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QUANTITATIVE APTITUDE

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SIERRA INNOVATIONS

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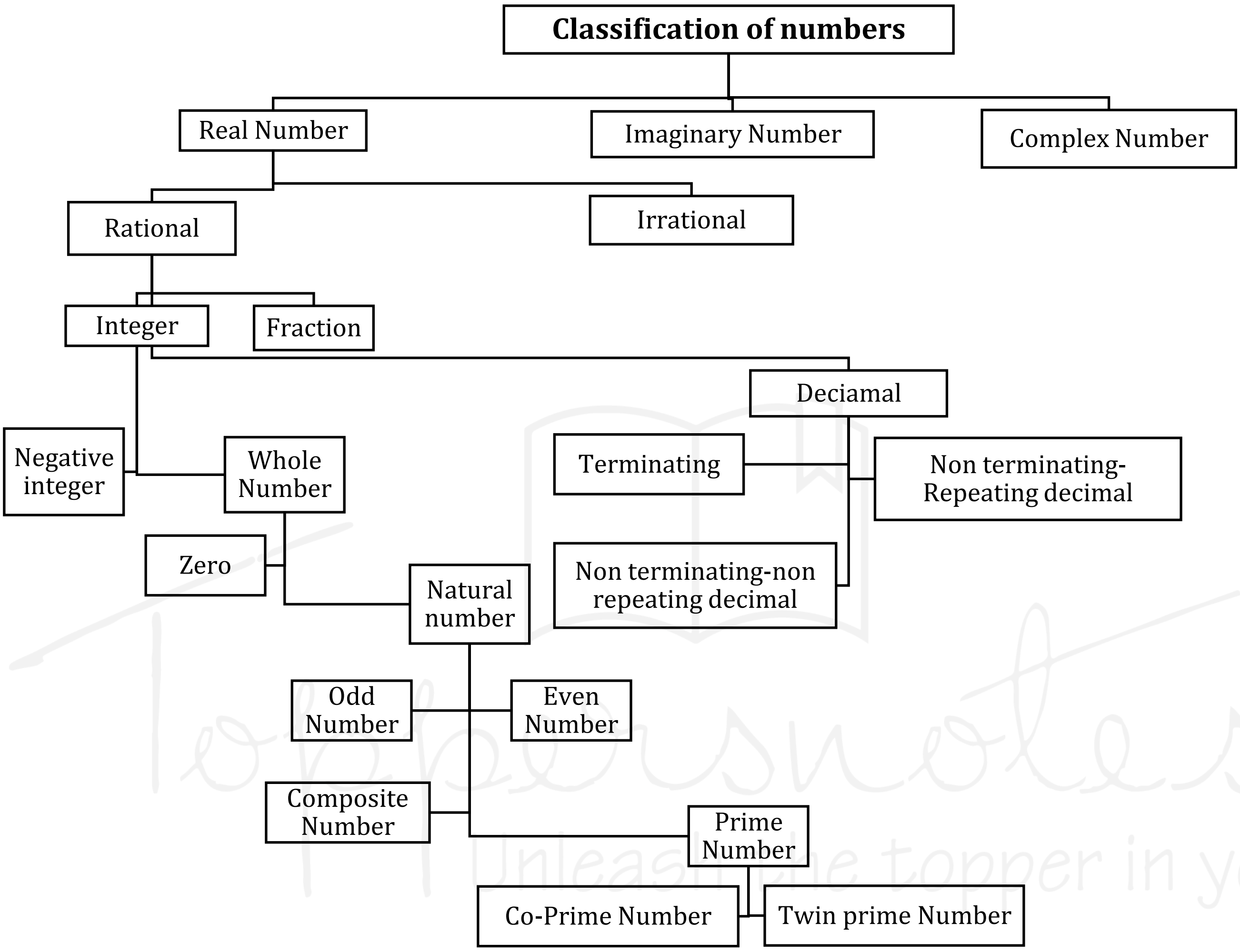
CHAPTER

Number System

- **Number System**: A number system is a **method of representing and working with numbers** using a defined set of symbols and rules.

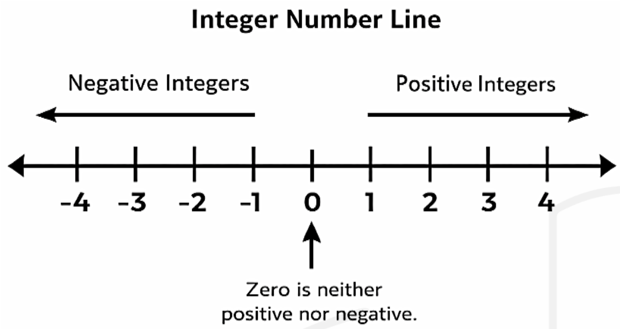


Types	Definition
Real Number	A real number is any number that can be represented on the number line. A real number is a number that includes all rational and irrational numbers and can be expressed as a point on the number line.
Rational Number	A rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Irrational Number	An irrational number is a number that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$
Fraction	A fraction is a number that represents a part of a whole or a ratio of two quantities. It is written in the form $\frac{a}{b}$.
Integer	An integer is a whole number that can be positive, negative, or zero, and does not include any fractional or decimal part

Negative Integer	Negative integers are whole numbers with a negative sign, such as $-1, -2, -3, \ldots$
Whole Number	A whole number is a non-negative integer, including zero.
Natural number	Natural numbers are the numbers starting from 1 and increasing by 1 each time. 1,2,3, 4,…
Odd Number	An odd number is a natural number that is not divisible by 2 or in the form of $2n + 1$
Even Numbers	An even number is a natural number that is completely divisible by 2 or in the form of $2n$.
Prime number	A prime number is a natural number greater than 1 that has exactly two distinct factors: 1 and the number itself. 2,3,5, 7. ➢ 2 is the smallest prime number. ➢ 2 is only even prime number. ➢ All prime number (except 2 and 3) can be written in the form of $6n + 1$ or $6n + 5$ where n is natural number however, the converse is not true. ➢ (3,5,7) is only group of three primes, which are consecutive odds. ➢ 101 is smallest 3-digit prime number. ➢ 997 is largest 3-digit prime number.
Composite Number	A composite number is a natural number greater than 1 that has more than two factors. A composite number can be divided exactly by 1, itself, and at least one other number. 4,6,8,9,10,12, 14… ➢ The smallest composite number is 4. ➢ The smallest odd composite number is 9 If a and b are any two odd primes then $a^2 + b^2$ and $a^2 - b^2$ is composite numbers.

