



**UKPSC – JE**

**Mechanical Engineers**

**Uttarakhand Public Service Commission (UKPSC)**

**Volume - 3**

**(Technical)**

**Fluid Mechanics**



# INDEX

S No.	Chapter Title	Page No.
1	Introduction	1
2	Pressure & it's Measurement	20
3	Buoyancy and Floatation	33
4	Fluid Kinematics	52
5	Fluid Dynamics	73
6	Laminar Fow in Pipes & Wall	97
7	Pipe Flow	116
8	Boundary Layer Theory	135
9	Dimensional Analysis	153
10	Impact of JET	176
11	Hydraulic Turbines	187
12	Pumps	211

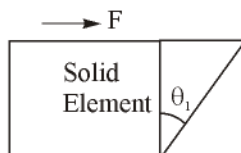
# 1 CHAPTER

## Introduction

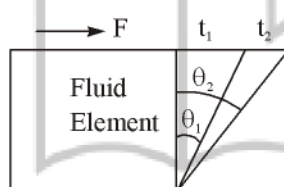
### THEORY

#### 1.1 FLUID

The substances which continuously deform under the application of a tangential or shear stress no matter how small is the value of tangential force is known as fluid.



$\theta$  is the deformation at time  $t_1$  and  $t_2$  if force ( $F$ ) is constant



$\theta_1$  at time  $t_1$ ;  $\theta_2$  at time  $t_2$  if  $t_2 > t_1$  then  $\theta_2 > \theta_1$  for fluid element

where,

$\theta_1$  = Deformation at time  $t_1$

$\theta_2$  = Deformation at time  $t_2$

#### 1.2 PROPERTIES OF FLUID

##### 1.2.1 Density or Mass Density

Mass density is define as the mass of the system corresponding to its volume.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$\text{Unit} = \left( \frac{\text{kg}}{\text{m}^3} \right)$$

$$\text{Dimensional formula} = [M^1 L^{-3} T^0]$$

For ideal gases

$$pV = mRT$$

$$p = \frac{m}{V} RT$$

$$P = \rho RT$$

where, R is the gas constant

$$R = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \text{ for air.}$$

**Note:**

- Density is absolute quantity.
- Density of gas is dependent on the temperature and pressure of the system.

### 1.2.2 Specific Gravity (S)

$$S = \frac{\text{density of substance}}{\text{density of standard fluid}}$$

**Note:**

- For liquid standard fluid is water (1000 kg/m<sup>3</sup>)
- For gases standard fluid is either air or hydrogen gas.
- Specific gravity of Hg is 13.6

### 1.2.3 Relative Density (R.D.)

$$\text{Relative density (R.D.)} = \frac{\text{density of one fluid}}{\text{density of another fluid}}$$

$$\text{R.D.} = \frac{\rho_1}{\rho_2}$$

### 1.2.4 Specific Weight or Weight Density (w or γ):

It is define as the ratio of weight of system to the volume of the system.

$$w = \frac{\text{weight}}{\text{volume}} = \frac{\rho g \nabla}{\nabla} = \rho g$$

$$\text{Unit} \left( \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}}{\text{sec}^2} \right) = \left( \frac{\text{kg}}{\text{m}^2 \cdot \text{sec}^2} \right)$$

$$\text{Dimensional formula} = [M^1 L^{-2} T^{-2}]$$

**Note:**

Specific weight is the relative quantity and it depends on the pressure, temperature and location.

### 1.2.5 Bulk Modulus and Compressibility

If original pressure is P in a liquid mass and its volume is ∇ and increase in pressure dP causes change in volume d∇ then

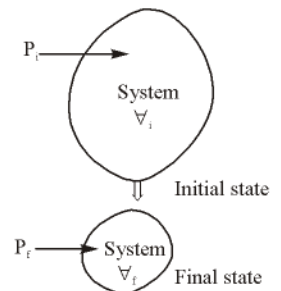
$$dP = P_f - P_i$$

$$d\nabla = \nabla_f - \nabla_i = -(\nabla_i - \nabla_f)$$

$$\text{Bulk modulus of elasticity (K)} = -\frac{dP}{(d\nabla/\nabla)}$$

Unit of K will same as pressure (Pa, N/m<sup>2</sup>)

[analogous to  $\frac{\text{stress}}{\text{strain}} = \text{Young's modulus of elasticity}$ ]



$$K = - \frac{dP}{\left(\frac{dV}{V}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}$$

$$\rho V = \text{mass} = \text{constant}$$

$$\rho dV + V d\rho = 0$$

$$\Rightarrow \frac{d\rho}{\rho} = - \frac{dV}{V}$$

$$\text{Compressibility} = \frac{1}{K} = \frac{1}{\rho} \frac{d\rho}{dP}$$

*Note* : If density does not change with pressure i.e.,  $\frac{d\rho}{dP} = 0$  (fluid is incompressible with respect to pressure).

### 1.2.6 Isothermal Bulk Modulus ( $K_T$ )

For ideal gas

$$P = \rho RT$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\Rightarrow \left(\frac{dP}{d\rho}\right)_{T=\text{Constant}} = RT$$

$$\Rightarrow K_T = \rho \frac{dP}{d\rho} = \rho \cdot RT = P$$

$\Rightarrow$  for isothermal condition in an ideal gas,

$$K_T = P$$

i.e.,

$$\boxed{\text{Isothermal bulk modulus} = \text{pressure}}$$

### 1.2.7 Adiabatic Bulk Modulus ( $K_A$ )

For adiabatic condition

$$pV^\gamma = \text{constant}$$

where,

$\gamma =$  adiabatic index

$$= \frac{C_p}{C_v} = \frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$$

$$\Rightarrow P \left(\frac{m}{\rho}\right)^\gamma = \text{constant}$$

$$\frac{P}{\rho^\gamma} = \text{constant (because mass 'm' is constant)}$$

$$\Rightarrow \boxed{P = C\rho^\gamma}$$

$$\frac{dP}{d\rho} = \rho C \gamma \rho^{\gamma-1}$$

$$K_A = \rho \frac{dP}{d\rho} = \gamma C \rho^\gamma = \gamma P$$

$$\Rightarrow \boxed{\text{Adiabatic bulk modulus} = \gamma P = \text{Adiabatic index} \times \text{Pressure}}$$

**Example 1 :** The bulk modulus of water is  $210 \times 10^4 \text{ kN/m}^2$ . What pressure is required to reduce the volume of water in a closed container by 2%; Also show that the increase in mass density because of the reduction in volume by 2% will be 2% only.

