



RRRB - JE

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ELECTRONICS

Railway Recruitment Board

Volume - 6

Communication



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1 CHAPTER

Fourier Series and Fourier Transform

THEORY

1. FOURIER SERIES :

A fourier series is a representation of a function $f(t)$ by the linear combination of elements of a complete set of infinite mutually orthogonal functions.

These elements must be mutually orthogonal.

Note: (i) Mutually orthogonal functions:

Two functions are said to be mutually orthogonal over an interval between t_1 and t_2 , if the integral of their product over this interval is zero.

i.e.
$$\int_{t_1}^{t_2} f(t)h(t) dt = 0$$

In general
$$\int_{t_1}^{t_2} f_i(t)f_k(t) dt = 0 \quad (i \neq k)$$

- (ii) It assure that one function $f(t)$ does not have any component of other function $g(t)$.
- (iii) The example of orthogonal functions are: Legendre polynomial, Jacobi polynomials, Trigonometric and exponential function.
- (iv) Orthogonalities in complex functions

$$\int_{t_1}^{t_2} f_1(t)f_2^*(t) dt = \int_{t_1}^{t_2} f_1^*(t)f_2(t) dt = 0$$

where, f_1^* and f_2^* are complex conjugate of $f_1(t)$ and $f_2(t)$ respectively.

2. DIRICHLET'S CONDITIONS

There are sufficient conditions that needs to be satisfied by a function $f(t)$ for its fourier series representation within the interval (t_1, t_2) . These are follows:

- (i) $x(t)$ is absolutely integrable, i.e. $\int_{-\infty}^{+\infty} |x(t)| dt < \infty$
- (ii) $x(t)$ is single valued and has only finite number of maxima and minima within any finite interval.
- (iii) $x(t)$ has a finite number of finite discontinuities within any finite interval.

Note: Fourier series is valid for periodic signals only.

3. TRIGONOMETRIC FOURIER SERIES :

A function $f(t)$ can be represented by a fourier series comprising the following sine and cosine functions:

$$f(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) \\ + \dots + a_n \cos(n\omega_0 t) + \dots + b_1 \sin(\omega_0 t) \\ + b_2 \sin(2\omega_0 t) + \dots + b_n \sin(n\omega_0 t) + \dots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)], \quad t_0 \leq t \leq t_0 + T$$

Where

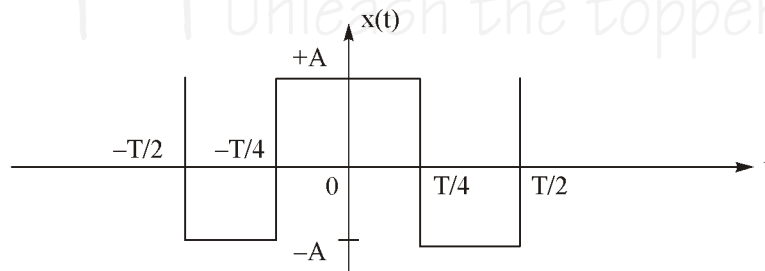
$$T = \frac{2\pi}{\omega_0}$$

$$a_0 = \frac{1}{T} \int_{\text{Over a one time period}} f(t) dt = \text{D.C. or average value}$$

$$a_n = \frac{2}{T} \int_{\text{over an one time period}} f(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{T} \int_{\text{over an one time period}} f(t) \sin(n\omega_0 t) dt$$

Example: Find the trigonometric fourier series representation of following figure



Solution:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad \dots(i)$$

Signal $x(t)$ is given by

$$x(t) = \begin{cases} -A & ; -\frac{T}{2} < t < -\frac{T}{4} \\ +A & ; -\frac{T}{4} < t < +\frac{T}{4} \\ -A & ; \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} x(t) \cos(n\omega_0 t) dt \\
 a_n &= \frac{2}{T} \left[\int_{-\frac{T}{2}}^{-\frac{T}{4}} -A \cos(n\omega_0 t) dt + \int_{-\frac{T}{4}}^{+\frac{T}{4}} A \cos(n\omega_0 t) dt + \int_{+\frac{T}{4}}^{+\frac{T}{2}} -A \cos(n\omega_0 t) dt \right] \\
 &= \frac{2A}{T} \left[-\int_{-\frac{T}{2}}^{-\frac{T}{4}} \cos(n\omega_0 t) dt + \int_{-\frac{T}{4}}^{+\frac{T}{4}} \cos(n\omega_0 t) dt - \int_{+\frac{T}{4}}^{+\frac{T}{2}} \cos(n\omega_0 t) dt \right] \\
 a_n &= \frac{2A}{T} \left[\left[-\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_{-\frac{T}{2}}^{-\frac{T}{4}} + \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_{-\frac{T}{4}}^{+\frac{T}{4}} + \left[-\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_{+\frac{T}{4}}^{+\frac{T}{2}} \right]
 \end{aligned}$$

By solving this, we get

$$a_n = \frac{8A}{n\omega_0 T} \sin\left(\frac{n\omega_0 T}{4}\right) - \frac{4A}{n\omega_0 T} \sin\left(\frac{n\omega_0 T}{2}\right)$$

$$a_n = \frac{8A}{2n\pi} \sin\left(\frac{n\pi}{2}\right) - \frac{4A}{2n\pi} \sin(n\pi) ; \omega_0 T = 2\pi$$

$$a_n = \frac{4A}{n\pi} \sin\left(\frac{n\pi}{2}\right) - 0$$

The second term in above expression is zero for all integer values of n.

$$a_n = \frac{4A}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

and $b_n = 0$ because given waveform is even function. and $a_0 = 0$ because given wave form is symmetrical about horizontal axis.