Co-Prime Numbers	Two or more numbers are called co-prime (or relatively prime) if their only common factor (HCF) is 1. ➢ 1 is neither prime or composite.
Twin Prime Numbers	Twin numbers (twin primes) are a pair of prime numbers whose difference is exactly 2. ➢ 5 is only prime which is in 2 twin prime pairs. (3, 5) (5, 7) ➢ Sum of twin prime number (expect 3 and 5) is always divisible by 12.
Decimal Number	A decimal number is a number that has a decimal point (.) and consists of a whole part and a fractional part. Ex- 3.5,12.75 etc
Terminating Decimal	A terminating decimal is a decimal number that ends after a finite number of digits after the decimal point. Ex- 2.5,0.75
Non-Terminating Repeating Decimal	A non-terminating repeating decimal is a decimal number that does not end and in which one or more digits repeat continuously after the decimal point. Ex- 0. 333..,0. 1212
Non-Terminating Non-Repeating Decimal	A non-terminating non-repeating decimal is a decimal number that does not end and does not repeat any digit or pattern after the decimal point. Ex-1.1412,3.14
Imaginary Number	An imaginary number is a number that can be written in the form $= bi$ b is a real number i is the imaginary unit, defined by

Complex Number	A complex number is a number that consists of two parts—a real part and an imaginary part—and can be expressed in the standard form $z = a + ib$ a is a real number, called the real part of z b is a real number, called the imaginary part of z i is the imaginary unit, defined by the property



How to check the given number is prime or not?

➢ To determine whether a number is prime, first find its square root and round it down to the nearest whole number. Then check whether the number is divisible by any prime number up to this value. If it is not divisible by any of them, the number is a prime number.

Between	Number or prime number
1-50	15
1-100	25
1-200	46

Ramanujan number

A Ramanujan number is a number that can be expressed as the sum of two positive cubes in two different ways. It is also known as the Hardy–Ramanujan number or taxi-cab number.

Smallest Ramanujan number = 1729

$1729 = 1^3 + 12^3 = 9^3 + 10^3$

Perfect number

A perfect number is a natural number that is equal to the sum of its proper divisors (that is, all its positive divisors excluding the number itself).

Ex: 4 is divisible by 1 and 2, so $1 + 2 = 3 \neq 4$; therefore,4 is not a perfect number.

6 is divisible by 1,2 and 3, so, $1 + 2 + 3 = 6 = 6$; therefore, 6 is a perfect number.

Key points

Even + Even = Even

Even × Even = Even

Even + Odd = odd

Even × Odd = Even

Odd + odd = Even

Odd × Odd = Odd

Sum Of 'n' odd number is odd, if n is odd.

Sum of 'n' odd number is even, if n is even.

Type 1: Question based on definitions

Ex: 173 is prime number of not?

Sol: The Square root of 173 is approximately 13.

The Prime number less than or equal to 13 are 2,3,5,7,11 and 13,

Since 173 is not divisible by any number, it is a prime number.

Ex: x, y and z are distinct prime number where $x < y < z$ if $x + y + z = 70$, then what is the value of z?

Sol: Now, sum is 70, means at least one of the numbers is even. A we know, only one even prime number exists, and that is 2.

2 is also the smallest prime number.

Means $x = 2$

Now, $70 - 2 = 68 = y + z$

By substituting the values of different prime numbers, we obtain the result.

$y = 31$ and $x = 37$

Ex: How many composite numbers are there from 53 to 97?

Sol: If we find the total number of integers between 53 and 97 and then subtract the number of prime numbers between them, we obtain the number of composite numbers.

Total number = $97 - 53 + 1 = 45$ (+1 when both numbers are included)

Total prime number from 53 to 97 is 10

So, composite number = 45 – 10 = 35

Ex: Which one of the following is TRUE?
(A) All irrational numbers are real
(B) All real numbers are irrational.
(C) Rational numbers are not real.
(D) Integers are not rational.
Sol: Irrational numbers form a subset of real numbers, so all irrational numbers are real.
Not all real numbers are irrational;
rational numbers are also real.
Rational numbers are real numbers.
Integers are a subset of rational numbers since any integer can be written as $\frac{p}{q}$(e.g., $5 = \frac{5}{1}$).
So, Answer is (A).

Special concept: based on sum of digits for special types of numbers

Number	Square	Sum of digits
11^2	121	3
111^2	12321	9
So, on		
111111111^2	12345678987654321	81

Ex: Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of digit of the resulting number.
Sol- using the concept
$111111111^2 \Rightarrow 81$

Type 2: Unit digit based

To find the units digit of an expression, consider only the unit digits of the numbers instead of evaluating the entire expression.
Unit digit of (a + b) = unit digit of a + unit digit of b
Unit digit of (a − b) = unit digit of a − unit digit of b
Unit digit of a × b = unit digit of a × unit digit of b

Ex: What are the unit digit of 435×433 .
Sol:Unit digit of a × b = unit digit of a × unit digit of b
$5 \times 3 = 15$ so, unit digit is 5

Cyclicity

Cyclicity in the number system means the repeating pattern of digits or remainders when a number is raised to higher powers. The unit digit remains unchanged for all powers.

$0 \to 0$ $1 \to 1$
$5 \to 5$ $6 \to 6$

Cycle of Length 2-The unit digit alternates between two values.
$4 \to 4, 6$
When the power is odd, the unit digit is 4, and when the power is even, the unit digit is 6.
$9 \to 9, 1$
When the power is odd, the unit digit is 9, and when the power is even, the unit digit is 1.
Cycle of Length 4- The unit digit repeats after four powers.
$2 \to 2, 4, 8, 6$ $3 \to 3, 9, 7, 1$
$7 \to 7, 9, 3, 1$ $8 \to 8, 4, 2, 6$
Let $N = x^y$
To find the unit digit of (N), we only need to consider the unit digit of the base number (x). The unit digit of an exponential expression can be determined by finding the remainder when the power is divided by 4.

Type 3: Cyclicity - Unit Digit Based on Arithmetic Equations

Ex: If $x = (164)^{169} + (333)^{337} - (727)^{726}$ then what is the unit digit of x?
Sol: In the expression, the **first term** has an odd power of 4, so the unit digit of the first term is 4. For the **second term**, dividing 337 by 4 gives a remainder of 1 so the unit digit of the second term is 3. For the **third term**, dividing 726 by 4 gives a remainder of 2; hence, the unit digit of the third term is 9.
So, unit digit of the expression $4 + 3 - 9 = -2$
If the unit digit comes out negative, add 10 to obtain the correct unit digit.Unit digit is $10 - 2 = 8$

Ex: What is the unit digit of $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 20^5$
Sol: In each term, the cyclicity is 1. Therefore, for every term, the units digit is the same as the number itself.
The Unit digit for 1 to 10 is zero.
$= (1 + 2 + 2..+9 + 0)$
$+ (1 + 2 + 3..+9 + 0) = 0$

Ex: Find the unit digit $x = 187^{280} \times 529^{320} \times 343^{236}$

Sol: If remainder is 0 then put power equal to the 4.