**Solution :** Given

$$k = 210 \times 10^4 \text{ kN/m}^2$$

$$\frac{dV}{V} = 0.02$$

From the definition of bulk modulus of elasticity

$$k = -\frac{dp}{\left(\frac{dV}{V}\right)} \quad \dots(1)$$

For 2% reduction in volume

$$\left(\frac{dV}{V}\right) = 0.02 \quad \dots(2)$$

$$\therefore 210 \times 10^4 \text{ kN/m}^2 = \frac{dP}{(0.02)}$$

$$\Rightarrow dP = 210 \times 10^4 \times 0.02 \text{ kN/m}^2$$

$$\therefore dP = 42,000 \text{ kN/m}^2$$

$$dP = 42 \text{ MN/m}^2$$

Thus additional pressure required to reduce the volume in closed container by 2% is 42 MN/m<sup>2</sup>

$\therefore$  For constant mass in the closed container,

$$\begin{aligned} \frac{d\rho}{\rho} &= -\left(\frac{dV}{V}\right) \\ &= -(-2\%) = 2\% \end{aligned}$$

Thus reduction in volume by 2% will lead to increase in mass density by 2%.

**Example 2 :** An empirical pressure-density relation for water in  $\frac{P}{P_a} = 3001 \left(\frac{\rho}{\rho_a}\right)^7 - 3000$  where suffix 'a' refers to atmospheric conditions. Determine the bulk modulus of elasticity and compressibility of water at 5 atmospheric pressure.

**Solution :** 
$$\frac{P}{P_a} = 3001 \left(\frac{\rho}{\rho_a}\right)^7 - 3000 \quad \dots(1)$$

differentiating both side.

$$\frac{dP}{P_a} = \frac{3001 \times 7 \cdot \rho^6 d\rho}{\rho_a^7} \Rightarrow \frac{dP}{d\rho} = \frac{3001 \times 7 \times \rho^6 \times P_a}{\rho_a^7} \quad \dots(2)$$

We know that,

$$K = \frac{-dP}{\left(\frac{dv}{v}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)} = \left(\frac{\rho dP}{d\rho}\right) \quad \dots(3)$$

Put the value of  $\frac{dP}{d\rho}$  from (2) in (3)

$$\therefore K = \frac{3001 \times 7 \times \rho^6 \cdot \rho}{\rho_a^7} P_a$$

$$\therefore K = 3001 \times 7 \left( \frac{\rho}{\rho_a} \right)^7 \times P_a \quad \dots(4)$$

Put the value of (1) in (4)

$$K = 3001 \times 7 \frac{\left( \frac{P}{P_a} + 3000 \right)}{3001} \times P_a$$

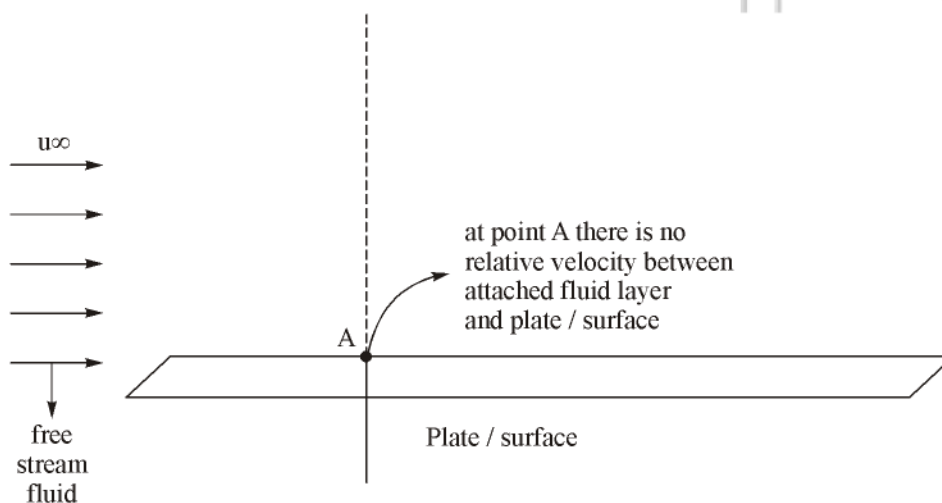
$$\begin{aligned} K &= 3001 \times 7 \frac{[5 + 3000]}{3001} \times P_a \\ &= 7 \times 101.325 \times 3005 \quad (\text{where } P_{\text{atm}} = 101.325 \text{ kN/m}^2) \\ &= 2.131 \times 10^6 \text{ KN/m}^2 \end{aligned}$$

$$\therefore \text{The compressibility} = \left( \frac{1}{K} \right) = \frac{1}{(2.131 \times 10^6)} = 4.693 \times 10^{-7} \text{ m}^2/\text{KN}$$

### 1.3 NO SLIP CONDITION

It has been established that the relative velocity between the solid surface and the adjacent fluid particle is zero when ever a viscous fluid flows over a solid surface. This is known as no-slip condition. This behavior of no-slip at the solid surface is not the same as the wetting of surface by the fluids. For example mercury flowing in a stationary glass tube will not wet the surface, but will have zero velocity at the wall of the tube. The wetting property results from surface tension, whereas the no-slip condition is a consequence of fluid viscosity.

In ideal fluid as there is no viscosity hence there will be no no-slip condition existing.



## 1.4 DYNAMIC VISCOSITY OR ABSOLUTE VISCOSITY

Dynamic viscosity is define as the property of the fluid which develop the internal resistance between two adjacent layer of fluid when they have a relative motion.

$$\tan \theta = \frac{du \Delta t}{dy}$$

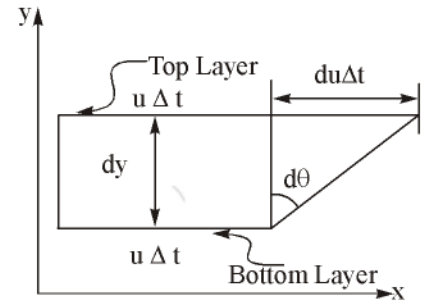
if,  $\Delta t \rightarrow 0$

$$\tan d\theta \approx d\theta$$

$$d\theta = \frac{du dt}{dy}$$

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Rate of change of shear strain  $\left(\frac{d\theta}{dt}\right) = \text{velocity gradient} \left(\frac{du}{dy}\right)$



## 1.5 NEWTON'S LAW OF VISCOSITY

According to Newton's Law of viscosity shear stress is directly proportional to the shear strain rate or velocity gradient in transverse direction.

