



ADRE

Grade III & IV

Assam Direct Recruitment Exam

Volume - 2

Logical Reasoning & General Mathematics



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CHAPTER

English Alphabet Test

The English alphabet test is based on the English letters or alphabet being arranged in a certain format. In this test, questions are solved based on the formation of words by the selected letters, finding the letters in the pair of letters and between 2 letters, etc.

Some important facts related to the English alphabet.

1. Capital/Small letters of the English alphabet

Block letters	A	B	C	D	E	
F	G					
	H	I	J	K	L	
M						
Lowercase letters	a	b	c	d	e	f
	g	h	i	j	k	l
m						
Block letters	N	O	P	Q	R	
S	T					
	U	V	W	X	Y	
Z						
Lowercase letters	n	o	p	q	r	
s	t					
	u	v	w	x	y	
z						

2. Vowels and consonants of the English alphabet

- (i) Vowel - There are 5 vowels in the English alphabet, which are as follows -
A, E, I, O, U

(ii) Consonants - There are 21

consonants in the English alphabet, which are as follows -

B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z

3. The place of letters in the English - Alphabet is the first 13 and the last 13 letters of the alphabet are called the first and second half of the alphabet respectively. This location depends on two sequences.

(i) First and second half of straight sequence

- In this sequence the letters from A to M are placed in the first half and the letters from N to Z are called the second half.

Left to Right

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

← First Half →

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

← Second Half →

(ii) The first second half of the

opposite order - In this sequence the letters from Z to N are called the first half and the letters from M to A are called the second half.

Z	Y	X	W	V	U	T	S	R	Q	P	O	N
1	2	3	4	5	6	7	8	9	10	11	12	13

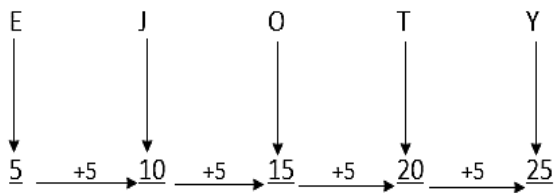
← First Half →

M	L	K	J	I	H	G	F	E	D	C	B	A
14	15	16	17	18	19	20	21	22	23	24	25	26

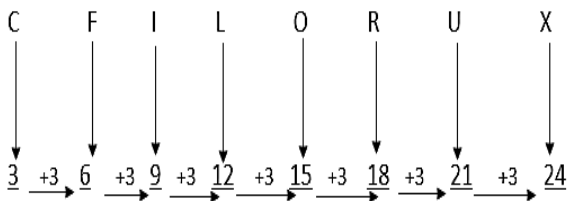
← Second half →

4. Finding the Position Order of Letters by EJOTY and CFILORUX

from Left



From Left



Opposite Letter - Each letter in the English alphabet has an opposite letter.

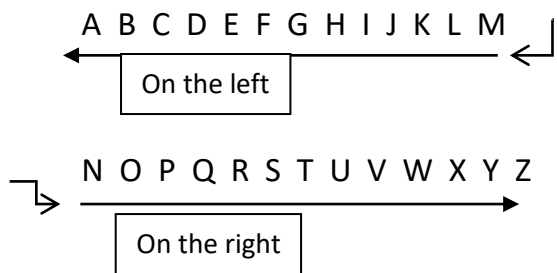
A	Z	$1 + 26 = 27$
B	Y	$2 + 25 = 27$
C	X	$3 + 24 = 27$
D	W	$4 + 23 = 27$
E	V	$5 + 22 = 27$
F	U	$6 + 21 = 27$
G	T	$7 + 20 = 27$
H	S	$8 + 19 = 27$
I	R	$9 + 18 = 27$
J	Q	$10 + 17 = 27$
K	P	$11 + 16 = 27$
L	O	$12 + 15 = 27$
M	N	$13 + 14 = 27$

If the letter of the English alphabet whose opposite letter is to be found, then the corresponding number of that letter is subtracted from 27. The number obtained after subtracting is the corresponding number of the opposite letter.

Finding the letter to the left and right of letters

- Whatever is on our right is the right of the same letters and whatever else is on

our left, is the left of those other letters. like -



Question Types

Type-I Questions Based on Alphabet Test

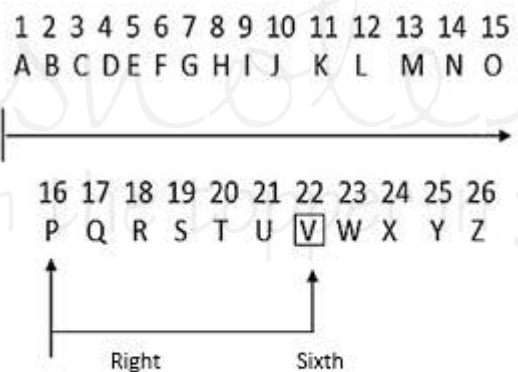
1. The position of the letters in the direct sequence shall be -

Example-1 Which letter is 6th to the right of the sixteenth letter from the left in the alphabet A B C D E F G H I J K L M N O P Q R S T U V W X Y Z ?

- (a) F
- (b) Q
- (c) U
- (d) V

Ans. (d)

Solution - As per question



*16th letter from left in English alphabet = P
Hence 6th letter to the right of P = V*

Alternative Method

n^{th} letter to the right of m^{th} letter from left in English alphabet = $(m+n)^{\text{th}}$ letter from left.

$m = 16$ and $n = 6$

$(16+6)^{\text{th}}$ letter from left = 22^{nd} letter from left = V

2. Position of letters in reverse order

Example - 2 If the English alphabet is written in reverse order, then which letter will be 13th to the left of the 3rd letter from the right?

- (a) C (b) P
(c) R (d) L

Ans. (b)

Solution - n^{th} letter to left of m letter from your right in reverse order of English alphabet = $(m + n)^{\text{th}}$ letter from right.

Here, $m = 3$ and $n = 13$

= $(3+13)^{\text{th}}$ letter from right = 16th letter from right = P Ans.

3. Position of letters in the first half of the reverse order

Under this, the first half of the letters of the English alphabet i.e. letters from A to M are written in reverse order and the remaining half of the letters are written as they are.

Example - 3

If the first half of the English alphabet is written in reverse order, then which letter will be to the left and seventh letter of the tenth letter from your right?

- (a) C (b) E
(c) D (d) J

Ans. (c)

Solution – According to question,

Solution – According to question,

13 12 11 10 9 8 7 6 5
M L K J I H G F E

4 3 2 1 14 15 16 17
D C B A N O P Q
7th ← To the left of → 10th

18 19 20 21 22 23 24 25 26
R S T U V W X Y Z

← From the left of →

The 10th letter from the right is Q and the seventh letter to the left of the letter Q is D.

So required letter = D

4. Position of letters in reverse order of multiple letter segments

Example - 4

If the first four letters of the English alphabet are written in reverse order, then 5 letters also in reverse order, again 6 letters also in reverse order, 5 letters also in reverse order, again 6 letters also in reverse order, again 7 letters are also written in reverse order and remaining letters are also written in reverse order, then which letter will be the seventh letter to the left of the eighth letter from the right?