Putting the values of a_n in equation (i)

$$x(t) = \frac{4A}{\pi} \left[\cos(\omega_0 t) - \frac{1}{3} \cos(3\omega_0 t) + \frac{1}{5} \cos(5\omega_0 t) \dots \dots \right] \text{Ans.}$$

Note: 1. Symmetry conditions

(i) If $x(t)$ = even function ; then, $b_n = 0$.

(ii) If $x(t)$ = odd function ; then $a_0 = 0$ and $a_n = 0$.

(iii) If $x(t)$ is symmetrical about horizontal axis, then $a_0 = 0$.

2. *Some trigonometric identity*

$$(i) \sin\left(\frac{n\pi}{2}\right) = \begin{cases} -1; & n = 3, 7, 11 \\ +1; & n = 1, 5, 9 \\ 0; & n = \text{even} \end{cases} \quad n = 0, 1, 2, 3, \dots$$

$$(ii) \cos\left(\frac{n\pi}{2}\right) = \begin{cases} 0; & n = \text{odd} \\ (-1)^{n/2}; & n = \text{even} \end{cases} \quad n = 0, 1, 2, 3, 4, \dots$$

$$(iii) \tan\left(\frac{n\pi}{2}\right) = \infty; \quad n = 1, 2, 3, \dots$$

$$(iv) \sin(n\pi) = 0; \quad n = 0, 1, \dots$$

$$(v) \cos(n\pi) = (-1)^n; \quad n = 0, 1, \dots$$

$$(vi) \tan(n\pi) = 0; \quad n = 0, 1, \dots$$

4. **POLAR FOURIER SERIES REPRESENTATION :**

A function $x(t)$ can be represented by a polar fourier series

$$x(t) = D_0 + \sum_{n=1}^{\infty} D_n (n\omega_0 t - \phi_n)$$

where, $D_0 = a_0 = \frac{1}{T} \int_{\text{over a time period}} x(t) dt = \text{D.C. value or average value.}$

$$D_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

5. **COMPLEX / EXPONENTIAL FOURIER SERIES**

A function $x(t)$ with period T can be represented by complex / exponential fourier series.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \exp(jn\omega_0 t) \quad \dots(ii)$$

where, $\omega_0 = \frac{2\pi}{T}$

$$C_n = \frac{1}{T} \int_{\text{over a time period}} x(t) \exp(-jn\omega_0 t) dt$$

Coefficient C_n are in general complex form, hence

$$C_n = |C_n| \exp(j\phi_n) \quad \dots(iii)$$

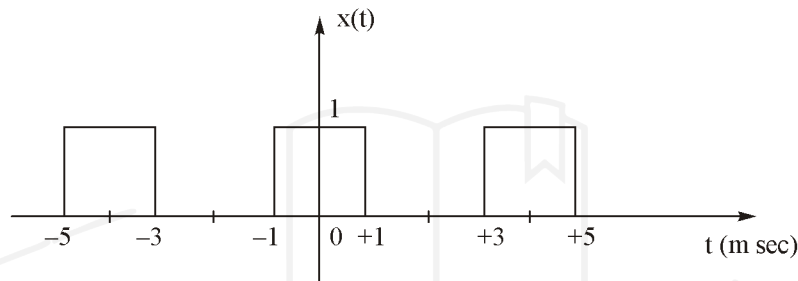
So, using equation (iii) in equation (ii)

$$x(t) = \sum_{n=-\infty}^{\infty} |C_n| \exp(j(n\omega_0 t + \phi_n))$$

The plot between $|C_n|$ versus n (or $n\omega_0$) is called magnitude spectrum and ϕ_n versus n (or $n\omega_0$) is called phase spectrum.

It is important to note that the spectrum of a periodic signal exists only at discrete frequency.

Example: For the unit amplitude rectangular pulse train shown in figure below, compute the fourier series coefficient.



Solution: Signal $x(t)$ has a period $T = 4$ millisecond and it is ON for half the period and OFF during the remaining half.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \exp(jn\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

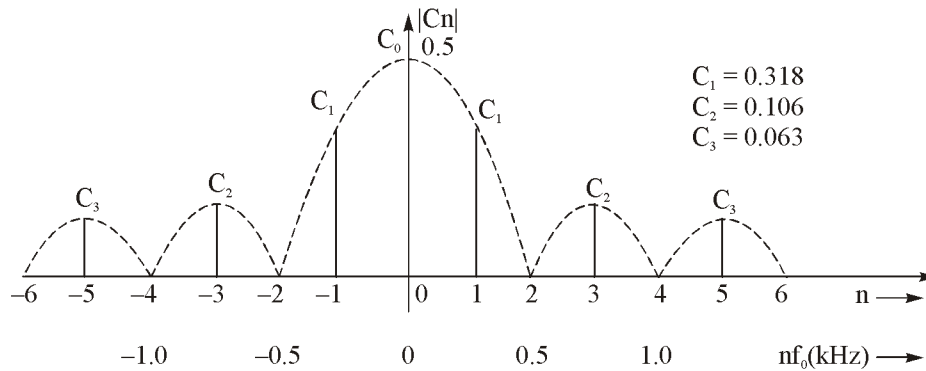
$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \exp(-jn\omega_0 t) dt$$