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

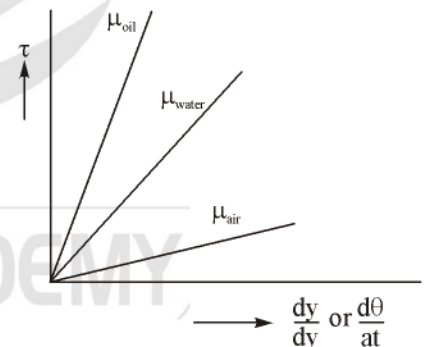
where,  $\mu$  is the dynamic viscosity of fluid.

*Note* : A fluid which obey Newton's Law of viscosity is known as Newtonian fluid.

### 1.5.1 Units of Dynamic Viscosity

- (i) Pa.sec in SI unit
- (ii)  $\frac{\text{kg}}{\text{m} \cdot \text{sec}}$  in M.K.S. unit
- (iii) 1 poise =  $10^{-1} \frac{\text{kg}}{\text{m} \cdot \text{sec}}$  in CGS

Dimensional formula =  $[M^1 L^{-1} T^{-1}]$



### 1.5.2 Effect of Temperature on Viscosity of Fluid

- a) In case of liquid, cohesive forces between molecules cause viscosity. Hence at higher temperature when molecules possesses more energy, cohesive force become weak. As a result, energized liquid molecule can move more freely, thus viscosity of liquid decreases with increase in temperature.

$$\mu_{\text{liq}} = \frac{\mu_0}{1 + \alpha t + \beta t^2}$$

$\alpha$  and  $\beta$  are constants,

$t$  = temperature in  $^{\circ}\text{C}$ .

$\mu_0$  = viscosity of liquid at  $0^{\circ}\text{C}$

For water

$$\mu_0 = 1.79 \times 10^{-3} \text{ poise}$$

$$\alpha = 0.03368$$

$$\beta = 0.000221$$

- b) In case of gases, viscosity is caused by molecular collisions. At high temperature, more molecular collisions per unit volume per unit time occurs. This results in greater resistance to flow. Thus, Viscosity increases with temperature

$$\mu_{\text{gas}} = \frac{a\sqrt{T}}{1 + \frac{b}{T}}$$

or

$$\mu_{\text{gas}} = \mu_0 + \alpha t - \beta t^2$$

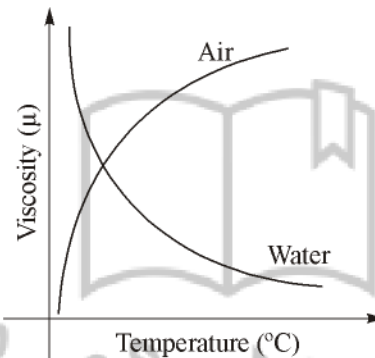
For Air

$$\mu_0 = 0.000017$$

$$\alpha = 0.000000056$$

$$\beta = 0.1189 \times 10^{-9}$$

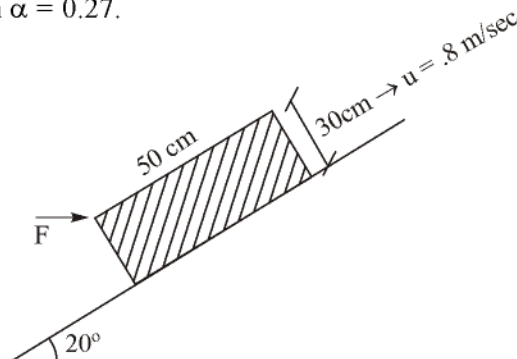
*Note :* At constant pressure, if temperature is increased 'μ' increases.



### 1.5.3 Effect of Pressure on Viscosity

For liquids, viscosity is practically independent of pressure except at extremely high pressure. For gases, dynamic viscosity is generally independent of pressure particularly (at low to moderate pressure) but kinematic viscosity decreases as density is proportional to pressure.

**Example 3 :** A 50cm × 30cm × 20cm block weighing 150N is to be moved upon inclined plane at constant velocity of 0.8 m/sec coefficient of friction  $\alpha = 0.27$ .



- Determine the force  $F$  required to be applied in horizontal direction to maintain this motion.
- If a 0.4 mm thick oil film with dynamic viscosity  $\mu = 0.012 \text{ Pa}\cdot\text{sec}$  is applied between the block and inclined surface determine the percentage reduction, in the required horizontal force.

**Solution : (a)**

$$\therefore \Sigma F_y = 0 \Rightarrow \text{because of constant volume} \Rightarrow (\text{acceleration} = 0)$$

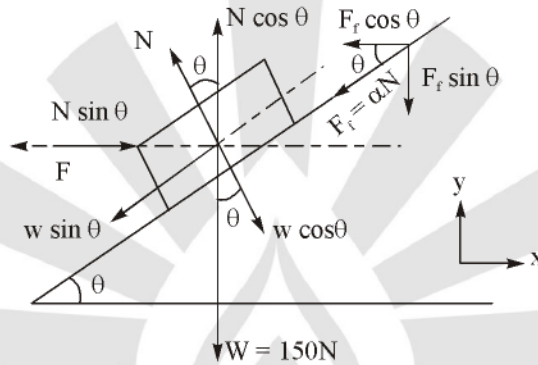
$$\therefore N \cos \theta - W - F_f \sin \theta = 0$$

$$N \cos \theta - W - \alpha N \sin \theta = 0$$

$$\therefore \frac{F_f}{N} = \alpha = \tan \theta \quad \dots(i)$$

$$\Rightarrow N(\cos \theta - \alpha \sin \theta) = W \quad F_f = N \cdot \alpha$$