- (a) O (b) L
(c) N (d) M

Ans. (d)

4 3 2 1 9 8 7 6 5 15 14
D C B A I H G F E O N

13 12 11 10 22 21 20 19
M L K J V U T S
From left 7th 8th

18 17 16 26 25 24 23
R Q P Z Y X W
From right

As per question,

The eighth letter from the right is S and the seventh letter to the left of the letter S is M. So required letter = M

5. Number of letters between 2 letters -

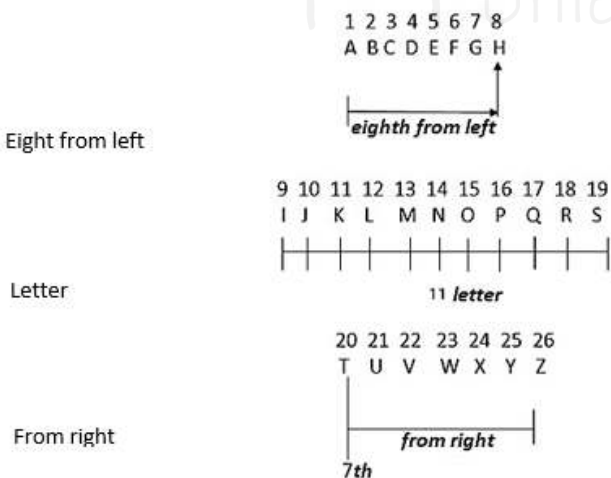
Example - 5

How many letters are there in the middle of the eighth letter from the left and the seventh letter from the right in the English alphabet?

- (a) 8
- (b) 9
- (c) 10
- (d) 11

Ans. (d)

Solution – As per Question



Hence, the eighth letter from the left is H and the seventh letter from the right is T and there are 11 letters in between these two.

6. Same position of letters when arranged alphabetically -

Example - If each letter in the word CADMP is arranged alphabetically, then the positions of how many letters will remain unchanged.

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Ans. (c)

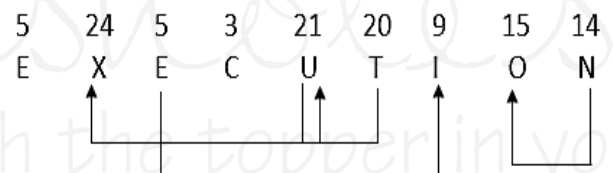
Solution

<i>Root Word</i>	C	A	D	M	P
<i>Alphabetical Order</i>	A	C	D	M	P

Hence such letters are D, M, and P.

Type-2 Letter- Pair Based Question

If there are as many letters between two letters of a word as there are between them in the English alphabet, then the pair thus formed between those two letters is called a letter pair.



Example - 7

How many such pairs of letters are there in the given word EXECUTION, which have as many letters between them in the word as there are between them in the English alphabet ?

- (a) 1
- (b) 2
- (c) 3
- (d) more than 3

Ans. (d)

Solution – According to question

So the required letter pair is UX, TU EI and NO i.e. 4.

Type - 3 Word Formation and Alphabet Arrangement

(i) Changing the letters of a meaningful word

Example - 8 If the letters of the first and second, third and fourth, fifth and sixth and so on are interchanged in the words, then what will be the tenth letter when counted from its right?

- (a) T (b) N
(c) U (d) A

Ans. (b)

Solution - The word formed by interchanging the letters of the given word COMMUNICATIONS



Hence, the required alphabet = N

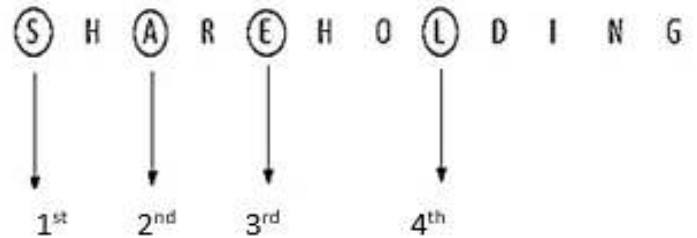
(ii) Making meaningful words from selected / consecutive letters of a semi-complete word

Example - 9 If anyone meaningful word can be formed from the first, third, fifth and eighth letters of the word SHARE HOLDING, then what will be its second letter? If no such word can be formed, give the answer 'X' and if more than one word can be formed, give the answer 'Y'

- (a) L (b) E (c) S
(d) X (e) Y

Ans. (e)

Solution - The word given according to the question,



New word SEAL = SALE, SEAL = 2 Meaningful word Two meaningful words formed from it are SALE and SEAL, so option Y will be correct.

Example - 10 How many meaningful words with four or more letters can be formed from the letters of the word DIALOGUE ?

- (a) 5 (b) 7 (c) 9 (d) 8

Ans. (c)

Solution - words with four or more letters can be formed from the letters of DIALOGUE.

For Example -

DIAL, GOAL, GOAD, GILD, GLUE, GUILD, LOUD, GUIDE, GUILLE etc.

(iii) Arrange the following numbers of different letters in such a way that a meaningful word is formed.

Example - 11

Arrange the numbers of different letters given below in such a way that a meaningful word is formed.

G	T	A	E	N	M
1	2	3	4	5	6

- (a) 1, 3, 2, 6, 4, 5 (b) 6, 3, 5, 1, 4, 2
(c) 1, 3, 2, 5, 4, 6 (d) 6, 3, 1, 5, 4, 2

Ans. (d)

Solution - On arranging the given letters in a meaningful order, the word MAGNET is formed which is obtained by arranging the numbers in the order 6, 3, 1, 5, 4, 2.

Type - 4 Letter Group Based Questions

Under this, some groups of three or four letters are given in the questions asked. The answer is to be found by arranging these letter groups according to the instructions given in the question.

Example – 12

If the letter S is put before them in all the given words, then how many words in the new arrangement will make meaningful English words?

- (a) Only SHE b) ANT and JUG
- (c) Only OLD (d) TIN and JUG
- (e) Only TIN

Ans. (c)

Solution - The following is the word in the new arrangement.

SHE, SOLD, SANT, STIN, SJUG

Hence in the given words, the letter S is preceded by only one meaningful word SOLD.

TYPE-5 RULE BASED ON INSTRUCTION

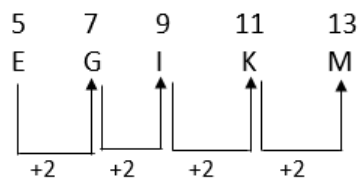
In this type of questions, related to English alphabets are given. Following these rules, it is to be seen that in which option the given letter is following the group.

Example - 13 A letter is missing between two letters.

- (a) EGIKM (b) MOQTU
- (c) MPQTU (d) MNOPQ

Ans. (a)

Solution -



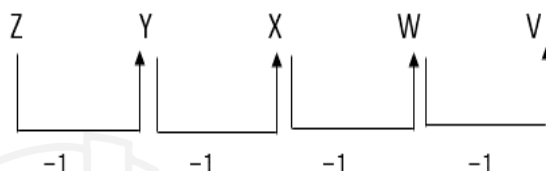
So in the EGIKM letter-pair, one letter is missed between every two letters.