For the first terms – 7^4 cyclicity is 1

For the second terms – power of 9 is even so the unit digit is 1

For the third terms - 3^4 cyclicity is 1

$Unit\ digit\ is\ =\ 1 \times 1 \times 1 = 1$

Ex: What is the unit digit in the expansion of $(57242)^{9\times7\times5\times3\times1}$?

Sol: We only check only for digit 2. Power will be divided by 4.

$= 2^{1\times(-1)\times1\times(-1)\times1} = 2^1$

So, the unit digit is 2

Type 4: Counting based Questions (a digit or page counting or key stokes)

1 to 9 → required digit is = 9
10 to 99 → 90×2 = 180

Ex: What is the number of digits required for numbering a book with 428 pages?

Sol: 1 to 9 → required digit is = 9

10 to 99 → 90×2 = 180

100 to 428 = (428-100+1) = 329→329×3=987

Total number of digits required = 9 +180 + 987 =1176

Type 5: Perfect Square Based Questions

How to check if a number is a perfect square (it shows the possibility)

1. The last two digits of any perfect square number must be among the squares from 1 to 24.
2. The units digit should not be 2, 3, 7, or 8.
3. The number of zeros in the number and the denominator should both be even.
4. After dividing a perfect square by 9, the remainder must be 0, 1, 4, or 7.

Ex: Is it possible that 562576 is a perfect square or not?

Sol: The number ends in 76, which is possible for a perfect square. The unit digit is 6, so it is possible for the number to be a perfect square.

Sum of digit of 562576 = 21

after dividing remainder is 3.

So, this number is not perfect.

Type 6 Decimal to fraction conversion

- The number of zeros in the denominator is equal to the number of digits after the decimal point.

  $0.abc = \frac{abc}{1000}$
- The number of 9 in the denominator is equal to the number of digits after the decimal point.

  $0.\overline{abc} = \frac{abc}{999}$
- When some digits are not covered by the overline

  $0.a\overline{bc} = \frac{abc - a}{990}$

  $0.ab\overline{cd} = \frac{abcd - ab}{9900}$
- Mix concept

  $a.b\overline{cd} = a + \frac{bcd - b}{990} = \frac{abcd - ab}{990}$

Ex: If $A = 0.3\overline{12}$, $B = 0.4\overline{15}$and $C = 0.30\overline{9}$ what is the value of $A + B + C$.

Sol:

$$A + B + C = \frac{312 - 3}{990} + \frac{415 - 4}{990} + \frac{309 - 30}{900}$$

$$A + B + C = \frac{720}{999} + \frac{279}{900}$$

$$A + B + C = \frac{10269}{9900} = \frac{1141}{1100}$$

Type 7: Number of zeros

- Zeros at the end of a number are determined by the number of 10s and in its factorization, primarily based on the pairs of 5s and 2s.
- **Factorial** is a mathematical operation defined for non-negative integers.

- **For** a positive integer n, the factorial of n, denoted by $n!$, is the product of all positive integers from 1 to n.
  $n! = n \times (n-1) \times (n-2) \times \cdots \times 1$
  $0! = 1,\ 1! = 1$
  - ✓ The unit digit of numbers after 4! is zero.
  - ✓ 4! and all factorials after it are divisible by 4.
  - ✓ Product of 'n' consecutive natural number is divisible by n
- **Power of number contained in a factorial:** Highest power of a prime number 'p' contained in n! is given by
  $= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$
- The product of n consecutive natural number is always divisible by n!

Ex: The three numbers 24, 25, and 26 are divisible by.
Sol: The product of n consecutive natural number is always divisible by n!
Means 24,25,26 is divisible by 3!

Ex: Find the number of trailing zeros in 100!
Sol: In 100! factors of 2 are abundant, we only count factors of 5.
Each multiple of 5 contributes at least one factor of 5. Numbers like 25, 50, 75, 100 contain an extra factor of 5 because $25 = 5^2$.
$\left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] = 20 + 4 + 0 = 24$
Number of trailing zeros = 24

Ex: **Find the number of trailing zeros in 2×4×6...×250.**
Sol: factors of 2 are abundant, we only count factors of 5.
$2 \times 4 \times 6 \ldots \times 250$
$= (2 \times 1) \times (2 \times 2) \ldots . (2 \times 125)$
$(2 \times 1) \times (2 \times 2) \ldots . (2 \times 125)$
$= 2^{125}(1 \times 2 \times \ldots 125)$
$2^{125}(1 \times 2 \times \ldots 125) = 2^{125} \times 125!$
$\left[\frac{125}{5}\right] + \left[\frac{125}{25}\right] + \left[\frac{125}{125}\right] = 25 + 5 + 1 = 31$
Number of trailing zeros = 31

Divisibility

Number	Divisibility Rule
2	Last digit is 0, 2, 4, 6, 8
3	Sum of digits divisible by 3
4	Last two digits form a number divisible by 4
5	Last digit is 0 or 5
25	Last two digit is 00 or divisibly by 25
6	Number divisible by both 2 and 3
7	Subtract twice the last digit from the remaining; result divisible by 7
8	Last three digits divisible by 8
9	Sum of digits divisible by 9
11	Difference between sum of digits at even and odd places is 0 or divisible by 11

Special cases
1. 1001 = 7×11×13
   $1001 \times abc = abcabc$
2. 10101 = 3×7×13×37
   $10101 \times ab = ababab$

Type 8: Based on Divisibility rules

Ex: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?
Sol: When any number is repeated n times, the combination of the 6-digit number is divisible by 7, 11, and 13.
96 times 9 will be divisible by 13, and only three 9s will remain.
$= \frac{999}{13} \Rightarrow R \rightarrow 11$

Ex: Find the greatest possible value of $(a+b)$ for which the 8-digit number 143b203a is divisible by 15.
Sol: Divisibility of 3 - if sum of its digits is divisible by 3.
Divisibility of 5- if the last number is either 0 or 5, the entire no. is divisible by 5.
Factors of 15 = (3 × 5) Hence, the number should be divisible by both 3 and 5.
So, a can be 0 or 5. But since the question asks for greatest value, a must be 5.
After that sum of the number is = 18+b
For greatest = b=9
So, $(a+b) = (9+5) = 14$

Ex: If the 9-digit number $72x8431y4$ is divisible by 36, find the value of $\left(\frac{x}{y}-\frac{y}{x}\right)$ for the smallest possible value of y, where x and y are natural numbers.

Sol: The divisibility rule of 36 is that the number must be divisible by 4 and 9.
Last two-digit number = $y4$, on putting $y = 2$, the last two-digit number becomes 24,
Therefore, the value of $y = 2$. (y is the smallest possible value)
For divisibility rule of 9,
sum of number = $31 + x$
So, $x = 5$
Now the value
$$\left(\frac{x}{y}-\frac{y}{x}\right) = \left(\frac{5}{2}-\frac{2}{5}\right) = 2\frac{1}{10}$$

Type 9: Divisibility of expressions

Expression	'n' is odd	'n' is even
$x^n - y^n$	$(x - y)$	$(x + y)(x - y)$
$x^n + y^n$	$(x + y)$	$Can't\ say$

Ex: What is the remainder if $(17^{26} - 11^{26})$ is divided by 42?