$$\therefore N = \frac{W}{(\cos \theta - \alpha \sin \theta)}$$



$$= \frac{150}{(\cos 20^\circ - 0.27 \sin 20^\circ)} = 177.023 \quad \dots(ii)$$

$$\Sigma F_x = 0$$

$$\therefore F - N \sin \theta - F_f \cos \theta = 0 \quad \dots(iii)$$

$$F = N \sin \theta + \alpha N \cos \theta$$

$$= N(\sin \theta + \alpha \cos \theta)$$

$$F = 177.023(\sin 20^\circ + 0.27 \cos 20^\circ)$$

$$F = 105.5 \text{ N} \quad \dots(iv)$$

$$(b) \therefore \left(\frac{du}{dy}\right) = \left(\frac{u}{d}\right)$$

$$\therefore \tau = \mu \left(\frac{du}{dy}\right) = \left(\frac{\mu u}{d}\right)$$

$$\therefore F_{\text{shear}} = (\text{Shear stress} \times \text{Area of contact})$$

$$= 0.012 \times \left(\frac{0.8}{0.4 \times 10^{-3}}\right) \times 50 \times 20 \times 10^{-4}$$

$$= 2.4 \text{ N} \quad \dots(v)$$

Applying N.S.L. along the plane

$$F_{\text{net}} = m \times a$$

$$F \cos 20^\circ - W \sin 20^\circ - \tau \times A = m \times 0$$

$$F \cos 20^\circ = W \sin 20^\circ + \frac{\mu V}{h} \cdot A$$

$$F \cos 20^\circ = 150 \sin 20^\circ + \frac{0.012 \times 0.8 \times 50 \times 20 \times 10^{-4}}{0.4 \times 10^{-3}}$$

$$F \cos 20^\circ = 53.70$$

$$F = 57.15 \text{ N}$$

$$\therefore \text{Percent reduction} = \left( \frac{105.5 - 57.149}{105.5} \right) \times 100$$

$$= 45.829\%$$

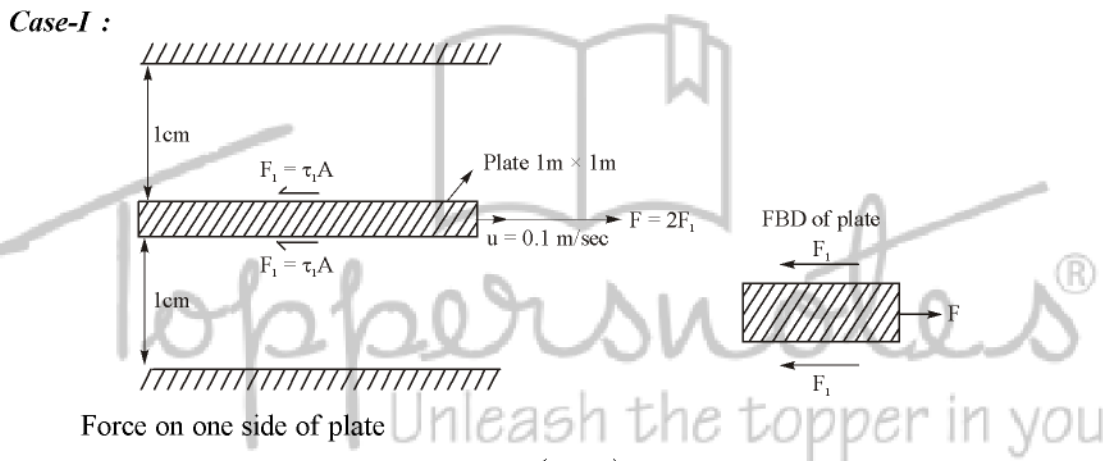
**Example 4 :** A thin square plate  $1\text{m} \times 1\text{m}$  is placed horizontally in a gap of height  $2\text{cm}$ . Filled with oil of viscosity  $10$  Poise and pulled at a constant velocity of  $0.10$  m/sec. Find the force on the plate.

The gap is now filled with another oil, when the plate is placed at a distance of  $0.5\text{cm}$ , from one of the surfaces of the gap and pulled with the same velocity, the force on the plate remains same as before. Find viscosity of new oil.

**Solution :** Data given:

$$\mu = 10 \text{ Poise} = 1.0 \frac{\text{N.s}}{\text{m}^2}$$

$$u = 0.1 \text{ m/sec.}$$



$$\Sigma F = m \vec{a} \quad (\vec{a} = 0)$$

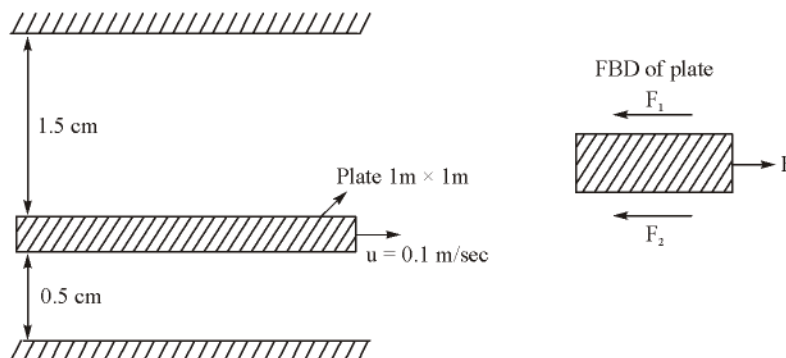
$$F_1 = \mu \left( \frac{du}{dy} \right) \times A = 1 \times \frac{0.1}{(1 \times 10^{-2})} \times 1 = 10 \text{ N}$$

$\therefore$

$$F = 2F_1 = 2 \times 10 = 20 \text{ Newton}$$

...(1)

**Case-II :**



$$\text{Force on top of plate} = \mu \left( \frac{dy}{dy} \right)_{\text{top}} \times \text{Area}$$

$$= \mu \frac{0.1}{1.5 \times 10^{-2}} \times 1 = \frac{20}{3} \mu \quad \dots(2)$$

Force on bottom of plate

$$= \mu \left( \frac{du}{dy} \right)_{\text{bottom}} \times \text{Area} = \mu \frac{0.1}{(0.5 \times 10^{-2})} \times 1 = 20\mu$$

$$\text{Total force in Case (2)} = \left( \frac{20}{3} \mu + 20\mu \right) = \left( \frac{80}{3} \mu \right) \quad \dots(3)$$

From (1) and (3)

$$\frac{80}{3} \mu = 20$$

$$\therefore \mu = \left( \frac{3}{4} \right) = 0.75 \left( \frac{\text{N.s}}{\text{m}^2} \right) \quad \text{Ans.}$$

**Example 5 :** Through a narrow gap of height  $h$ , a thin plate of large extent is pulled at a velocity  $V$ , at one side of the plate. Oil of viscosity  $\mu_1$ , and on other side  $\mu_2$ . Calculate the position of plate so that

- Shear force on two sides of the plate are equal.
- The pull required to drag to plate is minimum.