Example - 14 English letters are in the reverse order of the alphabet.

- (a) ABCDE (b) ZYXWV
- (c) KLMNO (d) PQRST

Ans. (b)

Solution



Hence, the letter pair Z Y X W V is in the opposite order of the alphabet.

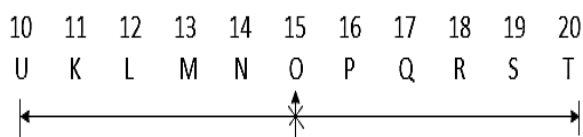
Example with Solution

1. Which letter is exactly between the letters J and T in the English alphabet?

- (a) N (b) O
- (c) P (d) Q
- (e) None of the letter

Ans. (b)

Solution



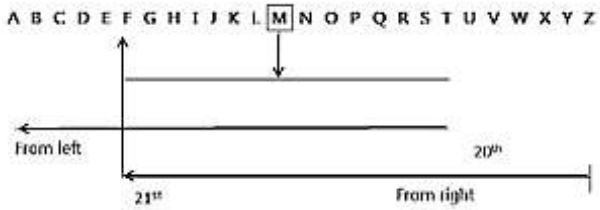
The letter O is exactly between J and T.

2. Which letter will be exactly between the 20th letter from the left and the 21st letter from the right in the English alphabet?

- (a) L (b) M
- (c) N (d) O
- (e) No letter

Ans. (b)

Solution



The 20th letter from the left is T and the 21st letter from the right is F and the letter M is exactly in between these two.

3. If the letters of the word BEAUTIFUL are rearranged and written alphabetically, then how many such letters will be there whose position will remain unchanged?

- (a) 1 (b) 3
- (c) 2 (d) more than three

Ans. (a)

Solution - According to the question,

<i>Root Word</i>	B E A U T I F U L
<i>Alphabetical Order</i>	A B E F I L T U U

Hence the position order of only one letter remains unchanged.

4. If each vowel of the word DOMAINS is changed to the letter that follows it according to the English alphabet and each consonant is changed to the letter that precedes it according to the English alphabet and then all the letters are alphabetically (from left to left) right), then which of the following will be at the third position from the right in the sequence so formed?

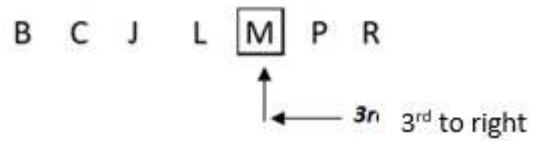
- (a) J (b) C
- (c) P (d) M (e) B

Ans. (d)

Solution

Given word → D O M A I
 N S →
 Changing the vowel → D P M B J
 N S

Changing the consonant C P L B J M R
 Now, alphabetizing these letters (from left to right)



5. Make a meaningful word using each letter of the word HTUTR only once. Your answer will be the fifth letter of the word formed. If more than one such word is formed, then your answer will be wrong and if no such word is formed then your answer will be correct.

- (a) H (b) R
- (c) U (d) X (e) Y

Ans. (a)

Solution - According to the question, meaningful words formed from the letter-group HTUTR = TRUTH

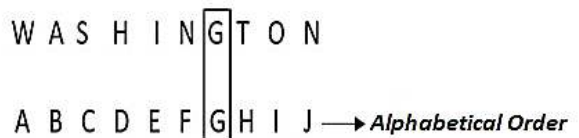
Hence, the fifth letter from the left in TRUTH = H

6. What is the letter in the word WASHINGTON, which when counted is the same number as in the alphabet?

- (a) N (b) T (c) O
- (d) G

Ans. (d)

Solution – According to question



So required letter = G

7. A set of letters is assigned a number to each. Putting them in a meaningful order, select the correct sequence from the letters of the given answers.

Y M L O S B C I
 1 2 3 4 5 6 7 8

- (a) 47685321 (b) 51264387
- (c) 21645387 (d) 56241387

Ans. (b)

Solution - By rearranging the letters of Y M L O S B C I, the word S Y M B O L I C will be formed. So the correct order is 5 1 2 6 4 3 8 7.

8. In the following question, a word is given followed by four alternatives. Out of the four alternatives, only one option can be made from the letters of the given root word. Select that option.

V E N T U R E S O M E

- (a) ROSTRUM (b) TRAVERSER
(c) SERMON (d) SEVENTEEN

Ans. (c)

Solution - The word SERMON can be formed from the letters of the given word.

9. Which letter can be put after the words of the following questions to form new words ?

STAG, ENGAG, DAMAG, SEWAG

- (a) A (b) S
(c) E (d) P

Ans. (c)

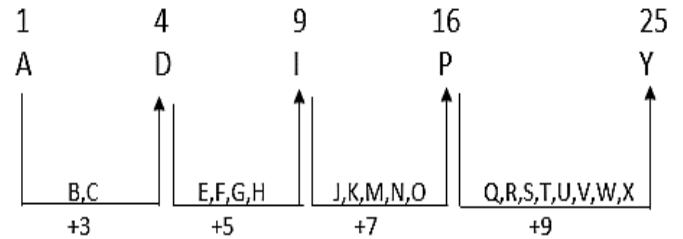
Solution - If the letter E is placed immediately after the words STAG, ENGAM, DAMAG, SEWAG, then completely new meaningful words STAGE, ENGAGE, DAMAGE, SEWAGE will be formed.

10. The number of letters skipped between two adjacent letters is increased by multiples of two.

- (a) ADIPY (b) JMRYG
(c) EHNTC (d) HKBWF

Ans. (a)

Solution - The rule given in series A is being followed.



So, the desired series = ADIP

Practice Questions

TYPE - 1

- Q. 1** Cannot be formed using the letters of the given word.

VENTURE

- (a) RENT (b) RATE
(c) TURN (d) TRUE

- Q. 2** IMPOSSIONABLE

- (a) IMPOSSIBLE (b) POSSIBLE
(c) IMPOSE (d) IMPASSIVE

- Q. 3** KOLMOGROV - SMIRNOV

- (a) MOORINGS (b) ROOMKIN
(c) GOVINDA (d) SLIMGROOM

- Q. 4** DISINTEGRATION

- (a) SIGNATURE (b) INTERN
(c) SINGER (d) RATION

- Q. 5** COMPETITION

- (a) TOTEM (b) POETIC
(c) COMPOSE (d) OPINE

- Q. 6** TERRORISM

- (a) MIRROR (b) ERROR
(c) MORE (d) ROAR

- Q. 7** COMMUNICATION
 (a) ACTION (b) UNION
 (c) NATION (d) UNISON
- Q. 8** TRANSLATION
 (a) TRANSIT (b) TRANSMIT
 (c) RATION (d) NATION
- Q. 9** TECHNOLOGICAL
 (a) LOGIC (b) LOAN
 (c) HATER (d) COLOGNE
- Q. 10** EXAMINATION
 (a) MINE (b) TIME
 (c) SAME (d) NOTE
- Q. 11** From the following alternatives, select the word which cannot be formed using the letters of the given word.
 BUILDINGS
 (a) BUILD (b) BINDS
 (c) SAND (d) DUBS

- Q. 12** From the following alternatives, select the word which cannot be formed using the letters of the given word.
 COMPLIMENTARY
 (a) TRIM (b) TRAPS
 (c) PRIME (d) TRAIN
- Q. 13** From the following alternatives, select the word which cannot be formed using the letters of the given word.
 PROMINENT
 (a) TONE (b) MINE
 (c) PROPS (d) MORE
- Q. 14** From the following alternatives, select the word which cannot be formed using the letters of the given word.
 CONVICTED
 (a) CITE (b) NODE
 (c) TONE (d) DESK

Answer Key

Directions - Find out the related words from the alternatives given below.