Sol: n is even. So, $(x + y)(x - y)$
$$= \frac{(17 + 11)(17 - 11)}{42} = \frac{28 \times 6}{42},$$
$Remainder = 0$

Key Point: If 'n' is odd and $a, b, c \dots z$ are consecutive natural number then $(a^n + b^n + \cdots + z^n)$ is divisible by $(a + b + c \dots z)$

Ex: $11^5 + 12^5 + 13^5$ is divisible by?

Sol- using concept
11+12+13 = 36
So, the expression is divisible by 36

Type 10: Digit and its Reverse Form 2-digit and 3 digit and its reverse form

1. **For 2-digit**
   Let, two-digit original number = $10x + y$, Reversed number = $10y + x$
   Sum of original and reserved number $(10x + y) + (10y + x) = 11(x + y)$
   Difference of original and reversed number $(10x + y) - (10y + x) = 9(x - y)$

2. **For 3-digit**
   Let, Hundred digit = x, Tenth digit = y, Unit digit = z So,
   original number = $100x + 10y + z$
   Hundred and unit digits interchange, new number = $100z + 10y + x$
   Difference of original and reverse number = $99(x - z)$

Ex: The sum of the two-digit number and the number obtained by inter-changing the digit is 99. If the difference of digits is 1, then the number is;

Sol: Let, number = $10x + y$
$(10x + y) + (10y + x) = 99$
$x + y = 9$
$x - y = 1$
After the equation $x = 5, y = 4$
So, the number is
$= 10 \times 5 + 4 = 54$

Ex: What will be the smallest natural number to be filled in the blank for the number $23x45678$ to be divisible by 22.

Sol: The divisibility rule for 22 is based on divisibility by both 2 and 11. A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is divisible by 11.
Odd positions sum: $3 + 4 + 6 + 8 = 21$
Even positions sum: $2 + x + 5 + 7 = 14 + x$
$21 - (14 + x) = 7 - x$
$7 - x = 11k$, for some integer k
For the smallest x, let's try k = 0
$7 - x = 0 \Rightarrow x = 7$
Thus, the smallest value of x that makes the number $23x45678$ divisible by 22 is 5.

Ex: How many numbers are there from 700 to 950 (including both) which are neither divisible by 3 nor by 7?

Sol:
Neither divisible by 3 nor by 7
= Total − (Divisible by 3 or 7) = Total − [N(3) + N(7) − N(21)]
Total number from 700 to 950 = 251
Divisible by 3 = $\frac{251}{3} \approx 83$
Divisible by 7 = $\frac{251}{7} \approx 35$
Divisible by 21= $\frac{251}{21} \approx 11$
Required number
$= 251 - (83 + 35 - 11) = 144$

Factorization

A factor of a number is a whole number that can be multiplied by another whole number to result in the original number. In other words, a factor divides the number evenly without leaving any remainder.

Let $N = a^p \times b^q \times c^r$

Total number of factors = $(p+1)(q+1)(r+1)$

Total number of even factors = $p(q+1)(r+1)$

Total number of odd factors = $(q+1)(r+1)$

Sum of all factors $= (a^0 + a^1 + \cdots a^p)(b^0 + b^1 + \cdots b^q)(c^0 + c^1 + \cdots + c^q)$

Sum of even factors $= (2^1 + 2^2 + \cdots + 2^p)(b^0 + b^1 + \cdots + b^q)(c^0 + c^1 + \cdots + c^r)$

Sum of odd factors $= 2^0(b^1 + b^2 + \cdots + b^q)(c^1 + c^2 + \cdots + c^r)$

Average of factor = $\frac{\text{sum of factor}}{\text{number of factor}}$

Sum of reciprocal of factors = $\frac{sum\ of\ factor}{given\ number}$

Number of prime factors $= p + q + r$

Number of distinct prime factor $= no\ of\ prime\ number\ in\ factor$

Number of composite numbers $= total - distinct\ prime\ number - 1$

- Every number has positive and negative factors.
- All numbers in the form $(prime)^2$ have 3 factors.

Type 11: Find the Number of Factors Based

Ex: Find all types of factors and the sum of all types of factors for $N = 3600$.

Sol: $N = 3600 = 2^4 \times 3^2 \times 5^2$

- Total number of factors $= 5 \times 3 \times 3 = 45$
- Number of even factors
  $2 \times (1800) = 2(2^3 \times 3^2 \times 5^2)$
  $= 4 \times 3 \times 3 = 36$
- Number of odd factors $= 3 \times 3 = 9$
- Number of prime factors$= 4 + 2 + 2 = 8$
- Number of distinct Prime factor (visible in factorization) $= 1 + 1 + 1 = 3$
- Number of Composite numbers
  $= 45 - 3 - 1 = 41$
- Perfect square $= (2^2)^2(3^2)^1(5^2)^1$
  $= 3 \times 2 \times 2 = 12$
- Perfect cube $= (2^3)^1 = (1 + 1) = 2$
- Sum of all factor
  $= (2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$
  $= 12493$
- Sum of even factor
  $= (2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 12090$
- Sum of odd factor
  $= (3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 403$
- Sum of prime $= 2 + 3 + 5 = 10$
- Sum of composite number
  $= total\ sum - (sum\ of\ prime + 1) =$
  $12493 - 10 - 1 = 12482$
- Perfect square

$= (2^0 + 2^2 + 2^4)(3^0 + 3^2)(5^0 + 5^1) = 5460$

Ex: Calculate the total number of prime factors in the expression$(4^{11} \times 5^5 \times 3^2 \times 13^2)$

Sol:

$(4^{11} \times 5^5 \times 3^2 \times 13^2) = 2^{22} \times 3^2 \times 5^5 \times 13^2$

Total number of prime factors

$= 22 + 2 + 5 + 2 = 31$

Ex: How many factors of $(30^{16} \times 16^{18} \times 20^{21})$ are perfect square as well as perfect cube?

Sol: $30^{16} \times 16^{18} \times 20^{21} = 2^{130} \times 3^{16} \times 5^{37}$

When asked to check for a perfect square or perfect cube, check if the power is a multiple of 6.

$2^{130} \times 3^{16} \times 5^{37}$
$= 2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$

$2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$
$= (2^6)^{21} \times 2^4 \times (3^6)^2 \times 3^4 \times (5^6)^6 \times 5^1$

The total number of factors that are both perfect squares and perfect cubes

$= (21 + 1)(2 + 1)(6 + 1) = 462$

Ex: How many factors of the number $2^8 \times 3^6 \times 5^4 \times 10^5$ are multiple of 120.