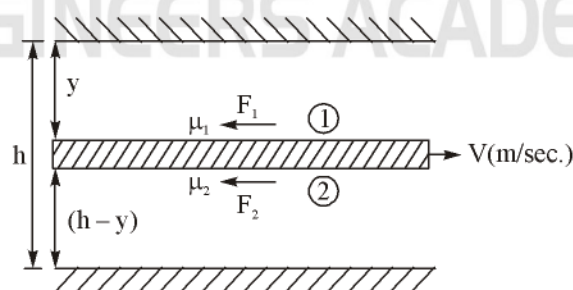
**Solution :**(a)  $\therefore$  Shear force on side (1) of plate

$$\Sigma F = ma \quad (a = 0)$$

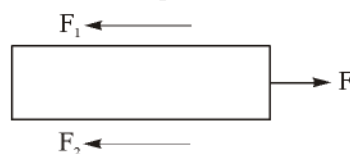
$$F_1 = \tau_1 A = \left( \frac{\mu_1 AV}{y} \right) \quad \dots(1)$$

Shear force on other side of plate

$$F_2 = \tau_2 A = \left( \frac{\mu_2 AV}{h-y} \right) \quad \dots(2)$$



FBD of plate



(a) From (1) and (2)

$$\frac{\mu_1 AV}{y} = \frac{\mu_2 AV}{(h-y)}$$

$$\Rightarrow \mu_1 h - \mu_1 y = \mu_2 y$$

$$\Rightarrow \mu_1 h = (\mu_1 + \mu_2)y$$

$$\therefore y = \frac{\mu_1 h}{(\mu_1 + \mu_2)} \quad \dots(3)$$

(b) Pull required to drag the plate

$$\therefore \text{total drag force } F_D = \frac{\mu_1 AV}{y} + \left( \frac{\mu_2 VA}{h-y} \right)$$

For  $F_D$  to be minimum.

$$\left( \frac{dF_D}{dy} \right) = 0$$

For minima of maxima.

$$\therefore \frac{dF_D}{dy} = -\frac{\mu_1 VA}{y^2} + \frac{\mu_2 AV}{(h-y)^2} = 0$$

$$\therefore \frac{\mu_2}{(h-y)^2} = \frac{\mu_1}{y^2}$$

$$\Rightarrow \left( \frac{\mu_2}{\mu_1} \right) = \left( \frac{h-y}{y} \right)^2 \quad (\text{where } \mu_1 \text{ and } \mu_2 \text{ are dynamic viscosities})$$

$$\therefore \frac{h-y}{y} = \sqrt{\left( \frac{\mu_2}{\mu_1} \right)}$$

$$\Rightarrow h\sqrt{\mu_1} - y\sqrt{\mu_1} = \sqrt{\mu_2}y$$

$$\Rightarrow h\sqrt{\mu_1} = y(\sqrt{\mu_1} + \sqrt{\mu_2})$$

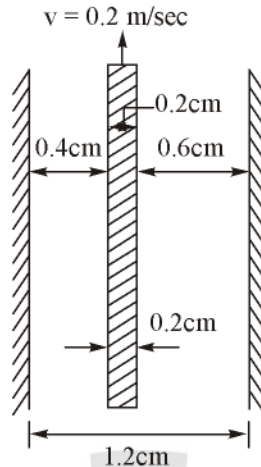
$$\Rightarrow y = \frac{h\sqrt{\mu_1}}{(\sqrt{\mu_1} + \sqrt{\mu_2})}$$

**Example 6 :** A horizontal gap 1.2 cm. Wide of infinite extent contains fluid of viscosity 1 N.S/m<sup>2</sup> and specific gravity 0.9. A metallic plate 1m × 1m × 0.2 cm is lifted up with a constant velocity of 0.2 m/sec through the gap. If the plate is at a distance of 0.4 cm, from one of the plane surface of the gap, find the vertical force required. Weight of plate is 50N.

**Solution :** As the given plate is submerged in the fluid of specific gravity (0.9)

Total forces acting on the plate are (a) Submerged weight in downward direction of plate (b) Drag force so total force required to life the plate

$$= \text{Submerged wt. of plate} + \text{total drag on the plate}$$



$$\begin{aligned}
 \text{Submerged wt. of the plate} &= \text{wt} - (\text{sp. gravity}) \times \gamma_w \times \text{vol. of plate} \\
 &= 50 - 0.9 \times 9.81 \times 10^3 \times (1 \times 1 \times 0.2 \times 10^{-2}) \\
 &= 32.342 \text{ N}
 \end{aligned}$$

and total drag force on the plate

$$\begin{aligned}
 \Rightarrow F_D &= \mu \cdot \left( \frac{du}{dy} \right) \times (\text{Area}) \\
 &= \left\{ 1 \cdot \left( \frac{0.2}{0.4 \times 10^{-2}} \right) \times 1 \times 1 \right\} + \left\{ 1 \cdot \frac{0.2}{0.6 \times 10^{-2}} \times 1 \times 1 \right\} \\
 &= 50 + \frac{100}{3} = 83.33 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence total force required to lift the with constant velocity} \\
 &= 32.342 + 83.33 \\
 &= 115.672 \text{ N.}
 \end{aligned}$$

## 1.6 RHEOLOGY

It is a branch of science in which we study different types of non-newtonian fluids on the basis of their behavior (flow behavior)

### 1.6.1 Non Newtonian Fluid

A fluid which do not obey Newton's law of viscosity is known as Non-Newtonian fluid. It obey power law relation

$$\tau = A \left( \frac{du}{dy} \right)^n + B$$

where

A = consistency index

n = flow behavior index

or

$$\tau = A \left( \frac{du}{dy} \right)^{n-1} \left( \frac{du}{dy} \right)$$

(μ<sub>aparent</sub>)

#### a) Pseudo Plastic Fluid :

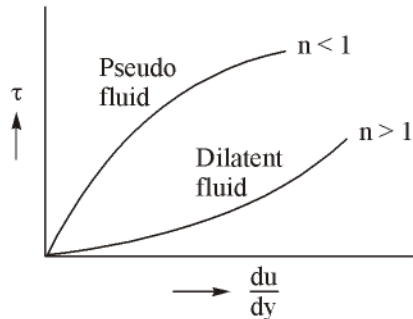
For B = 0 and n < 1 and fluid is also known as shear thinning fluid.