$$= \frac{1}{T} \int_{-T/4}^{T/4} \exp(-jn\omega_0 t) dt$$

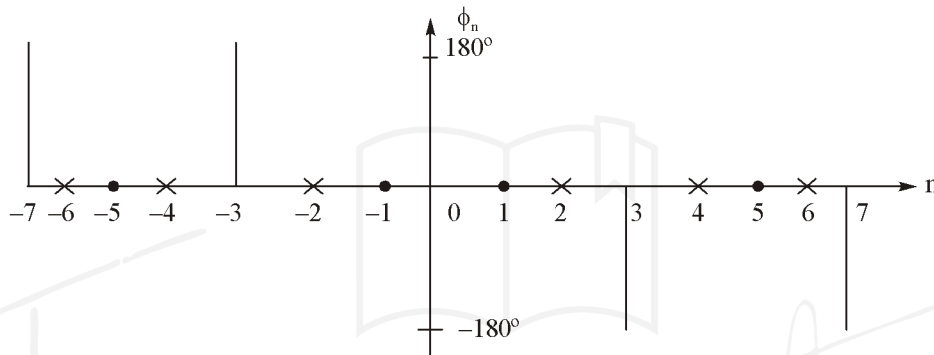
$$= \frac{1}{T} \frac{2 \sin\left(\frac{n\pi}{2}\right)}{n\omega_0}$$

$$= \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

All the fourier coefficients are real but could be positive and negative. Hence ϕ_n is either zero or $\pm\pi$ for all n.



(a) Magnitude spectrum.



(b) Phase spectrum.

- C_0 , the average or the DC value of pulse train is $\frac{1}{2}$.
- Spectrum exists only at discrete frequencies, $f = nf_0$ with $f_0 = 250$ Hz. Such a spectrum is called the discrete spectrum
- The envelope consists of several lobes and the magnitude of each lobe keeps decreasing with increase in frequency.
- The magnitude spectrum is symmetric and phase spectrum is antisymmetric. This is because $x(t)$ is real.
- ϕ_n at $n = \pm 2, \pm 4$ etc. is undefined at $|C_n| = 0$ for these n. This is indicated with a cross on the phase spectrum plot.

6. SINC FUNCTION

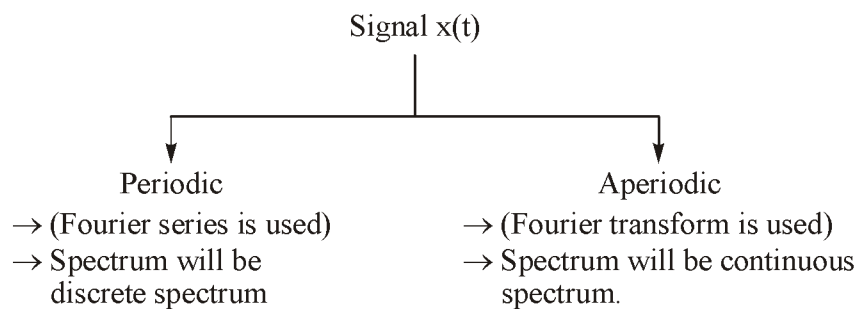
$$\text{sinc } \lambda = \text{sinc } (\lambda) = \frac{\sin(\pi\lambda)}{(\pi\lambda)}$$

7. SAMPLING FUNCTION

$$S_a(\lambda) = \frac{\sin \lambda}{\lambda}$$

8. FOURIER TRANSFORM

Fourier transform is used to find the frequency component in time domain signal.



Fourier transform of $x(t)$ is $X(f)$. $X(f)$ is defined as

$$x(t) \Leftrightarrow X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (\text{Analysis equation})$$

$$X(f) = F[x(t)]$$

$X(f)$ is, in general, a complex quantity.

$$X(f) = X_R(f) + jX_I(f)$$

$$= |X(f)| e^{j\phi(f)}$$

where,

$$X_R(f) = \text{Real part of } X(f)$$

$$X_I(f) = \text{Imaginary part of } X(f)$$

$$|X(f)| = \text{Magnitude of } X(f)$$

$$= \sqrt{(X_R(f))^2 + (X_I(f))^2}$$

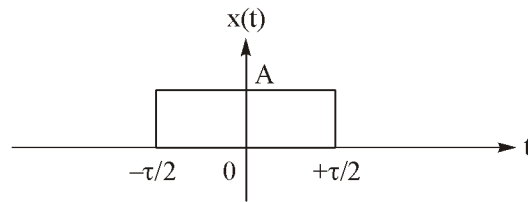
$$\theta(f) = \arg[X(f)] = \tan^{-1} \left[\frac{X_I(f)}{X_R(f)} \right]$$

The plot between $|X(f)|$ versus f , is known as magnitude spectrum and $\theta(f)$ versus f , is known as the phase spectrum.

Inverse fourier transform (IFT) is defined as

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} dt \quad (\text{Synthesis equation})$$

Example: Find the fourier transform of figure.