Sol: $N = 2^8 \times 3^6 \times 5^4 \times 10^5 = 2^{13} \times 3^6 \times 5^9$

$\frac{N}{120} = \frac{2^{13} \times 3^6 \times 5^9}{2^3 \times 3^1 \times 5^1} = 2^{10} \times 3^5 \times 5^8$

Number of factors $= 11 \times 6 \times 9 = 594$

Remainder

Let suppose number N when divided by divisor D, leaves the remainder as R and quotient as Q.

$N = D \times Q + R$

Dividend = Divisor × quotient + Remainder

If a gives r remainder when divided by n, then ka gives kr remainder when divided by n.

If a and b give r_1 and r_2 remainders respectively when divided by n then.

$a + b$ divided by n, gives remainder $r_1 + r_2$

$a \times b$ divided by n, gives remainder $r_1 + r_2$

Type 12: Question based on remainder theorem

Ex: A number being divided by 52 gives a remainder 45. If the number is divided by 13, the remainder will be?

Sol: Since 13 is a multiple of 52, we can directly divide the remainder of one by the divisor.

$= \frac{45}{13} \Rightarrow R \to 6$

Ex: A number when divided by 12 leaves 5 remainder. What is the remainder if square of this number is divided by 8.

Sol: When a mathematical operation is performed on a number, the same operation can also be performed on the remainder.

$= \frac{5^2}{8} = \frac{25}{8} \Rightarrow R \to 1$

Ex: If the dividend is 45, the divisor is 8, and the quotient is 5, find the remainder.

Sol- $N = D \times Q + R$

$45 = 8 \times 5 + Remainder$

Remainder $= 45 - 40 = 5$

Type 13: Important concept of remainder

$\frac{(x+a)^n}{x} \Rightarrow R \to a^n$

$\frac{(x+1)^n}{x} \Rightarrow R \to 1$

$\frac{(x-1)^n}{x} \Rightarrow R \to (1)^n$

$R = 1 \; when \; n \; is \; even$

$R = -1 \; when \; n \; is \; odd$

Special case:

$\frac{4}{6} \Rightarrow R \to 4$

$\frac{4^n}{6} \Rightarrow R \to 4$

Ex: What is the remainder when 2^{75} is divided by 14?

Sol:

$\frac{2^{75}}{15} = \frac{(2^4)^{18}.2^3}{15} = \frac{(16-1)^{18} \times 8}{15} \Rightarrow R \to 8$

Ex: What is the remainder if 2^{192} is divided by 6?

Sol: Using the concept

$\frac{2^{192}}{6} = \frac{4^{96}}{6} \Rightarrow R \to 4$

Ex: What is the remainder when 37^{47} is divided by 19.

Sol:

$\frac{37^{47}}{19} = \frac{(38-1)^{47}}{19} \Rightarrow R \to -1$

Remainder cannot be negative, so the remainder is subtracted from the divisor.

$R \to -1 \Rightarrow R \to 19 - 1 = 18$

Type 14: Fermat's little theorem

$If \; \frac{a^{P-1}}{P}, \quad \Rightarrow R \to 1$

Conditions

1. P is prime number.
2. a, P are co-prime.

Ex: What is the remainder when 21^{47} is divided by 47?

Sol: 21 and 47 are coprime number so we can use Fermat's little theorem.

$\frac{21^{47}}{47} = \frac{21^{46} \times 21}{47} \Rightarrow R \to 21$

Type 15: Euler's (totient) theorem

$\frac{A^X}{d} \Rightarrow R \to 1$

Where A and d are co-prime.

X – number of numbers less than d and co-prime to d.

$X = d\left(1 - \frac{1}{P^1}\right)\left(1 - \frac{1}{P^2}\right) \dots \dots \dots$

Where P^1 and P^2 are distinct prime factors of d.

Ex: **If φ is the Euler's totient functions, then $\varphi(92)$ is.**

Sol: $92 = 2^2 \times 23$

$$\varphi(92) = 92\left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{23}\right) = 44$$

Ex: **7^{82} is divided by 11, R=?**

Sol: To find the remainder when $7^{82}$is divided by 11, we can use Fermat's Little Theorem, which states that if p is a prime number and $a$is an integer not divisible by p, then

$$\varphi(11) = 11\left(1 - \frac{1}{11}\right) = 10$$

82 is divided by 10, and the remainder is

$2.\frac{7^2}{11} \Rightarrow R \to 5$

Type 16: Wilson's theorem

If P is prime number, then,

$\frac{(P-1)!}{P} \quad \Rightarrow R \to P - 1$

$\frac{(P-2)!}{P} \quad \Rightarrow R \to 1$

Ex: **What will be the remainder when 568! is divided by 569.**

Sol- Using the Wilson theorem

$\frac{568!}{596} = \frac{(569-1)!}{P} \quad \Rightarrow R \to 569 - 1$

$= 568$

2 CHAPTER Simplification

Simplification means reducing an expression to its simplest and most compact form by applying standard mathematical rules.

Order to solve

——	Vinculum/Line/Bar bracket
B	Bracket
O	Of
D	Division
M	Multiplication
A	Addition
S	Subtraction

Types of brackets and solving order

( )	Small bracket
{ }	Curly bracket
[ ]	Square bracket

Important Exam – Oriented Types

Type 1: VBODMAS based questions

Ex; The value of

$$\mathbf{441 \div \left[270 \div \tfrac{3}{7} \text{ of } 35 + \left(17 \div \tfrac{1}{3}\right) - \left(8\tfrac{1}{2} - \tfrac{5}{2}\right)\right]}$$

is:

Sol: Using concept of BODMAS

$= 441 \div \left[270 \div \left(\frac{3}{7} \times 35\right) + (17 \times 3) - (17/2 - 5/2)\right]$

$= 441 \div \left[270 \div (3 \times 5) + 51 - \left(\frac{12}{2}\right)\right]$

$= 441 \div [270 \div 15 + 51 - 6]$

$= 441 \div 63 = 7$

Ex: Simplify the following expression.

$$\mathbf{\frac{3\frac{2}{3} - 5\frac{7}{9} \div \frac{1}{3} \times 1\frac{2}{13}}{\frac{2}{3} \text{ of } 1\frac{1}{3} \div \frac{1}{3}}}$$

Sol: Using concept of BODMAS

$= \dfrac{3\frac{2}{3} - 5\frac{7}{9} \div \frac{1}{3} \times 1\frac{2}{13}}{\frac{2}{3} \text{ of } 1\frac{1}{3} \div \frac{1}{3}}$

$= \dfrac{\left[\frac{11}{3} - \frac{52}{9} \times 3 \times \frac{15}{13}\right]}{\frac{2}{3} \times \frac{4}{3} \times 3}$