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| Q.1 (d) | Q.2 (d) | Q.3 (c) | Q.4 (a) |
| Q.5 (c) | Q.6 (d) | Q.7 (d) | Q.8 (d) |
| Q.9 (c) | Q.10 (c) | Q.11 (c) | Q.12 (b) |
| Q.13 (c) | Q.14 (d) | | |

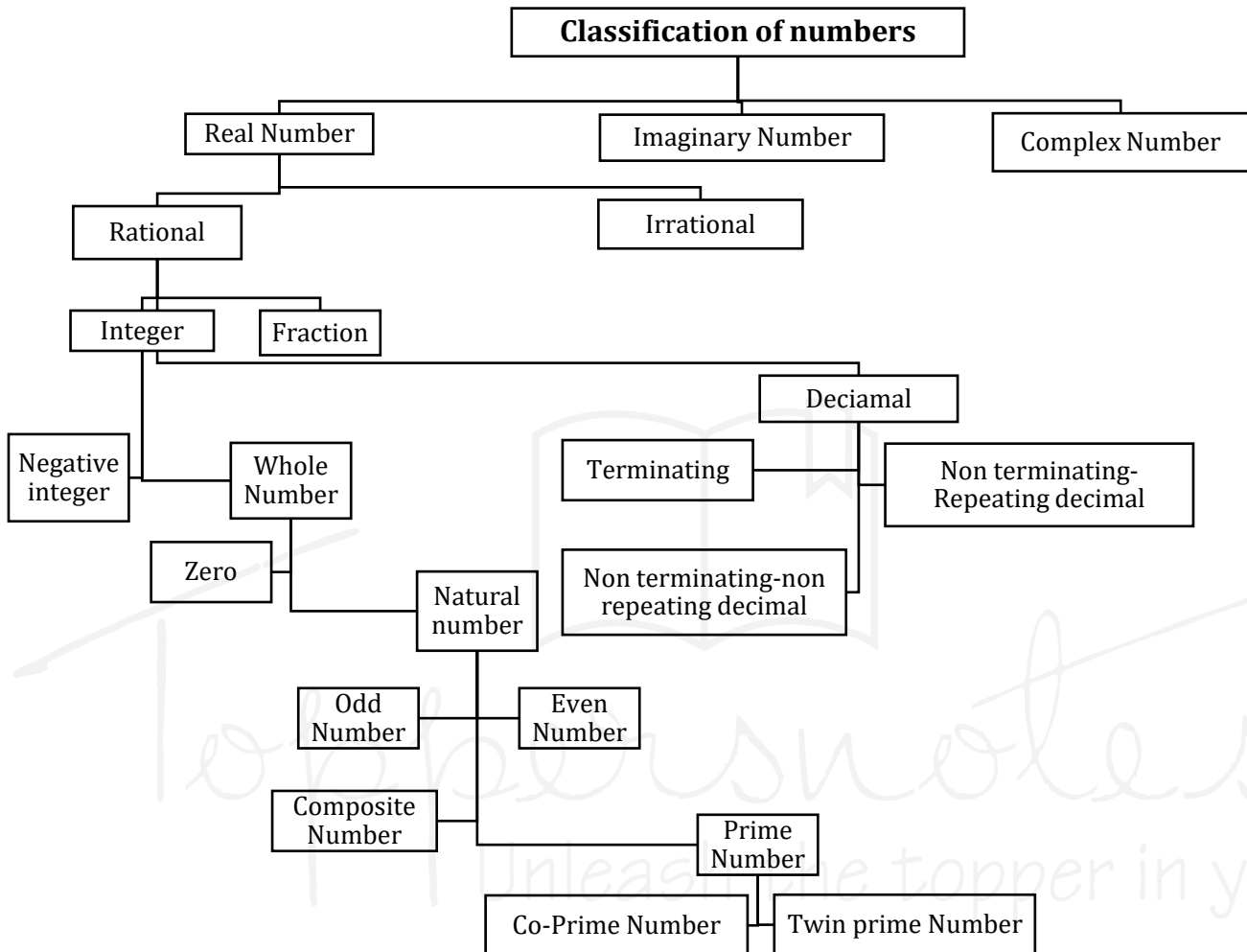
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CHAPTER

Number System



- **Number System:** A number system is a **method of representing and working with numbers** using a defined set of symbols and rules.



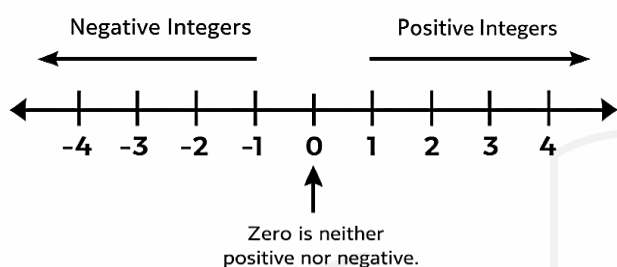
Types	Definition
Real Number	A real number is any number that can be represented on the number line. A real number is a number that includes all rational and irrational numbers and can be expressed as a point on the number line.
Rational Number	A rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Irrational Number	An irrational number is a number that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$
Fraction	A fraction is a number that represents a part of a whole or a ratio of two quantities. It is written in the form $\frac{a}{b}$.
Integer	An integer is a whole number that can be positive, negative, or zero, and does not include any fractional or decimal part