Solution:

$$x(t) \Leftrightarrow X(f)$$

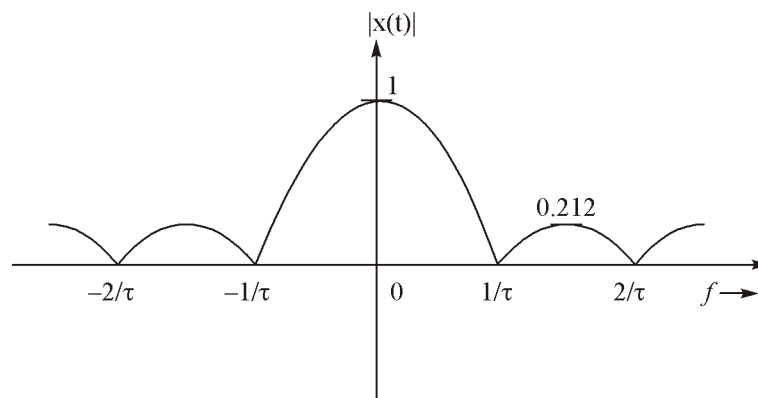
$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\ &= \int_{-\tau/2}^{+\tau/2} A \cdot e^{-j2\pi ft} dt \\ &= A \left[\frac{e^{-j2\pi ft}}{-j2\pi f} \right]_{-\tau/2}^{+\tau/2} \\ &= A\tau \operatorname{sinc}(\tau f) \end{aligned}$$

Note:

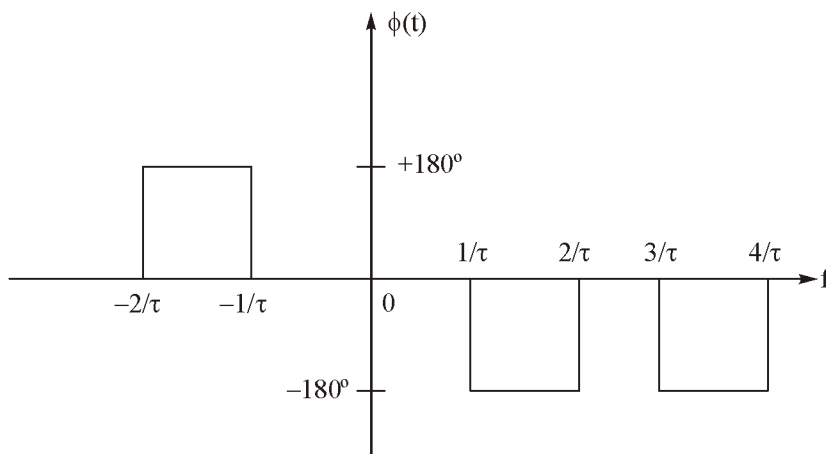
$$\operatorname{sinc}[x] = \frac{\sin(\pi x)}{\pi x} = \begin{cases} 1 & ; \quad x = 0 \\ 0 & ; \quad x = \pm 1, \pm 2, \dots \end{cases}$$

$$\operatorname{sinc}(f\tau) = \begin{cases} 1 & ; \quad f\tau = 0 \Rightarrow f = 0 \\ 0 & ; \quad f\tau = \pm 1, \pm 2, \pm 3, \dots \\ & ; \quad f = \pm \frac{1}{\tau}, \pm \frac{2}{\tau}, \pm \frac{3}{\tau}, \dots \end{cases}$$

$$X(f) = |X(f)| e^{j\theta(f)}$$



(a) Magnitude spectrum

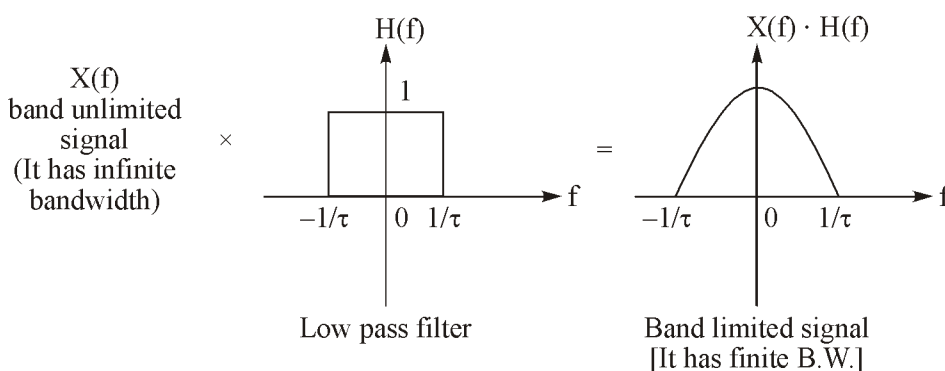


(b) Phase spectrum

- The fourier transform of $x(t)$, $X(f)$ contains all the possible frequencies.
- During the interval, $\frac{m}{\tau} < |f| < \frac{m+1}{\tau}$, with m odd, $\text{sinc}(f\tau)$ is negative. As the magnitude spectrum is always positive, negative value of $\text{sinc}(f\tau)$ are taken care of by making $\theta(f) = \pm 180^\circ$.
- Signal bandwidth (B.W) = Highest positive frequency – Lowest positive frequency
 $= \infty - 0 = \infty$

for proper transmission of a signal, channel B.W. > signal B.W.

- Signal B.W. is ∞ . So, it is not possible to transmit the signal. Signal B.W. should be finite.
- Before transmission, the above signal should be band limited by band limiting process.
- To band limit a signal, all the its significant frequency components has to be retained and insignificant frequency component has to be eliminated.
- Significant frequency contains almost of 95% to 99% of total strength of given signal.



$$\text{B.W.} = \frac{1}{\tau}$$

$$\tau \neq 0$$

For transmission, significant frequency are given high importance for effective utilization of available channel bandwidth.