$= \dfrac{11 - 60}{3} \times \dfrac{3}{8} = -6\dfrac{1}{8}$

Ex: Evaluate: $\mathbf{[7 + 7 \times (7 + 7 \div 7)] + 7 \div 7}$

Sol: Using concept of BODMAS

$= [7 + 7 \times (7 + 7 \div 7)] + 7 \div 7$

$= [7 + 7 \times (7 + 1)] + 1$

$= 63 + 1 = 64$

Ex: The simplified value of the expression

$$\mathbf{\frac{1.0025 + 6.25 \times 10^{-6}}{0.0025 + 0.95}}$$

Sol:

$= \dfrac{1.0025 + 6.25 \times 10^{-6}}{0.0025 + 0.95}$

$= \dfrac{1.0025 + 0.00000625}{0.0025 + 0.95}$

$= \dfrac{1.00250625}{0.9525} = 1.0525$

Equations based

Ex: What is $\mathbf{12\%}$ **of** $\mathbf{4\%}$ **of** $\mathbf{7\%}$ **of** $\mathbf{2 \times 10^6}$**?**

Sol: $10^6 = (10^2)^3$

Number $= 2 \times 100^3 \times \dfrac{12}{100} \times \dfrac{4}{100} \times \dfrac{7}{100}$

$= 2 \times 100 \times 100 \times 100 \times \dfrac{12}{100} \times \dfrac{4}{100} \times \dfrac{7}{100}$

$= 2 \times 12 \times 4 \times 7 = 672$

Ex: Simplify the following expression

$\mathbf{25 - [16 - \{14 - (18 - \overline{8 + 3})\}]}$

Sol: Using concept of BODMAS

$= 25 - [16 - \{14 - (18 - \overline{8 + 3})\}]$

$= 25 - [16 - \{14 - (18 - 11)\}]$

$= 25 - [16 - \{14 - 7\}] = 25 - [16 - 7]$

$= 25 - 9 = 16$

Type 2: Based on algebraic formula

Ex: The value of expression

$$\frac{\mathbf{428 \times 428 \times 428 + 348 \times 348 \times 348}}{\mathbf{428 \times 428 - 428 \times 348 + 348 \times 348}}$$

Sol: Use the identity

$$\frac{a^3+b^3}{a^2-ab+b^2}=a+b$$

$a=428, b=348$

$a+b=428+348=776$

Ex: Simplify the following $\mathbf{25^3-75^3+50^3}$

Sol- $a=25, b=50, c=-75$

$a+b+c=25+50-75=0$

$a^3+b^3+c^3=3abc$

$3abc=3\times 25\times 50\times(-75)=-281250$

Ex: The value of expression is

$$\frac{[\mathbf{68.4^2+31.6^2}]}{[(\mathbf{684+316})^2+(\mathbf{684-316})^2]}$$

Sol: $a=684, \quad b=316$

$$=\frac{1}{100}\times\frac{[a^2+b^2]}{[(a+b)^2+(a-b)^2]}$$

$$=\frac{1}{100}\times\frac{[a^2+b^2]}{2\times[a^2+b^2]}=0.005$$

Ex: Simplify

$$\frac{\mathbf{(8.3)^3+(9.2)^3+(6.1)^3-3\times 8.3\times 9.2\times 6.1}}{\mathbf{(8.3)^2+(9.2)^2+(6.1)^2-8.3\times 9.2-9.2\times 6.1-6.1\times 8.3}}$$

Sol- We know that,

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}=(a+b+c)$$

$a=8.3, b=9.2, c=6.1$

$a+b+c=8.3+9.2+6.1=23.6$

Type 3: Comparison of fractions

Proper fraction

A proper fraction is a fraction in which the numerator is smaller than the denominator. In such fractions, the value is always less than 1.

$\frac{3}{5}, \frac{7}{12}, \frac{9}{10} \ldots. etc$

To compare

- **Step 1:** Take the difference of Numerator and Denominator of each of the fractions
- **Step 2:** The differences should be the same. If they are not equal, first find the LCM of the differences and then multiply each fraction so that all differences become equal.
- **Step 3:** Fraction of smaller numerator will be least and fraction with greater numerator will be greatest.

Improper fraction

A fraction whose numerator is greater than or equal to its denominator is called an improper fraction.

To compare

- **Step 1:** Take the difference of numerator and denominator of each of the fractions.
- **Step 2:** The differences should be the same. If they are not equal, first find the LCM of the differences and then multiply each fraction so that all differences become equal.
- **Step 3:** Fraction of smaller numerator will be greatest and fractions with greater numerator will be smallest.

Key Point:

- **When denominator is equal:** Fractions with greater numerator will be greatest and vice-versa.
- **When numerator is equal:** Fractions with smaller denominator will be greatest and vice-versa.
- If we increase numerator and decrease denominator, then resultant fraction will be greater.
- If we decrease numerators and increase denominator, then resultant fraction will be smaller.

Ex: If we take fractions $\mathbf{0.75}, \frac{3}{5}, \frac{7}{25}, \mathbf{1.57}$ in descending order, then what will be the third fraction from the beginning?

Sol: convert fractions into decimals to compare. 0.75, 0.6, 0.28, 1.57

Descending order – 1.57, 0.75, 0.6, 0.28

The answer is – 0.6

Ex: Find the greatest among the following-
$\mathbf{\frac{7}{8}, \frac{6}{7}, \frac{8}{9}}$

Sol: Fraction is proper fraction, and the difference is 1. Fraction of smaller numerator will be least and fraction with greater numerator will be greatest.

$\frac{8}{9} > \frac{7}{8} > \frac{6}{7}$

Ex: Find the difference between the greatest and smallest value among the fractions. $\mathbf{\frac{5}{11}, \frac{5}{7}, \frac{3}{8}, \frac{6}{13}}$

Sol: L.C.M. of 11, 7, 8, 13 = 8008

$= \left(\frac{5}{11}, \frac{5}{7}, \frac{3}{8}, \frac{6}{13}\right) \times 8008$

$= 5 \times 728, \quad 5 \times 1144, \quad 3 \times 1001, \quad 6 \times 616$

$= 3640, \quad 5720, \quad 3003, \quad 3696$

Least fraction $= \frac{3}{8}$

Greatest fraction $= \frac{5}{7}$

Difference $= \left(\frac{5}{7} - \frac{3}{8}\right) = \frac{19}{56}$

Ex: Find the greatest among the following -
$\mathbf{\frac{9}{8}, \frac{8}{7}, \frac{7}{6}}$

Sol: Fraction is improper fraction, and the difference is 1. Fraction of smaller numerator will be greatest and fractions with greater numerator will be smallest.