*Example:* Milk, blood, ketch up

**b) Dilatant Fluids (Shear Thickening Fluid) :**

$B = 0; n > 1$ ; Dilatant fluid is known as shear thickening fluid.

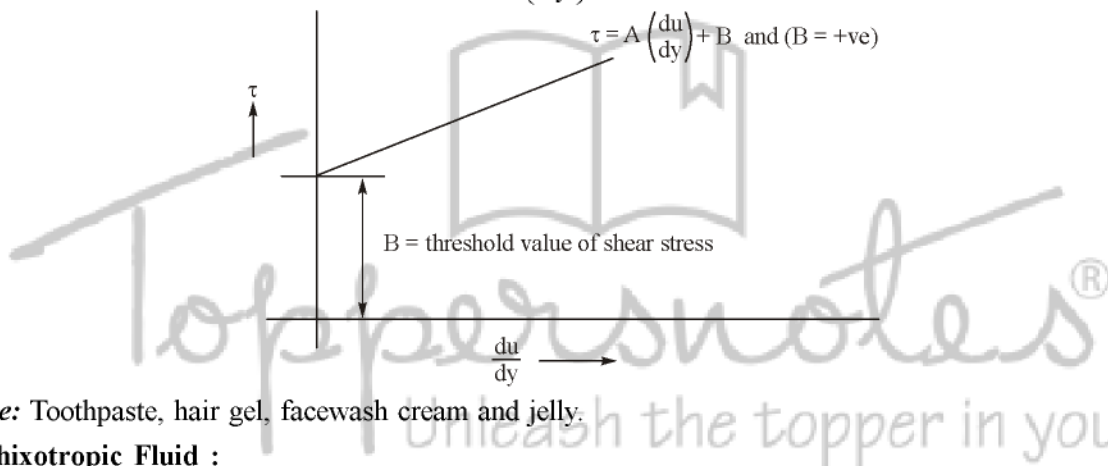
*Example:* honey, saturated solution of sugar, rice-starch solution.



**c) Bingham Plastic Fluids (Ideal Plastic Fluid):**

$$n = 1 ; B \neq 0$$

$$\tau = A \left( \frac{du}{dy} \right) + B \text{ and } (B = +ve)$$

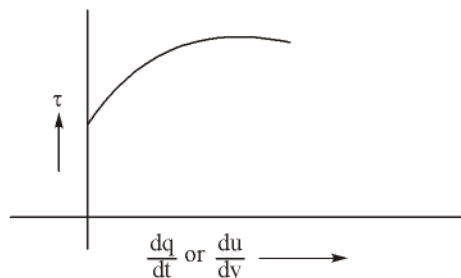


*Example:* Toothpaste, hair gel, facewash cream and jelly.

**d) Thixotropic Fluid :**

Thixotropic fluid is defined as the apparent viscosity increases with respect to time.

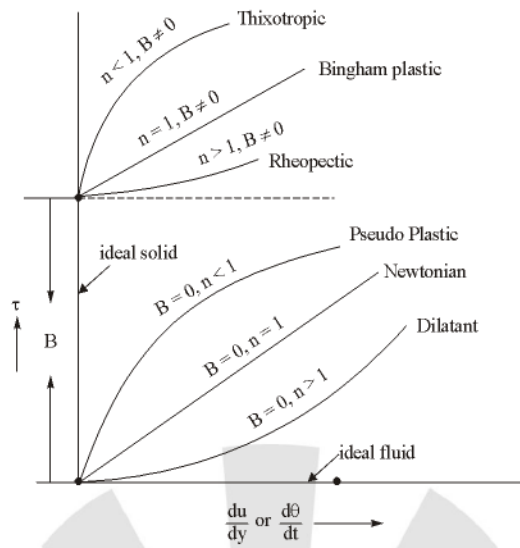
*Example:* Drilling water, castrol oil and antibiotic.



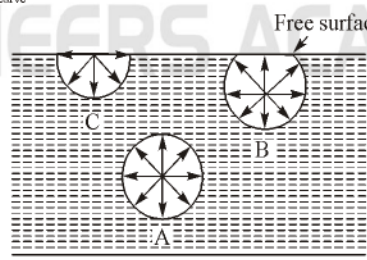
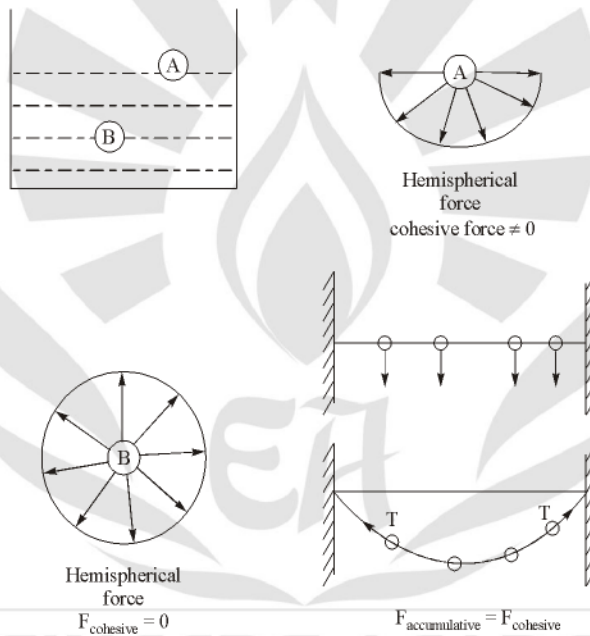
**e) Rheopectic Fluid :**

Apparent viscosity decreases with respect to time, such fluid is known as rheopectic fluid.