Negative Integer	Negative integers are whole numbers with a negative sign, such as $-1, -2, -3, \dots$	Co-Prime Numbers	Two or more numbers are called co-prime (or relatively prime) if their only common factor (HCF) is 1. ➤ 1 is neither prime or composite.
Whole Number	A whole number is a non-negative integer, including zero.	Twin Prime Numbers	Twin numbers (twin primes) are a pair of prime numbers whose difference is exactly 2. ➤ 5 is only prime which is in 2 twin prime pairs. (3, 5) (5, 7) ➤ Sum of twin prime number (except 3 and 5) is always divisible by 12.
Natural number	Natural numbers are the numbers starting from 1 and increasing by 1 each time. 1,2,3, 4,...	Decimal Number	A decimal number is a number that has a decimal point (.) and consists of a whole part and a fractional part. Ex- 3.5,12.75 etc
Odd Number	An odd number is a natural number that is not divisible by 2 or in the form of $2n + 1$	Terminating Decimal	A terminating decimal is a decimal number that ends after a finite number of digits after the decimal point. Ex- 2.5,0.75
Even Numbers	An even number is a natural number that is completely divisible by 2 or in the form of $2n$.	Non-Terminating Repeating Decimal	A non-terminating repeating decimal is a decimal number that does not end and in which one or more digits repeat continuously after the decimal point. Ex- 0. 333.,0. 1212
Prime number	A prime number is a natural number greater than 1 that has exactly two distinct factors: 1 and the number itself. 2,3,5, 7. ➤ 2 is the smallest prime number. ➤ 2 is only even prime number. ➤ All prime number (except 2 and 3) can be written in the form of $6n + 1$ or $6n + 5$ where n is natural number however, the converse is not true. ➤ (3,5,7) is only group of three primes, which are consecutive odds. ➤ 101 is smallest 3-digit prime number. ➤ 997 is largest 3-digit prime number.	Non-Terminating Non-Repeating Decimal	A non-terminating non-repeating decimal is a decimal number that does not end and does not repeat any digit or pattern after the decimal point. Ex- 1.1412,3.14
Composite Number	A composite number is a natural number greater than 1 that has more than two factors. A composite number can be divided exactly by 1, itself, and at least one other number. 4,6,8,9,10,12, 14... ➤ The smallest composite number is 4. ➤ The smallest odd composite number is 9 If a and b are any two odd primes then $a^2 + b^2$ and $a^2 - b^2$ is composite numbers.	Imaginary Number	An imaginary number is a number that can be written in the form $= bi$ b is a real number i is the imaginary unit, defined by

Complex Number	<p>A complex number is a number that consists of two parts—a real part and an imaginary part—and can be expressed in the standard form</p> $z = a + ib$ <p>a is a real number, called the real part of z</p> <p>b is a real number, called the imaginary part of z</p> <p>i is the imaginary unit, defined by the property</p>
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Integer Number Line



How to check the given number is prime or not?

- To determine whether a number is prime, first find its square root and round it down to the nearest whole number. Then check whether the number is divisible by any prime number up to this value. If it is not divisible by any of them, the number is a prime number.

Between	Number or prime number
1-50	15
1-100	25
1-200	46

Ramanujan number

A Ramanujan number is a number that can be expressed as the sum of two positive cubes in two different ways. It is also known as the Hardy–Ramanujan number or taxi-cab number.

Smallest Ramanujan number = 1729

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Perfect number

A perfect number is a natural number that is equal to the sum of its proper divisors (that is, all its positive divisors excluding the number itself).

Ex: 4 is divisible by 1 and 2, so $1 + 2 = 3 \neq 4$; therefore, 4 is not a perfect number.

6 is divisible by 1, 2 and 3, so, $1 + 2 + 3 = 6 = 6$; therefore, 6 is a perfect number.

Key points

Even + Even = Even

Even × Even = Even

Even + Odd = odd

Even × Odd = Even

Odd + odd = Even

Odd × Odd = Odd

Sum Of 'n' odd number is odd, if n is odd.

Sum of 'n' odd number is even, if n is even.

Type 1: Question based on definitions



Ex: 173 is prime number or not?

Sol: The Square root of 173 is approximately 13.

The Prime number less than or equal to 13 are 2, 3, 5, 7, 11 and 13,

Since 173 is not divisible by any number, it is a prime number.

Ex: x, y and z are distinct prime number where $x < y < z$ if $x + y + z = 70$, then what is the value of z ?

Sol: Now, sum is 70, means at least one of the numbers is even. As we know, only one even prime number exists, and that is 2.

2 is also the smallest prime number.

Means $x = 2$

Now, $70 - 2 = 68 = y + z$

By substituting the values of different prime numbers, we obtain the result.

$y = 31$ and $x = 37$

Ex: How many composite numbers are there from 53 to 97?

Sol: If we find the total number of integers between 53 and 97 and then subtract the number of prime numbers between them, we obtain the number of composite numbers.

Total number = $97 - 53 + 1 = 45$ (+1 when both numbers are included)

Total prime number from 53 to 97 is 10

So, composite number = $45 - 10 = 35$

Ex: Which one of the following is TRUE?

- (A) All irrational numbers are real
- (B) All real numbers are irrational.
- (C) Rational numbers are not real.
- (D) Integers are not rational.

Sol: Irrational numbers form a subset of real numbers, so all irrational numbers are real.

Not all real numbers are irrational; rational numbers are also real.

Rational numbers are real numbers.

Integers are a subset of rational numbers since any integer can be written as $\frac{p}{q}$ (e.g., $5 = \frac{5}{1}$).

So, Answer is (A).

Special concept: based on sum of digits for special types of numbers

Number	Square	Sum of digits
11^2	121	3
111^2	12321	9
So, on		
111111111^2	12345678987654321	81

Ex: Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of digit of the resulting number.

Sol- using the concept

$$111111111^2 \Rightarrow 81$$



Type 2: Unit digit based

To find the units digit of an expression, consider only the unit digits of the numbers instead of evaluating the entire expression.

Unit digit of $(a + b) =$ unit digit of $a +$ unit digit of b

Unit digit of $(a - b) =$ unit digit of $a -$ unit digit of b

Unit digit of $a \times b =$ unit digit of $a \times$ unit digit of b

Ex: What are the unit digit of 435×433 .

Sol: Unit digit of $a \times b =$ unit digit of $a \times$ unit digit of b

$$5 \times 3 = 15 \text{ so, unit digit is } 5$$

Cyclicity

Cyclicity in the number system means the repeating pattern of digits or remainders when a number is raised to higher powers. The unit digit remains unchanged for all powers.

$$0 \rightarrow 0 \quad 1 \rightarrow 1$$

$$5 \rightarrow 5 \quad 6 \rightarrow 6$$

Cycle of Length 2-The unit digit alternates between two values.

$$4 \rightarrow 4, 6$$

When the power is odd, the unit digit is 4, and when the power is even, the unit digit is 6.

$$9 \rightarrow 9, 1$$

When the power is odd, the unit digit is 9, and when the power is even, the unit digit is 1.

Cycle of Length 4- The unit digit repeats after four powers.

$$2 \rightarrow 2, 4, 8, 6$$

$$3 \rightarrow 3, 9, 7, 1$$

$$7 \rightarrow 7, 9, 3, 1$$

$$8 \rightarrow 8, 4, 2, 6$$

$$\text{Let } N = x^y$$

To find the unit digit of (N) , we only need to consider the unit digit of the base number (x) .

The unit digit of an exponential expression can be determined by finding the remainder when the power is divided by 4.

Type 3: Cyclicity - Unit Digit Based on Arithmetic Equations



Ex: If $x = (164)^{169} + (333)^{337} - (727)^{726}$ then what is the unit digit of x ?