9. PROPERTIES OF FOURIER TRANSFORM

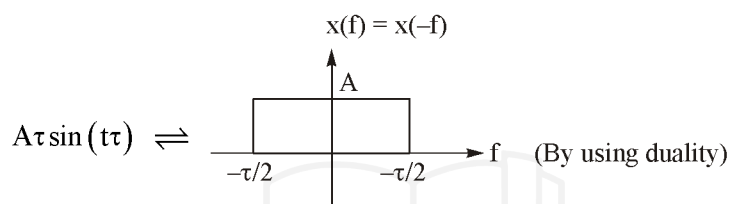
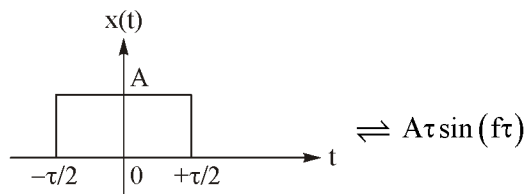
(i) Duality property :

$$\text{If } x(t) \Leftrightarrow X(f)$$

$$\text{then, } X(t) \Leftrightarrow x(-f)$$

Example: Find out the fourier transform of $\sin c(100t)$.

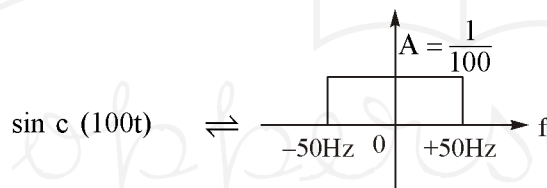
Solution :



Put
and

$$\tau = 100$$

$$A = 1$$



Example: Find out the fourier transform of $\delta(t)$.

Solution:

$$F[\delta(t)] = \int_{-\infty}^{+\infty} \delta(t) e^{-j2\pi ft} dt$$

$$F[\delta(t)] = \int_{-\infty}^{+\infty} \delta(t) dt$$

$$[x(t)\delta(t) = x(0)\delta(t)]$$

$$F[\delta(t)] = 1$$

$$\left[\int_{-\infty}^{+\infty} \delta(t) dt = \text{Area under curve} = 1 \right]$$

$$\delta(t) \Leftrightarrow 1$$

Example: Find the fourier transform of 1.

Solution:

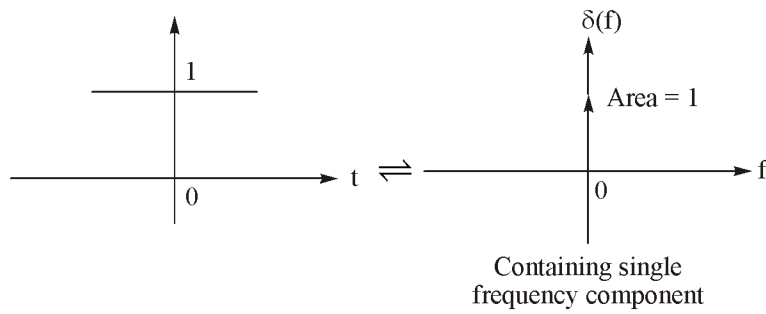
$$\delta(t) \Leftrightarrow 1$$

$$1 \Leftrightarrow \delta(-f)$$

[by using duality property]

$$1 \Leftrightarrow \delta(f)$$

[\delta(f) is an even function]



(ii) Frequency Shifting Property :

If $x(t) \Leftrightarrow X(f)$

then, $x(t)e^{j2\pi f_0 t} \Leftrightarrow X(f - f_0)$

and $x(t)e^{-j2\pi f_0 t} \Leftrightarrow X(f + f_0)$

Example: Find out the fourier transform of $e^{j2\pi f_0 t}$ and $e^{-j2\pi f_0 t}$.

Solution:

$$1 \Leftrightarrow \delta(f)$$

$$1 \cdot e^{j2\pi f_0 t} \Leftrightarrow \delta(f - f_0)$$

$$1 \cdot e^{-j2\pi f_0 t} \Leftrightarrow \delta(f + f_0) \quad \text{[by using frequency shifting property]}$$

$$F[e^{j2\pi f_0 t}] = \delta(f - f_0)$$

$$F[e^{-j2\pi f_0 t}] = \delta(f + f_0)$$

Example: Find the fourier transform of $A \cos(2\pi f_0 t)$.

Solution.

$$e^{j2\pi f_0 t} \Leftrightarrow \delta(f - f_0)$$

$$e^{-j2\pi f_0 t} \Leftrightarrow \delta(f + f_0)$$

By adding both equation

$$e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \Leftrightarrow \delta(f - f_0) + \delta(f + f_0)$$

$$2 \cos(2\pi f_0 t) \Leftrightarrow \delta(f - f_0) + \delta(f + f_0)$$

$$\cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$$

By multiplying A on both side,

$$A \cos(2\pi f_0 t) \Leftrightarrow \frac{A}{2} [\delta(f - f_0) + \delta(f + f_0)]$$

$$A \cos(2\pi f_0 t) \Leftrightarrow \frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0)$$

$$A \cos(2\pi f_0 t) \Leftrightarrow \begin{array}{c} \begin{array}{c} \frac{A}{2} \delta(f + f_0) \\ \uparrow \\ -f_0 \end{array} \quad \begin{array}{c} \frac{A}{2} \\ \uparrow \\ f_0 \end{array} \delta(f - f_0) \end{array}$$

Fourier transform of $A \cos(2\pi f_0 t)$ has two frequency component at $+f_0$ and $-f_0$.

Example: Find the fourier transform of $x(t) \cos(2\pi f_0 t)$ when fourier transform of $x(t)$ is $X(f)$.