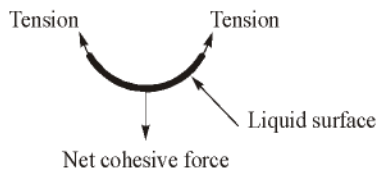
*Example:* Gypsum in water and Printer ink.



## 1.7 SURFACE TENSION



Net down force on c is shown here



There is a tensile force acting on the surface. As a result of net downward force, surface will be pulled down causing a curvature to the surface. This in turn develops a tension in the surface which is similar to the tension in a stretched membrane. This is known as surface tension. It is an internal force.

It is the force per unit length with unit of N/m or J/m<sup>2</sup>

Dimensional formula : [M<sup>1</sup>L<sup>0</sup>T<sup>-2</sup>]

$$\sigma_{\text{water}} = 0.073 \frac{\text{N}}{\text{m}} \quad \text{at } 20^\circ\text{C}$$

$$\sigma_{\text{Hg}} = 4.56 \frac{\text{N}}{\text{m}}$$

For same volume, sphere has minimum surface area.

It is a property of liquid by virtue of which it minimizes or tries to minimum its surface area is known as surface tension.

$$\sigma = \frac{F_T}{\ell}$$

Where  $F_T \rightarrow$  Tangential force

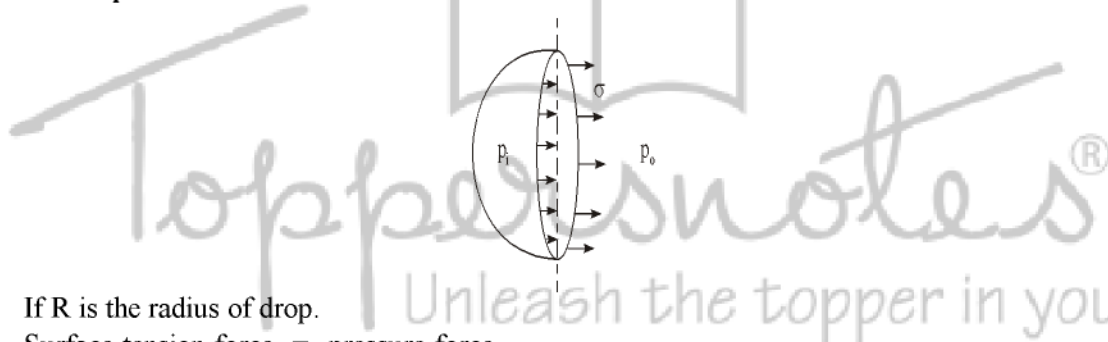
$$\sigma = \frac{F_T}{\ell} \times \frac{\Delta x}{\Delta x}$$

$$\sigma = \frac{\text{work done}}{\text{change in surface area}}$$

So surface tension is also define as a required work done to change in unit surface area.

### 1.7.1 Application of Surface Tension

#### (a) Water Droplet :



If  $R$  is the radius of drop.

Surface tension force = pressure force

Then

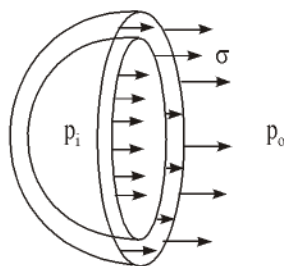
$$F_S = F_P$$

$$\sigma \cdot 2\pi R = \Delta P \pi R^2 \quad (\Delta P = P_i - P_o)$$

$$\Delta P = \left( \frac{2\sigma}{R} \right)$$

$$\Delta P = \frac{4\sigma}{D} \quad (\text{where } D = 2R)$$

#### (b) Soap Bubble :



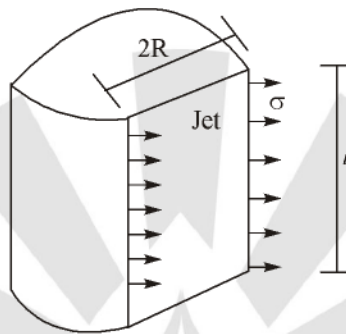
By force balancing in horizontal direction.

$$\sigma \cdot 2\pi R + \sigma \cdot 2\pi R = \Delta P \pi R^2 \quad (\text{in bubble there is two face})$$

$$\Delta P = \frac{4\sigma}{R}$$

$$\Delta P = \frac{8\sigma}{D}$$

(c) Jet :



Force balancing in horizontal direction

$$\sigma \cdot l + \sigma \cdot l = \Delta P \cdot 2Rl$$

$$2\sigma l = \Delta P \cdot 2Rl$$

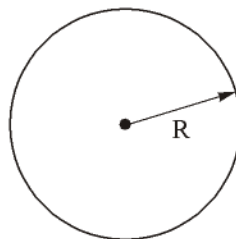
$$\Delta P = \frac{\sigma}{R} = \frac{2\sigma}{D} \quad (\text{where } d \text{ is the diameter of pipe})$$

**Example 8 :** Spherical droplet of radius  $R$  is split into  $n$ -smaller droplet of equal size. Find the work required? Given that surface tension is equal to  $\sigma$ .

**Solution :** We know that surface tension is work done per unit increase in surface area.

$$\therefore \frac{n \times \frac{4}{3}\pi r^3}{3} = \frac{4}{3}\pi R^3$$

$$r = \left(\frac{R^3}{n}\right)^{\frac{1}{3}} = \frac{R}{(n)^{\frac{1}{3}}} \quad \dots(I)$$



$$\therefore \text{Increase in surface area} = (4\pi r^2) \times n - 4\pi R^2 = 4\pi [nr^2 - R^2]$$

$$= 4\pi \left[ n \left( \frac{R}{n^{1/3}} \right)^2 - R^2 \right]$$

$$= 4\pi R^2 \left[ n^{(1-2/3)} - 1 \right] = 4\pi R^2 \left[ n^{1/3} - 1 \right]$$

We have, 
$$\sigma = \frac{\text{work done}}{\text{(Increase in surface area)}}$$

$\therefore$  Work done =  $\sigma \times$  increase in surface area

$\Rightarrow$  
$$W = 4\sigma \pi R^2 \left[ n^{1/3} - 1 \right]$$

## 1.8 CAPILLARITY

Capillarity effect is a consequence of cohesion and adhesion. It is defined as the rise or fall of a liquid in a small diameter tube inserted into liquid. The rise is called capillary rise and fall is called capillary depression. The diameter of the tube is less than 1cm.