Sol: In the expression, the **first term** has an odd power of 4, so the unit digit of the first term is 4. For the **second term**, dividing 337 by 4 gives a remainder of 1 so the unit digit of the second term is 3. For the **third term**, dividing 726 by 4 gives a remainder of 2; hence, the unit digit of the third term is 9.

So, unit digit of the expression $4 + 3 - 9 = -2$. If the unit digit comes out negative, add 10 to obtain the correct unit digit. Unit digit is $10 - 2 = 8$.

Ex: What is the unit digit of $1^5 + 2^5 + 3^5 + 4^5 + \dots + 20^5$

Sol: In each term, the cyclicity is 1. Therefore, for every term, the units digit is the same as the number itself.

The Unit digit for 1 to 10 is zero.

$$= (1 + 2 + 2.. + 9 + 0)$$

$$+ (1 + 2 + 3.. + 9 + 0) = 0$$

Ex: Find the unit digit $x = 187^{280} \times 529^{320} \times 343^{236}$

Sol: If remainder is 0 then put power equal to the 4.

For the first terms - 7^4 cyclicity is 1

For the second terms - power of 9 is even so the unit digit is 1

For the third terms - 3^4 cyclicity is 1

Unit digit is $= 1 \times 1 \times 1 = 1$

Ex: What is the unit digit in the expansion of $(57242)^{9 \times 7 \times 5 \times 3 \times 1}$?

Sol: We only check only for digit 2. Power will be divided by 4.

$$= 2^{1 \times (-1) \times 1 \times (-1) \times 1} = 2^1$$

So, the unit digit is 2

Type 4: Counting based Questions (a digit or page counting or key strokes)



1 to 9 \rightarrow required digit is = 9

10 to 99 $\rightarrow 90 \times 2 = 180$

Ex: What is the number of digits required for numbering a book with 428 pages?

Sol: 1 to 9 \rightarrow required digit is = 9

10 to 99 $\rightarrow 90 \times 2 = 180$

100 to 428 $= (428 - 100 + 1) = 329 \rightarrow 329 \times 3 = 987$

Total number of digits required = $9 + 180 + 987 = 1176$

Type 5: Perfect Square Based Questions



How to check if a number is a perfect square (it shows the possibility)

1. The last two digits of any perfect square number must be among the squares from 1 to 24.
2. The units digit should not be 2, 3, 7, or 8.
3. The number of zeros in the number and the denominator should both be even.
4. After dividing a perfect square by 9, the remainder must be 0, 1, 4, or 7.

Ex: Is it possible that 562576 is a perfect square or not?

Sol: The number ends in 76, which is possible for a perfect square. The unit digit is 6, so it is possible for the number to be a perfect square.

Sum of digit of 562576 = 21

after dividing remainder is 3.

So, this number is not perfect.

Type 6 Decimal to fraction conversion



- The number of zeros in the denominator is equal to the number of digits after the decimal point.

$$0.abc = \frac{abc}{1000}$$

- The number of 9 in the denominator is equal to the number of digits after the decimal point.

$$0.\overline{abc} = \frac{abc}{999}$$

- When some digits are not covered by the overline

$$0.a\overline{bc} = \frac{abc - a}{990}$$

$$0.a\overline{bcd} = \frac{abcd - ab}{9900}$$

- Mix concept

$$a.\overline{bcd} = a + \frac{bcd - b}{990} = \frac{abcd - ab}{990}$$

Ex: If $A = 0.3\overline{12}$, $B = 0.4\overline{15}$ and $C = 0.30\overline{9}$ what is the value of $A + B + C$.

Sol:

$$A + B + C = \frac{312 - 3}{990} + \frac{415 - 4}{990} + \frac{309 - 30}{900}$$

$$A + B + C = \frac{720}{999} + \frac{279}{900}$$

$$A + B + C = \frac{10269}{9900} = \frac{1141}{1100}$$



Type 7: Number of zeros

- Zeros at the end of a number are determined by the number of 10s and in its factorization, primarily based on the pairs of 5s and 2s.

- **Factorial** is a mathematical operation defined for non-negative integers.

- For a positive integer n , the factorial of n , denoted by $n!$, is the product of all positive integers from 1 to n .

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 1$$

$$0! = 1, 1! = 1$$

- ✓ The unit digit of numbers after $4!$ is zero.
- ✓ $4!$ and all factorials after it are divisible by 4.
- ✓ Product of ' n ' consecutive natural number is divisible by n
- **Power of number contained in a factorial:** Highest power of a prime number ' p ' contained in $n!$ is given by

$$= \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots$$
- The product of n consecutive natural number is always divisible by $n!$

Ex: The three numbers 24, 25, and 26 are divisible by.

Sol: The product of n consecutive natural number is always divisible by $n!$

Means 24,25,26 is divisible by 3!

Ex: Find the number of trailing zeros in 100!

Sol: In $100!$ factors of 2 are abundant, we only count factors of 5.

Each multiple of 5 contributes at least one factor of 5. Numbers like 25, 50, 75, 100 contain an extra factor of 5 because $25 = 5^2$.

$$\left[\frac{100}{5} \right] + \left[\frac{100}{25} \right] + \left[\frac{100}{125} \right] = 20 + 4 + 0 = 24$$

Number of trailing zeros = 24

Ex: Find the number of trailing zeros in $2 \times 4 \times 6 \dots \times 250$.

Sol: factors of 2 are abundant, we only count factors of 5.

$$2 \times 4 \times 6 \dots \times 250$$

$$= (2 \times 1) \times (2 \times 2) \dots (2 \times 125)$$

$$(2 \times 1) \times (2 \times 2) \dots (2 \times 125)$$

$$= 2^{125} (1 \times 2 \times \dots \times 125)$$

$$2^{125} (1 \times 2 \times \dots \times 125) = 2^{125} \times 125!$$

$$\left[\frac{125}{5} \right] + \left[\frac{125}{25} \right] + \left[\frac{125}{125} \right] = 25 + 5 + 1 = 31$$

Number of trailing zeros = 31

Divisibility

Number	Divisibility Rule
2	Last digit is 0, 2, 4, 6, 8
3	Sum of digits divisible by 3
4	Last two digits form a number divisible by 4
5	Last digit is 0 or 5
25	Last two digit is 00 or divisibly by 25
6	Number divisible by both 2 and 3
7	Subtract twice the last digit from the remaining; result divisible by 7
8	Last three digits divisible by 8
9	Sum of digits divisible by 9
11	Difference between sum of digits at even and odd places is 0 or divisible by 11

Special cases

$$1. 1001 = 7 \times 11 \times 13$$

$$1001 \times abc = abcabc$$

$$2. 10101 = 3 \times 7 \times 13 \times 37$$

$$10101 \times ab = ababab$$

Type 8: Based on



Divisibility rules

Ex: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

Sol: When any number is repeated n times, the combination of the 6-digit number is divisible by 7, 11, and 13.

