the time required by the system to reach and stay within a specified tolerance band (2% or 5%) of its final value. The settling time for a second order control system is approximated as four times the time constant of the system for 2% of tolerance band.

i.e. $$t_s = \frac{4}{\xi\omega_n} \text{ (for 2\% band)}$$

and for 5% tolerance band, settling time is three times the time constant of the system.

i.e. $$t_s = \frac{3}{\xi\omega_n} \text{ (for 5\% band)}$$

2.6 TIME RESPONSE OF SECOND ORDER CONTROL SYSTEM FOR UNIT RAMP INPUT

The output response of second order control system for ramp input is given by

$$c(t) = t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\left[2\xi\sqrt{1-\xi^2}\cos\omega_n\sqrt{1-\xi^2}\,t + (2\xi^2-1)\sin\omega_n\sqrt{1-\xi^2}\,t\right]$$

Putting $$2\xi\sqrt{1-\xi^2} = \sin\phi$$

$$2\xi^2 - 1 = \cos\phi$$

$$c(t) = t - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t + \phi\right]$$

We know, the error signal is given by

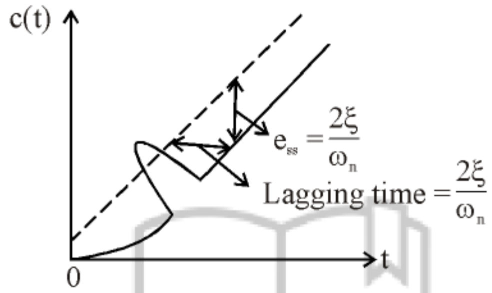
$$e(t) = r(t) - c(t) = \frac{2\xi}{\omega_n} - \frac{e^{-\xi\omega_n t}}{\omega_n\sqrt{1-\xi^2}}\sin\left[\omega_n\sqrt{1-\xi^2}\,t + \phi\right]$$

And steady state error is given by

$$e_{ss}(t) = \frac{2\xi}{\omega_n}$$

The steady state error can be decreased either by decreasing ξ or increasing ω_n

The response of a second order control system subjected to unit ramp function is drawn in figure below.



From above figure it is clear that steady state error is $\frac{2\xi}{\omega_n}$ and output response lags by a time of $\frac{2\xi}{\omega_n}$.

2.7 TIME RESPONSE OF SECOND ORDER CONTROL SYSTEM FOR UNIT IMPULSE INPUT

The output response is given by:

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t\right)$$

Therefore, the time response for different values of ξ.

Static Error Coefficients for Different Inputs

Input	Laplace Transform of Input	Type '0'		Type '1'		Type '2'	
		Static error coefficient	e_{ss}	Static error coefficient	e_{ss}	Static error coefficient	e_{ss}
Unit step	$\frac{1}{s}$	$K_p = K$	$\frac{1}{1+K}$	$K_p = \infty$	0	$K_p = \infty$	0
Unit ramp	$\frac{1}{s^2}$	$K_v = 0$	∞	$K_v = K$	$\frac{1}{K}$	$K_v = \infty$	0
Unit parabolic	$\frac{1}{s^3}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = 0$	$\frac{1}{K}$

From the above table we observe that

(i) For type '0' system the ramp and parabolic inputs are not acceptable.

(ii) For type '1' system the parabolic input is not applicable.

(iii) For type '2' system all the three inputs are acceptable.

2.8 SENSITIVITY

Sensitivity gives an assessment of the system performance as affected due to parameter variations. The use of feedback in a control system reduces the effect of parametric variations.

Let A is a variable in control system which changes its value and this change is considered due to parametric variation of K such as gain. Then sensitivity of control system is expressed as

$$\text{Sensitivity} = \frac{\%\text{ Change in A}}{\%\text{ Change in K}}$$

$$S_K^A = \frac{\partial A / A}{\partial K / K}$$

Sensitivity should be as minimum as possible for stability.

(a) Sensitivity of overall transfer function M(s) with respect to forward path transfer function G(s).

(i) For open loop system $S_G^T = 1$

(ii) For close loop system $S_G^M = \dfrac{1}{1+G(s)H(s)}$

(b) Sensitivity of overall transfer function M(s) with respect to feedback path H(s)

$$S_H^M = \frac{-G(s)H(s)}{1+G(s)H(s)}$$

Comparing above sensitivity relations it is concluded that a closed loop control system is more sensitive to variations in feedback path parameters H(s) than the variations in forward path parameters.

Therefore feedback elements should be more rigid as compared to that of forward path elements.

❍❍❍