$\frac{9}{8} < \frac{8}{7} < \frac{7}{6}$

Cross multiplication method

Ex: Find the greatest among the following –
$\mathbf{\frac{8}{11}, \frac{15}{19}, \frac{4}{5}, \frac{13}{21}}$

Sol: Cross multiply pairwise

$\frac{8}{11} \times \frac{15}{19} \Rightarrow 152 < 165 \Rightarrow \frac{8}{11}$ (Eliminated)

$\frac{15}{19} \times \frac{4}{5} \Rightarrow 75 < 76 \Rightarrow \frac{15}{19}$ (Eliminated)

$\frac{4}{5} \times \frac{13}{21} \Rightarrow 84 > 65 \Rightarrow \frac{13}{21}$ (Eliminated)

$\therefore \frac{4}{5}$ is the greatest fraction.

Ex: Find the greatest fraction
$\mathbf{\frac{8}{11}, \frac{13}{17}, \frac{21}{29}, \frac{34}{47}}$

Sol: Cross multiply pairwise

$\frac{8}{11} \times \frac{13}{17} \Rightarrow 136 < 143 \Rightarrow \frac{8}{11}$ (Eliminated)

$\frac{13}{17} \times \frac{21}{29} \Rightarrow 377 > 357 \Rightarrow \frac{21}{29}$ (Eliminated)

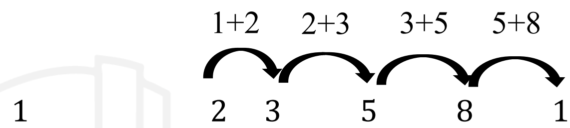
$\frac{13}{17} \times \frac{34}{47} \Rightarrow 611 > 578 \Rightarrow \frac{34}{47}$ (Eliminated)

$\therefore \frac{13}{17}$ is the greatest fraction.

Type 4: Ladder fraction

When the expression is in the form 1 + …

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{2}}}}$$



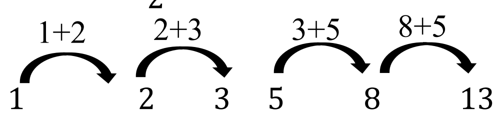
Step 1: First write the term $\frac{1}{3}$. Write ‘1’ first and then write ‘3’. As many times as ‘1’ is given in the question immediately preceding number will be added to the next number.

Step 2: And finally, write the last number as a fraction with the number immediately preceding it.

$Ans = \frac{13}{8}$

When the expression is in the form 1/ …

$$\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{2}}}}$$



Step 1: First write the term $\frac{1}{3}$. Write ‘1’ first and then write ‘3’. As many times as ‘1’ is given in the question immediately preceding number will be added to the next number

Step 2: And finally, write the second last number as a fraction with the number immediately succeeding it.

$Ans = \frac{8}{13}$

Ex: Find the value of the following

$$\boldsymbol{x = 4 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \frac{4}{5}}}}}$$

Sol: Follow the arrows

1 3 2 4

+ × + × + × + ×

4 5 9 32 73 324

$$x = \frac{324}{73}$$

Ex: Find the value of the following

$$\boldsymbol{x = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - \frac{2}{3}}}}}$$

Sol:

3-2 3-1 -2-1 -3-(-2)

2 3 1 -2 -3 -1

$$x = \frac{-1}{-3} = \frac{1}{3}$$

Ex: Find the value of $\boldsymbol{a + b + c + d}$.

$$\boldsymbol{\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \frac{1}{d}}}} = \frac{17}{60}}$$

Sol:

$\frac{60}{17} = 3 + \frac{9}{17} \quad \rightarrow a = 3$

$\frac{17}{9} = 1 + \frac{8}{9} \quad \rightarrow b = 1$

$\frac{9}{8} = 1 + \frac{1}{8} \quad \rightarrow c = 1$

$\frac{8}{1} = 8 = d$

$a + b + c + d = 3 + 1 + 1 + 8 = 13$

Ex: Find the value of the following

$$\boldsymbol{3 + \cfrac{5}{5 + \cfrac{3}{1 + \frac{7}{5}}}}$$

Sol-

×1+7 ×5+5 ×3+4

7 5 $\frac{12}{3} = 4$ $\frac{25}{5} = 5$ 19

$$A = \frac{19}{5}$$

Type 5: Special Series based questions

$$\frac{1}{2 \times 3}$$

In the given fraction, the difference between 2 and 3 is 1, which is equal to the numerator. Hence, the fraction can be written as

$$\frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{4}{3 \times 5 \times 7} = \frac{1}{3 \times 5} - \frac{1}{5 \times 7}$$

In the given fraction, the difference between 3 and 7 is 4, which is equal to the numerator. Hence, the fraction can be written as

$$\frac{6}{3 \times 5 \times 7 \times 9} = \frac{1}{3 \times 5 \times 7} - \frac{1}{5 \times 7 \times 9}$$

Ex: Find the value of $\boldsymbol{x}$.

$$\boldsymbol{x = \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110}}$$

Sol:

$$x = \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110}$$

$$= \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10} + \frac{1}{10 \times 11}$$

$$x = \frac{1}{5} - \frac{1}{11} = \frac{6}{55}$$

Ex: Find the value of A.

$$\boldsymbol{A = \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \cdots\cdots\cdots + \frac{19}{9^2 \times 10^2}}$$

Sol:

$$A = \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \cdots\cdots\cdots + \frac{19}{9^2 \times 10^2}$$

$$= \frac{5}{4 \times 9} + \frac{7}{9 \times 16} + \cdots\cdots\cdots + \frac{19}{81 \times 100}$$

$$A = \frac{1}{4} - \frac{1}{100} = \frac{6}{25}$$

3 CHAPTER

LCM & HCF

LCM- least common multiple

- The smallest number that is exactly divisible by each of the given numbers is called their LCM.
  Example:
  Factor of 12 – 12,24,36, 48, …
  Factor of 16 - 16,32,48,64, …
  Common multiple – 48, 96...
  Least common multiple – 48

HCF- highest common factor

- The largest number that exactly divides each of the given numbers is called their HCF.
  Example
  Factors of 24 - 1, 2, 3, 4, 6, 8, 12, 24
  Factors of 36 - 1, 2, 3, 4, 6, 9, 12, 18, 36
  Common factors of 24 and 36 - 1, 2, 3, 4, 6, 12
  Highest common factor – 12

Important Exam – Oriented Types

Type 1: Least common Multiple – based

Find the smallest number that is exactly divisible by x, y and z.
$= LCM\ of\ (x, y, z)$

1. Find the smallest number that is divided by x, y and z and leaves remainder r in each case.
   $= LCM\ of\ (x, y, z) + r$
2. Find the smallest number that is divisible by x, y, and z, and leaves remainders of a, b, and c, respectively.
   $= LCM\ of\ (x, y, z) - k$
   $Where\ \ k = (x - a) = (y - b) = (z - c)$

Ex: Find the smallest number which when divided 5,6,7 and 8 gives remainder 3 in each case.
Sol: Smallest number $=$ LCM of $(x, y, z) + r$
$= LCM(5,6,7,8) + 3 = 840 + 3 = 843$

Ex: Find the smallest number that, when divided by 25, 15, and 30, leaves remainders of 21, 11, and 26, respectively.
Sol:
$k = (25 - 21) = (15 - 11) = (30 - 26) = 4$
$= LCM\ of\ (25,15,30) - 4 = 146$

Ex: When the numbers 12, 16, 18, 20, and 25 divide the least number x, the remainder in each case is 4, but x is divisible by 7. What is the digit at the thousands' place in x
Sol:
$LCM = LCM\ of\ (12,16,18,20,25) = 3600$
Given, 3600 should be divisible by 7.
Let $(3600k + 4)$ is divisible by 7.
So, we can write
$$\frac{3600k + 4}{7}$$
Put $k = 1,2,3,4,5..$ if we put $k = 5$ then expression is divisible by 7.
Number $= 3600 \times 5 + 4 = 18004$
Digit at thousands' place in 18004 is 8.