96 times 9 will be divisible by 13, and only three 9s will remain.

$$\frac{999}{13} \Rightarrow R \rightarrow 11$$

Ex: Find the greatest possible value of $(a + b)$ for which the 8-digit number $143b203a$ is divisible by 15.

Sol: Divisibility of 3 - if sum of its digits is divisible by 3.

Divisibility of 5- if the last number is either 0 or 5, the entire no. is divisible by 5.

Factors of 15 = (3×5) Hence, the number should be divisible by both 3 and 5.

So, a can be 0 or 5. But since the question asks for greatest value, a must be 5.

After that sum of the number is = $18+b$

For greatest = $b=9$

$$\text{So, } (a + b) = (9 + 5) = 14$$

Ex: If the 9-digit number $72x8431y4$ is divisible by 36, find the value of $\left(\frac{x}{y} - \frac{y}{x}\right)$ for the smallest possible value of y , where x and y are natural numbers.

Sol: The divisibility rule of 36 is that the number must be divisible by 4 and 9.

Last two-digit number = $y4$, on putting $y = 2$, the last two-digit number becomes 24, Therefore, the value of $y = 2$. (y is the smallest possible value)

For divisibility rule of 9,
sum of number = $31 + x$

So, $x = 5$

Now the value

$$\left(\frac{x}{y} - \frac{y}{x}\right) = \left(\frac{5}{2} - \frac{2}{5}\right) = 2\frac{1}{10}$$



Type 9: Divisibility of expressions

Expression	'n' is odd	'n' is even
$x^n - y^n$	$(x - y)$	$(x + y)(x - y)$
$x^n + y^n$	$(x + y)$	Can't say

Ex: What is the remainder if $(17^{26} - 11^{26})$ is divided by 42?

Sol: n is even. So, $(x + y)(x - y)$

$$= \frac{(17 + 11)(17 - 11)}{42} = \frac{28 \times 6}{42}$$

Remainder = 0

Key Point: If ' n ' is odd and $a, b, c \dots z$ are consecutive natural number then $(a^n + b^n + \dots + z^n)$ is divisible by $(a + b + c \dots z)$

Ex: $11^5 + 12^5 + 13^5$ is divisible by?

Sol- using concept

$$11 + 12 + 13 = 36$$

So, the expression is divisible by 36

Type 10: Digit and its Reverse Form 2-digit and 3 digit and its reverse form



1. For 2-digit

Let, two-digit original number = $10x + y$, Reversed number = $10y + x$

Sum of original and reserved number

$$(10x + y) + (10y + x) = 11(x + y)$$

Difference of original and reversed number $(10x + y) - (10y + x) = 9(x - y)$

2. For 3-digit

Let, Hundred digit = x , Tenth digit = y , Unit digit = z So,

$$\text{original number} = 100x + 10y + z$$

Hundred and unit digits interchange,

$$\text{new number} = 100z + 10y + x$$

Difference of original and reverse number = $99(x - z)$

Ex: The sum of the two-digit number and the number obtained by inter-changing the digit is 99. If the difference of digits is 1, then the number is;

Sol: Let, number = $10x + y$

$$(10x + y) + (10y + x) = 99$$

$$x + y = 9$$

$$x - y = 1$$

After the equation $x = 5, y = 4$

So, the number is

$$= 10 \times 5 + 4 = 54$$

Ex: What will be the smallest natural number to be filled in the blank for the number $23x45678$ to be divisible by 22.

Sol: The divisibility rule for 22 is based on divisibility by both 2 and 11. A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is divisible by 11.

Odd positions sum: $3 + 4 + 6 + 8 = 21$

Even positions sum: $2 + x + 5 + 7 = 14 + x$

$$21 - (14 + x) = 7 - x$$

$$7 - x = 11k, \quad \text{for some integer } k$$

For the smallest x , let's try $k = 0$

$$7 - x = 0 \Rightarrow x = 7$$

Thus, the smallest value of x that makes the number $23x45678$ divisible by 22 is 5.

Ex: How many numbers are there from 700 to 950 (including both) which are neither divisible by 3 nor by 7?

Sol:

Neither divisible by 3 nor by 7

$$= \text{Total} - (\text{Divisible by 3 or 7}) = \text{Total} - [N(3) + N(7) - N(21)]$$

$$\text{Total number from 700 to 950} = 251$$

$$\text{Divisible by 3} = \frac{251}{3} \approx 83$$

$$\text{Divisible by 7} = \frac{251}{7} \approx 35$$

$$\text{Divisible by 21} = \frac{251}{21} \approx 11$$

Required number

$$= 251 - (83 + 35 - 11) = 144$$

Factorization

A factor of a number is a whole number that can be multiplied by another whole number to result in the original number. In other words, a factor divides the number evenly without leaving any remainder.

$$\text{Let } N = a^p \times b^q \times c^r$$

$$\text{Total number of factors} = (p + 1)(q + 1)(r + 1)$$

$$\text{Total number of even factors} = p(q + 1)(r + 1)$$

$$\text{Total number of odd factors} = (q + 1)(r + 1)$$

$$\text{Sum of all factors} = (a^0 + a^1 + \dots + a^p)(b^0 + b^1 + \dots + b^q)(c^0 + c^1 + \dots + c^r)$$

$$\text{Sum of even factors} = (2^1 + 2^2 + \dots + 2^p)(b^0 + b^1 + \dots + b^q)(c^0 + c^1 + \dots + c^r)$$

$$\text{Sum of odd factors} = 2^0(b^1 + b^2 + \dots + b^q)(c^1 + c^2 + \dots + c^r)$$

$$\text{Average of factor} = \frac{\text{sum of factor}}{\text{number of factor}}$$

$$\text{Sum of reciprocal of factors} = \frac{\text{sum of factor}}{\text{given number}}$$

$$\text{Number of prime factors} = p + q + r$$

$$\text{Number of distinct prime factor} = \text{no of prime number in factor}$$

$$\text{Number of composite numbers} = \text{total} - \text{distinct prime number} - 1$$

- Every number has positive and negative factors.
- All numbers in the form $(\text{prime})^2$ have 3 factors.