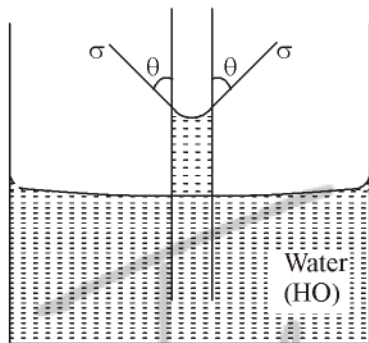


Fig. (1)

$\theta =$  acute angle  
 $\theta < 90^\circ$   
 $\sigma =$  surface tension

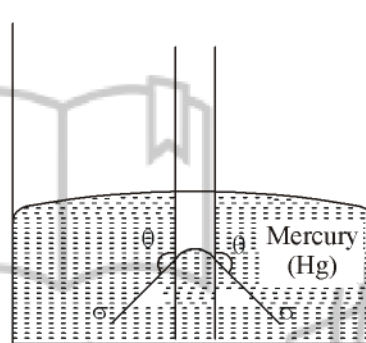
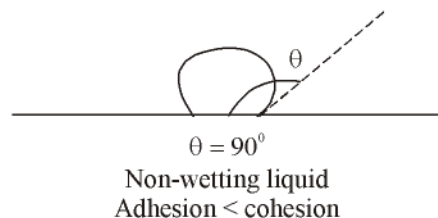
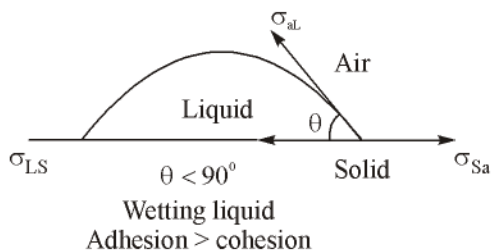


Fig. (2)

$\sigma =$  surface tension  
 $\theta > 90^\circ$   
 $\theta$  is obtuse angle

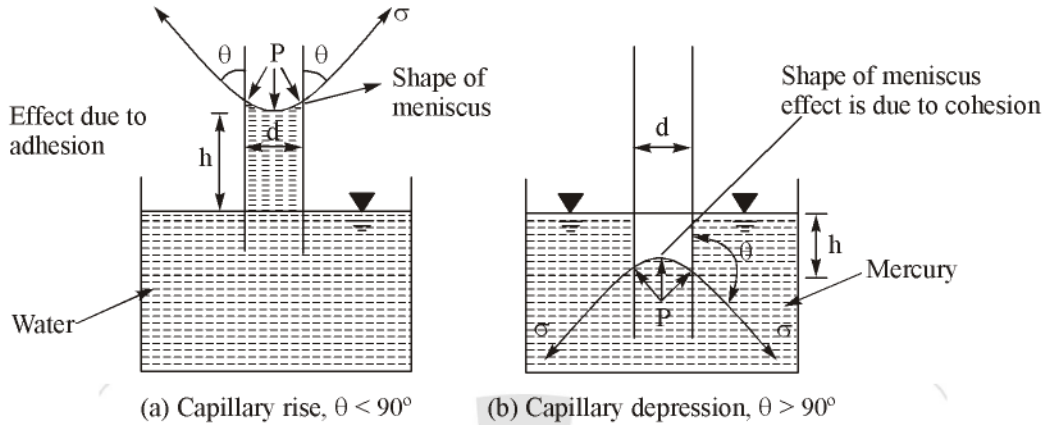
### 1.8.1 Angle of Contact

$\theta < 90^\circ$	cohesion < adhesion	liquid wets solid surface	top surface of water is concave	liquid rises in capillary tube
$\theta > 90^\circ$	cohesion > adhesion	liquid does not wets solid surface	top surface of water is convex	liquid falls in the capillary tube



$$\cos \theta = \frac{\sigma_{sa} - \sigma_{LS}}{\sigma_{aL}} \quad \text{Young's equation}$$

### 1.8.2 Expression of Capillary Rise



(a) Capillary rise,  $\theta < 90^\circ$

(b) Capillary depression,  $\theta > 90^\circ$

#### Capillary rise and fall in glass tube

Let,

$d$  = diameter of glass tube open at both ends.

$h$  = height of the liquid in the tube

$\sigma$  = surface tension of liquid

$\theta$  = angle of contact between liquid and glass tube.

Under the state of equilibrium, the weight of liquid of height  $h$  in the tube is balanced by the forces at the surface of the liquid in the tube due to surface tension.

$$\text{Weight of liquid of height } h \text{ in the tube} = mg = \rho \nabla g = \frac{\pi}{4} d^2 h \rho g$$

where,  $\rho$  = density of liquid.

Vertical component of the force due to surface tension =  $\sigma \times \pi d \times \cos \theta$

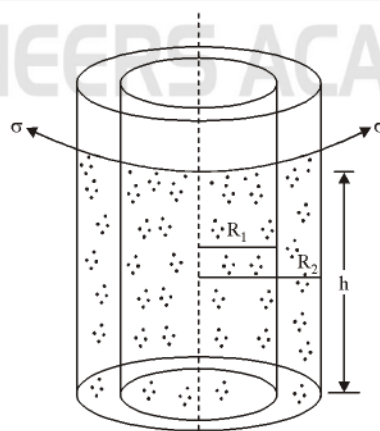
For equilibrium

$$\therefore \frac{\pi}{4} d^2 h \rho g = \sigma \pi d \cos \theta$$

$$h = \frac{4\sigma \cos \theta}{\rho g d} = \frac{4\sigma \cos \theta}{\gamma d}$$

Note : For clean water take  $\theta = 0$

### 1.8.3 Annular Capillary Tube



$\sigma$  same because liquid same so  $\theta$  also same.

Force Balancing in Vertical Direction

$$\sigma \cos \theta \times 2\pi R_1 + \sigma \cos \theta \times 2\pi R_2 = \text{weight of the liquid}$$

$$= \rho \nabla g$$

$$= \rho [\pi R_1^2 - \pi R_2^2] hg$$

$$2\sigma \cos \theta (R_1 + R_2) = \rho \pi (R_1 + R_2) (R_1 - R_2) hg$$

$$h = \frac{2\sigma \cos \theta}{\rho g (R_1 - R_2)}$$

○○○