Solution: Given that

$$x(t) \Leftrightarrow X(f)$$

$$x(t)e^{j2\pi f_0 t} \Leftrightarrow X(f - f_0)$$

[by using shifting property]

$$x(t)e^{-j2\pi f_0 t} \Leftrightarrow X(f + f_0)$$

By adding both,

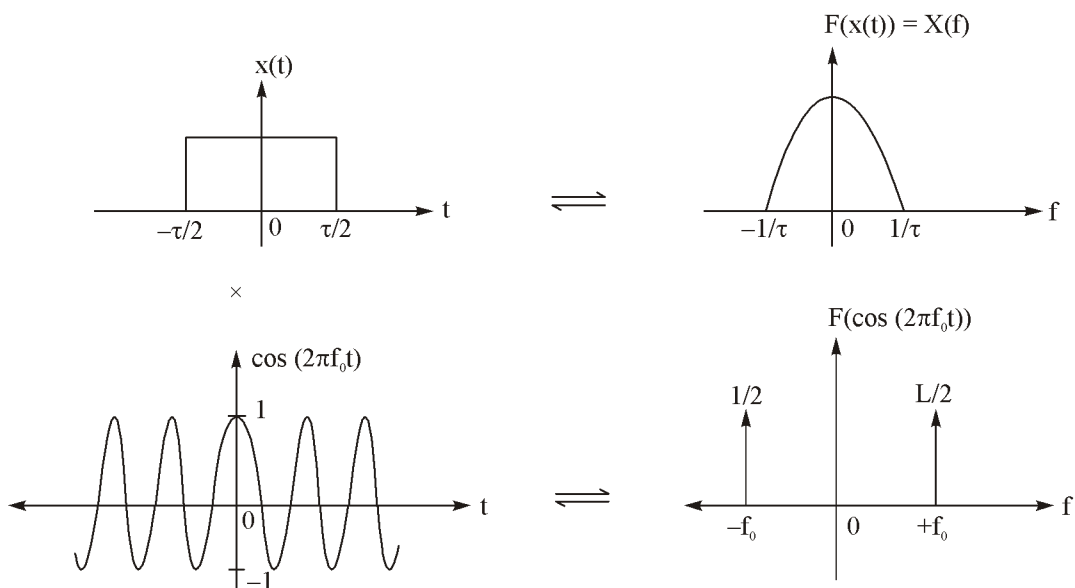
$$x(t)e^{j2\pi f_0 t} + x(t)e^{-j2\pi f_0 t} \Leftrightarrow X(f - f_0) + X(f + f_0)$$

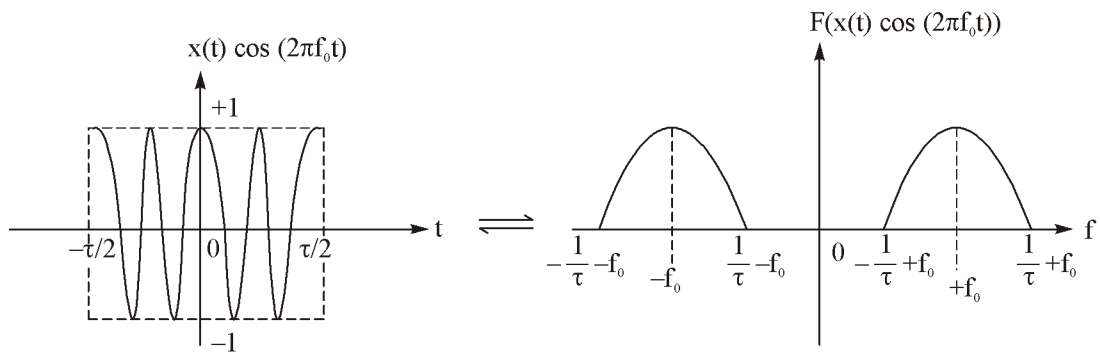
$$x(t)[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}] \Leftrightarrow X(f - f_0) + X(f + f_0)$$

$$x(t)[2 \cos(2\pi f_0 t)] \Leftrightarrow X(f - f_0) + X(f + f_0)$$

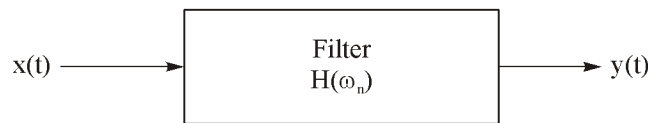
$$x(t) \cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}[X(f - f_0) + X(f + f_0)]$$

It means that when the signal $x(t)$ is multiplied with cosine signal, then fourier transform of $x(t)$ is shifted by $+f_0$ and $-f_0$.





10. RESPONSE OF A LINEAR SYSTEM



$$H(\omega_n) = |H(\omega_n)| e^{-j\theta(\omega_n)}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} X_n e^{j2\pi n t / T_0} \quad \text{(in terms of fourier series)}$$

then

$$y(t) = \sum_{n=-\infty}^{\infty} H(\omega_n) X_n e^{j2\pi n t / T_0}$$

11. NORMALIZED POWER IN FOURIER SERIES

(i) Normalized power in trigonometric fourier series

$$S = a_0^2 + \sum_{n=1}^{\infty} \frac{a_n^2}{2} + \sum_{n=1}^{\infty} \frac{b_n^2}{2}$$

(ii) Normalized power in polar fourier series

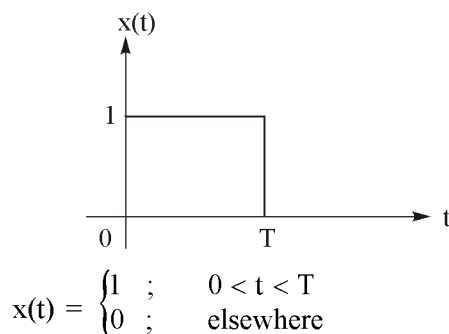
$$S = C_0^2 + \sum_{n=1}^{\infty} \frac{C_n^2}{2}$$

(iii) Normalized power in complex / exponential fourier series

$$S = \sum_{n=-\infty}^{\infty} D_n D_n^*$$

Example: Find the fourier transform of rectangular pulse.