Type 11: Find the Number of Factors Based



Ex: Find all types of factors and the sum of all types of factors for $N = 3600$.

$$\text{Sol: } N = 3600 = 2^4 \times 3^2 \times 5^2$$

- Total number of factors = $5 \times 3 \times 3 = 45$
- Number of even factors
 $2 \times (1800) = 2(2^3 \times 3^2 \times 5^2)$
 $= 4 \times 3 \times 3 = 36$
- Number of odd factors = $3 \times 3 = 9$
- Number of prime factors = $4 + 2 + 2 = 8$
- Number of distinct Prime factor (visible in factorization) = $1 + 1 + 1 = 3$
- Number of Composite numbers
 $= 45 - 3 - 1 = 41$
- Perfect square = $(2^2)^2(3^2)^1(5^2)^1$
 $= 3 \times 2 \times 2 = 12$

- Perfect cube = $(2^3)^1 = (1 + 1) = 2$
- Sum of all factor
 $= (2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$
 $= 12493$
- Sum of even factor
 $= (2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 12090$
- Sum of odd factor
 $= (3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 403$
- Sum of prime = $2 + 3 + 5 = 10$
- Sum of composite number
 $= \text{total sum} - (\text{sum of prime} + 1) = 12493 - 10 - 1 = 12482$
- Perfect square
 $= (2^0 + 2^2 + 2^4)(3^0 + 3^2)(5^0 + 5^1) = 5460$

Ex: Calculate the total number of prime factors in the expression $(4^{11} \times 5^5 \times 3^2 \times 13^2)$

Sol:

$$(4^{11} \times 5^5 \times 3^2 \times 13^2) = 2^{22} \times 3^2 \times 5^5 \times 13^2$$

Total number of prime factors

$$= 22 + 2 + 5 + 2 = 31$$

Ex: How many factors of $(30^{16} \times 16^{18} \times 20^{21})$ are perfect square as well as perfect cube?

$$\text{Sol: } 30^{16} \times 16^{18} \times 20^{21} = 2^{130} \times 3^{16} \times 5^{37}$$

When asked to check for a perfect square or perfect cube, check if the power is a multiple of 6.

$$2^{130} \times 3^{16} \times 5^{37}$$

$$= 2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$$

$$2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$$

$$= (2^6)^{21} \times 2^4 \times (3^6)^2 \times 3^4$$

$$\times (5^6)^6 \times 5^1$$

The total number of factors that are both perfect squares and perfect cubes

$$= (21 + 1)(2 + 1)(6 + 1) = 462$$

Ex: How many factors of the number $2^8 \times 3^6 \times 5^4 \times 10^5$ are multiple of 120.

$$\text{Sol: } N = 2^8 \times 3^6 \times 5^4 \times 10^5 = 2^{13} \times 3^6 \times 5^9$$

$$\frac{N}{120} = \frac{2^{13} \times 3^6 \times 5^9}{2^3 \times 3^1 \times 5^1} = 2^{10} \times 3^5 \times 5^8$$

$$\text{Number of factors} = 11 \times 6 \times 9 = 594$$

Remainder

Let suppose number N when divided by divisor D , leaves the remainder as R and quotient as Q .

$$N = D \times Q + R$$

Dividend = Divisor \times quotient + Remainder

If a gives r remainder when divided by n , then ka gives kr remainder when divided by n .

If a and b give r_1 and r_2 remainders respectively when divided by n then.

$a + b$ divided by n , gives remainder $r_1 + r_2$

$a \times b$ divided by n , gives remainder $r_1 + r_2$

Type 12: Question based on remainder theorem

Ex: A number being divided by 52 gives a remainder 45. If the number is divided by 13, the remainder will be?

Sol: Since 13 is a multiple of 52, we can directly divide the remainder of one by the divisor.

$$= \frac{45}{13} \Rightarrow R \rightarrow 6$$

Ex: A number when divided by 12 leaves 5 remainder. What is the remainder if square of this number is divided by 8.

Sol: When a mathematical operation is performed on a number, the same operation can also be performed on the remainder.

$$= \frac{5^2}{8} = \frac{25}{8} \Rightarrow R \rightarrow 1$$

Ex: If the dividend is 45, the divisor is 8, and the quotient is 5, find the remainder.

$$\text{Sol- } N = D \times Q + R$$

$$45 = 8 \times 5 + \text{Remainder}$$

$$\text{Remainder} = 45 - 40 = 5$$

Type 13: Important concept of remainder

$$\frac{(x+a)^n}{x} \Rightarrow R \rightarrow a^n$$

$$\frac{(x+1)^n}{x} \Rightarrow R \rightarrow 1$$

$$\frac{(x-1)^n}{x} \Rightarrow R \rightarrow (1)^n$$

$$R = 1 \text{ when } n \text{ is even}$$

$$R = -1 \text{ when } n \text{ is odd}$$

Special case:

$$\frac{4}{6} \Rightarrow R \rightarrow 4$$

$$\frac{4^n}{6} \Rightarrow R \rightarrow 4$$

Ex: What is the remainder when 2^{75} is divided by 14?

Sol:

$$\frac{2^{75}}{15} = \frac{(2^4)^{18} \cdot 2^3}{15} = \frac{(16-1)^{18} \times 8}{15} \Rightarrow R \rightarrow 8$$

Ex: What is the remainder if 2^{192} is divided by 6?

Sol: Using the concept

$$\frac{2^{192}}{6} = \frac{4^{96}}{6} \Rightarrow R \rightarrow 4$$

Ex: What is the remainder when 37^{47} is divided by 19.

Sol:

$$\frac{37^{47}}{19} = \frac{(38-1)^{47}}{19} \Rightarrow R \rightarrow -1$$

Remainder cannot be negative, so the remainder is subtracted from the divisor.

$$R \rightarrow -1 \Rightarrow R \rightarrow 19 - 1 = 18$$

Type 14: Fermat's little theorem

$$\text{If } \frac{a^{p-1}}{p}, \Rightarrow R \rightarrow 1$$

Conditions

1. P is prime number.
2. a, P are co-prime.

Ex: What is the remainder when 21^{47} is divided by 47?

Sol: 21 and 47 are coprime number so we can use Fermat's little theorem.

$$\frac{21^{47}}{47} = \frac{21^{46} \times 21}{47} \Rightarrow R \rightarrow 21$$

Type 15: Euler's (totient) theorem

$$\frac{A^X}{d} \Rightarrow R \rightarrow 1$$

Where A and d are co-prime.

X - number of numbers less than d and co-prime to d .

$$X = d \left(1 - \frac{1}{p^1}\right) \left(1 - \frac{1}{p^2}\right) \dots \dots \dots$$

Where P^1 and P^2 are distinct prime factors of d .

Ex: If φ is the Euler's totient functions, then $\varphi(92)$ is.

Sol: $92 = 2^2 \times 23$

$$\varphi(92) = 92 \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{23}\right) = 44$$

Ex: 7^{82} is divided by 11, R=?

Sol: To find the remainder when 7^{82} is divided by 11, we can use Fermat's Little Theorem, which states that if p is a prime number and a is an integer not divisible by p , then

$$\varphi(11) = 11 \left(1 - \frac{1}{11}\right) = 10$$

82 is divided by 10 , and the remainder is

$$2 \cdot \frac{7^2}{11} \Rightarrow R \rightarrow 5$$

Type 16: Wilson's theorem



If P is prime number, then,

$$\frac{(P-1)!}{P} \Rightarrow R \rightarrow P-1$$

$$\frac{(P-2)!}{P} \Rightarrow R \rightarrow 1$$

Ex: What will be the remainder when $568!$ is divided by 569 .

Sol- Using the Wilson theorem

$$\frac{568!}{596} = \frac{(569-1)!}{P} \Rightarrow R \rightarrow 569-1$$
$$= 568$$



ToppersNotes
Unleash the topper in you