Type 2: Highest Common factor – Based

1. Find the largest number which is divisible by x, y and z exactly.
   $= HCF\ of\ (x, y, z)$
2. Largest number which can divide x, y and z and leaves remainder r.
   $= HCF\ of\ (x, y, z) + r$
   If r is absent.
   $= HCF\ of\ |x - y|, |y - z|, |z - x|$
3. Largest number which can be divide x, y, z and leaves remainder a, b, c respectively.
   $= HCF\ of\ (x - a)(y - b)(z - c)$

Ex: If $X = 2^3 \times 3^{10} \times 5^1$ and $Y = 2^5 \times 3^1 \times 7^1$, then find the HCF of X and Y?
Sol: The largest number that exactly divides each of the given numbers is called their HCF.
$HCF = 2^3 \times 3^1$

Ex: Find the greatest possible length which can be used to measure exactly the length 7m,3m 85cm, 12m 95cm.

Sol:

$= \text{HCF of } (700, 385, 1295)$

$= \text{HCF of } (x-a)(y-b)(z-c)$

$\text{HCF of } (700-385)(1295-385)(1295-700)$

$= \text{HCF of } (315)(1225)(595)$

$\text{HCF of } (315)(1225)(595) = 35 \text{ cm}$

Type 3: LCM and HCF of fractions

$$\text{LCM of } \left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{LCM of } (a, c, e)}{\text{HCF of } (b, d, f)}$$

$$\text{HCF of } \left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{HCF of } (a, c, e)}{\text{LCM of } (b, d, f)}$$

Key point – The fraction must be in its simplest form.

Ex: What is the LCM of 3.6,1.8 and 0.144?

Sol:

$$\frac{36}{10} = \frac{18}{5}, \quad \frac{18}{10} = \frac{9}{5}, \quad \frac{144}{1000} = \frac{18}{125}$$

$$\text{LCM of } \left(\frac{18}{5}, \frac{9}{5}, \frac{18}{125}\right) = \frac{\text{LCM of } (18,9,18)}{\text{HCF of } (5,5,125)}$$

$$= \frac{18}{5} = 3.6$$

LCM & HCF of powers

$a^p + 1, \quad a^q + 1$

$\text{LCM of } (a^p + 1)(a^q + 1) = a^{\text{LCM of } (p,q)} + 1$

$\text{HCF of } (a^p + 1)(a^q + 1) = a^{\text{HCF of } (p,q)} + 1$

Ex: What is the HCF $(2^{36} - 1)$ and $(2^{45} - 1)$.

Sol:

$HCF\ of\ (a^p + 1)(a^q + 1) = a^{HCF\ of\ (p,q)} + 1$

$\text{HCF of} (2^{36} - 1)(2^{45} - 1)$

$= 2^{\text{HCF of } (36,45)} + 1 = 2^9 + 1 = 511$

Type 4: Relationship between LCM and HCF Based

Let the 2 numbers are

$N_1 = ha, N_2 = hb \rightarrow$ 'a' and 'b' are coprime

$\rightarrow h(a, b) = 1$

$\text{LCM} \times \text{HCF} = N_1 \times N_2$

$\text{LCM} = hab$

$\text{sum} = (a+b)h$

$\text{difference} = (a-b)h$

$\text{Product} = h^2ab$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{ab}{1}$$

Ex: Two numbers are in the ratio of 5:7. the product of their LCM and HCF is 12635. Then the sum of the number will be.

Sol: Let HCF = x, then numbers are 5x, 7x

$5x \times 7x = 12635$

$x = 19$

$\text{sum} = (a+b)h = (5+7)19 = 228$

Ex: The sum of and difference between the LCM and HCF of two numbers are 512 and 496, respectively. If one number is 72, then the other number is.

Sol: Let second number is N_1

$LCM + HCF = 512$

$LCM - HCF = 496$

After solving the equations

$LCM = 504, \quad HCF = 8$

$72 \times N_2 = 504 \times 8$

$N_2 = 56$

Type 5: LCM and HCF questions based on ratios

Ex: Three numbers are in the ratio $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$. The difference between the largest and the smallest number is 21. The sum of all the numbers is.

Sol:

largest number − smallest number = 21

$$\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9$$

Now, let the ratio be x then, numbers are $6x, 8x$ and $9x$

$9x - 6x = 21$

$x = 7$

Sum of all three numbers

$= (6 + 8 + 9) \times 7 = 161$

Type 6: Questions based on bells and traffic signals

Ex: There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 p.m., they together turn green. At what time will they change to green next, simultaneously?

Sol: The three traffic signals turn from green to red at and interval 25 second, 39 second, and 60 seconds.

$LCM(25,39,60) = 3900\, sec = 65\, min$

$Total = 2\, times = 65 + 65 = 130\, min$

Time = 4:10 P.M.

Ex: Four ropes of lengths 102m,119m,153m and 204 m are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?

Sol:

HCF= equal length in which ropes can be cut

HCF(102,119,153,204) = 17

Total parts = $\frac{102}{17} + \frac{119}{17} + \frac{153}{17} + \frac{204}{17}$

$= 6 + 7 + 9 + 12 = 34$

Type 7: LCM and HCF of polynomials

Ex: What is the HCF of $(x^8 - y^8)$ and $(x^7 - y^7 + x^5y^2 - x^2y^5)$.

Sol:

$(x^8 - y^8) = (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

$(x^7 - y^7 + x^5y^2 - x^2y^5)$

$= x^7 + x^5y^2 - y^7 - x^2y^5$

$x^7 + x^5y^2 - y^7 - x^2y^5 = (x^2 + y^2)(x^5 - y^5)$

$(x^2 + y^2)(x^5 - y^5)$

$= (x^2 + y^2)(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

HCF of $(x^8 - y^8)(x^7 - y^7 + x^5y^2 - x^2y^5)$

$= (x^2 + y^2)(x - y)$

HCF $= (x^2 + y^2)(x - y)$

$= x^3 - x^2y + xy^2 - y^3$