Solution: Rectangular pulse is defined as

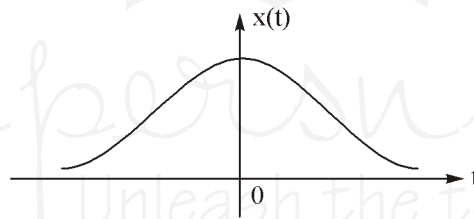


$$\begin{aligned}
 \text{Fourier transform of } x(t) &= X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\
 &= \int_0^T e^{-j2\pi ft} dt = \left[\frac{e^{-j2\pi ft}}{-j2\pi f} \right]_0^T \\
 &= -\frac{1}{j2\pi f} [e^{-j2\pi fT} - 1] \\
 &= -\frac{1}{j2\pi f} \frac{[e^{-j2\pi fT/2} - e^{j2\pi fT/2}]}{e^{j2\pi fT/2}} \\
 &= T e^{-j\omega T/2} \operatorname{sinc} \left(\frac{\omega T}{2\pi} \right)
 \end{aligned}$$

Example: Find the fourier transform of Gaussian pulse.

Solution: A Gaussian pulse is defined as

$$x(t) = e^{-\pi t^2}$$



$$\text{Fourier transform of } x(t) = F[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$F[x(t)] = \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-[\pi t^2 + j\omega t]} dt$$

Put $\pi t^2 + j\omega t = \left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}} \right)^2 + \frac{\omega^2}{4\pi}$

$$\begin{aligned}
 F[x(t)] &= \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}} \right)^2} e^{-\omega^2/4\pi} dt \\
 &= e^{-\pi t^2} \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}} \right)^2} dt
 \end{aligned}$$

Assuming $\sqrt{\pi} t + \frac{j\omega}{2\sqrt{\pi}} = y$

$$\sqrt{\pi} dt = dy$$

$$dt = \frac{dy}{\sqrt{\pi}}$$

$$F[x(t)] = e^{-\pi t^2} \int_{-\infty}^{\infty} e^{-y^2} dy / \sqrt{\pi} = \frac{e^{-\pi t^2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{e^{-\pi t^2}}{\sqrt{\pi}} 2 \int_0^{\infty} e^{-y^2} dy \quad (\text{because } e^{-y^2} \text{ is an even function})$$

$$= \frac{2}{\sqrt{\pi}} e^{-\pi t^2} \int_0^{\infty} e^{-y^2} dy$$

$$= \frac{\cancel{2}}{\cancel{\sqrt{\pi}}} e^{-\pi t^2} \cdot \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$$

$$\left[\because \int_0^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2} \right]$$

$$F[e^{-\pi t^2}] = e^{-\pi t^2}$$

$$e^{-\pi t^2} \Leftrightarrow e^{-\pi f^2}$$

12. SOME IMPORTANT PROPERTIES OF FOURIER TRANSFORM

(i) Time Scaling Property

If $x(t) \Leftrightarrow X(f)$

Then $x(at) \Leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$

where, $a = \text{any real constant.}$

(ii) Time Shifting Property

If $x(t) \Leftrightarrow X(f)$

Then $x(t - b) \Leftrightarrow X(f)e^{-j2\pi fb}$

(iii) Time Differentiation Property

If $x(t) \Leftrightarrow X(f)$

Then $\frac{d}{dt}x(t) \Leftrightarrow (j2\pi f)X(f)$

(iv) Area Under the Curve

$$\text{If } x(t) \rightleftharpoons X(f)$$

$$\text{then } \int_{-\infty}^{\infty} x(t) dt = X(0) = \text{Area under the curve } x(t)$$

$$\text{and } \int_{-\infty}^{\infty} X(f) df = x(0) = \text{Area under the curve } X(f)$$

13. CONVOLUTION

It is a mathematical operation which is used to express the input / output relationship in a linear time invariant system.

It is represented by *

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t - \tau) d\tau$$

(a) Convolution Theorem in Time Domain

$$\text{If } x_1(t) \rightleftharpoons X_1(f)$$

$$x_2(t) \rightleftharpoons X_2(f)$$

$$\text{then } x_1(t) * x_2(t) \rightleftharpoons X_1(f) X_2(f)$$

(b) Convolution Theorem in Frequency Domain

$$\text{If } x_1(t) \rightleftharpoons X_1(f)$$

$$x_2(t) \rightleftharpoons X_2(f)$$

$$\text{then } x_1(t) x_2(t) \rightleftharpoons X_1(f) * X_2(f)$$

14. ENERGY SIGNAL

Energy signal has finite energy and zero average power.

$x(t)$ is non periodic signal or time limited signal, then energy is expressed as

$$E = \int_{-\infty}^{\infty} x^2(t) dt$$

$$\text{If } x(t) \rightleftharpoons X(f), \text{ then}$$

$$E = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

$$\begin{aligned} |X(f)|^2 &= \frac{E}{\Delta f} = \text{Energy spectral density} = \text{Energy density spectrum.} \\ &= \text{energy per unit B.W.} = \psi(f) \end{aligned}$$

15. POWER SIGNAL

A power signal has finite power and infinite energy.

All periodic signal is power signal.

$$0 < P < \infty, E = \infty$$

where, P = average power and E is energy of signal.

If $x(t)$ is periodic signal, then

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

If complex fourier series of $x(t) = \sum_{n=-\infty}^{\infty} C_n \exp(jn\omega_0 t)$

then power (in terms of complex fourier series coefficient,

$$= \sum_{n=-\infty}^{\infty} |C_n|^2$$

○○○

Toppernotes
Unleash the topper in you

PRACTICE SHEET

OBJECTIVE QUESTIONS

- Let $x(t)$ be a real signal with the Fourier transform $X(f)$. Let $X'(f)$ denote the complex conjugate of $X(f)$. Then
 - $X(-f) = X'(f)$
 - $X(-f) = X(f)$
 - $X(-f) = -X(f)$
 - $X(-f) = -X'(f)$
- Let the transfer function of a network be $H(f) = |H(f)| e^{j\theta(f)}$. If a signal $x(t)$ is applied to such a network, the output $Y(t)$ is given by
 - $2 x(t)$
 - $x(t-2)$
 - $2x(t-2)$
 - $2x(t-4\pi)$
- Let $m(t) = \cos[(2\pi \times 10^3)t]$ be the message signal and $c(t) = 15 \cos [(2\pi \times 10^6)t]$ be the carrier. $c(t)$ and $m(t)$ are to generate an AM signal. The modulation index of the generated AM signal is 0.5. The ratio of the upper sideband power to the carrier power
 - $\frac{1}{2}$
 - $\frac{1}{4}$
 - $\frac{1}{8}$
 - $\frac{1}{16}$
- The trigonometric Fourier series of a periodic time function can have only
 - cosine terms
 - sine terms
 - cosine and sine terms
 - DC and cosine terms
- Which of the following cannot be the Fourier series expansion of a periodic signal?
 - $x(t) = 2 \cos t + 3 \cos 3t$
 - $x(t) = 2 \cos \pi t + 7 \cos t$
 - $x(t) = \cos t + 0.5$
 - $x(t) = 2 \cos 1.5\pi t + \sin 3.5 \pi t$
- The Fourier transform of given figure with $\omega = 1$ rad/sec

 - $\left(\cos \frac{\pi}{180} + \sin \frac{\pi}{180} \right)$
 - $\left(4 \cos \frac{\pi}{180} + \sin \frac{\pi}{180} \right)$
 - $\left(2 \cos \frac{\pi}{180} + 4 \sin \frac{\pi}{180} \right)$
 - $\left(2 \sin \frac{\pi}{180} + 4 \cos \frac{\pi}{180} \right)$

ANSWERS AND EXPLANATIONS

1. *Ans. (a)*

$$x(t) \leftrightarrow X(f)$$

we know that

$$x(-t) \leftrightarrow X(-f)$$

$$\text{and } x^*(t) \leftrightarrow X^*(-f)$$

∴ $x(t)$ is a real signal

$$\therefore x(t) = X^*(t)$$

$$\therefore x(f) = X^*(-f)$$

$$\text{or } x(-f) = X^*(f)$$

2. *Ans. (c)*

$$\text{Given } H(f) = 2e^{-j4\pi f}$$

$$\text{and } x(t) \leftrightarrow X(f)$$

$$\therefore \text{o/p } Y(t) \leftrightarrow X(f) * h(f)$$

Taking Fourier transform we get

$$Y(f) = X(f) H(f)$$

(according to convolution property of F.T.)

$$Y(f) = X(f) 2e^{-j\pi \times 4f}$$

$$Y(f) = 2X(f)e^{-j2\pi f \times 2}$$

taking in verse F.T. we get

$$Y(t) = 2F \cdot T^{-1}[X(f)e^{-j\pi f \times 2}] \quad \dots(i)$$

we know that

Time shifting property

$$x(t-t_0) \leftrightarrow X(f)e^{-j2\pi ft_0} \quad \dots(ii)$$

so from (1) and (2) we have

$$Y(t) = 2x(t - 2)$$

3. *Ans. (d)*

$$\frac{\text{Upper side band power}}{\text{Carrier power}} = \frac{\mu^2}{4} = \frac{(0.5)^2}{4} = \frac{1}{16}$$

4. *Ans. (d)*

The trigonometric Fourier series of periodic time function can have DC and cosine terms.

5. *Ans. (b)*

$x(t) = (2 \cos \pi t + 7 \cot t)$ does not satisfy the Dirichlet condition, therefore, it can't be expanded

as Fourier series.

6. *Ans. (d)*

Signal is

$$f(t) = t^2; -1 \leq t \leq +1$$

Fourier transfer of signal is,

$$\begin{aligned} F[f(t)] &= \int_{-1}^{+1} t^2 e^{-j\omega t} dt \\ &= \frac{1}{j\omega} (e^{j\omega} - e^{-j\omega}) + \frac{2}{\omega^2} (e^{-j\omega} + e^{j\omega}) \\ &\quad + \frac{2}{j\omega^3} (e^{-j\omega} - e^{j\omega}) \end{aligned}$$

if $\omega = 1$ rad/sec.

$$F(f(t)) = 2(e^{-j} + e^j) + j(e^j - e^{-j})$$

$$= 4 \cos \frac{\pi}{180^\circ} + 2 \sin \frac{\pi}{180^\circ}$$