



Rajasthan – CET

Graduate Level

Common Eligibility Test (CET)

Volume - 5

Logical Reasoning & Mental Ability



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CHAPTER

English Alphabet Test



- This chapter includes questions based on the English alphabet (A–Z). Candidates should know the positions of all 26 letters and basic related concepts clearly.
- **Letters are of two types:**
 - ✓ Vowels – A, E, I, O, U (There are 5 vowels in the English alphabet.)

- ✓ Consonants – B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z (There are 21 consonants in the English alphabet.)
- **The alphabet is divided into two halves:**
 - ✓ First Half – A to M (The first half contains 13 letters, i.e., positions 1 to 13.)
 - ✓ Second Half – N to Z (The second half contains 13 letters, i.e., positions 14 to 26.)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Short trick:

- **EJOTY (इजोटी)**

5	10	15	20	25
E	J	O	T	Y

- **CFILORUX (सिफिलोरक्स)**

3	6	9	12	15	18	21	24
C	F	I	L	O	R	U	X

- **Finding letters from the right side can be simplified using the formula:**

- ✓ Position from left = 27 – Position from right

- **Trick to Remember Opposite Letters**

Pair	AZ	BY	CX	DW	EV	FU	GT
Trick	AZ	BYe	Cracks	DeW	EVening	Few / Uff	G.T. Road

Pair	HS	IR	JQ	KP	LO	MN
Trick	High School	Indian Railway	Jaipur Queen	KanPur	Life OK	MaN

Type-1 Position of a Letter in the English Alphabet

- If counting in the **same direction** (left to left or right to right), **subtract** both positions.
- If counting in **opposite directions** (left to right or right to left), **add** both positions.

Ex: In the English alphabet, what is the 10th letter to the left of the 21st letter from the left?

Ans: Using the trick: If counting in the same direction (left to left or right to right), subtract both positions.

English alphabet = 21-10= 11th alphabet = K

Ex: In the English alphabet, what is the 9th letter to the left of the 11th letter from the right?

Ans: In these types of question first we calculate from the left and then subtract from the 27.

Alphabet = $11 + 9 = 20^{\text{th}}$ from the left

Original alphabet = $27 - 20 = 7^{\text{th}}$ alphabet = G

Type-2 Forming Letter Pairs

- The letter pair can be formed in both forward and backward directions.
- Multiple pairs can be created from a single word.
- After forming a pair with a letter, you can form another pair with the same letter if they are at the same distance according to the English alphabet.

Ex: How many such pairs of letters are there in the word 'COMBINE', each of which has as many letters between them in the word (both forward and backward direction) as they have between them in the English Alphabet?

Ans: First of all, we will write number of positions of 'COMBINE' word according to English alphabet i.e. C is written as 3 and O will be written as 15 and so on.

C	O	M	B	I	N	E
3	15	13	2	9	14	5

Here we can see that only B and E is making one pair.

Type-3 Letter Problems

Ex: If the first and eighth letters of the word 'REPRESENTATIVE' are swapped, then the second and ninth letters, and so on, are also swapped, what will be the fourth letter to the left of the 6th letter from the left in the new arrangement?

Ans:

Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Original Letter	R	E	P	R	E	S	E	N	T	A	T	I	V	E
New Letter	N	T	A	T	I	V	E	R	E	P	R	E	S	E

Fourth letter to the left of the 6th letter from the left = $6 - 2 = 4^{\text{th}} = \text{T}$

Type-4 Arrangement of English Words

- Arranging English words in alphabetical or dictionary order is called arrangement of words.

Ex: Arrange the following words according to English dictionary arrangement:

(A) Epitaxy (B) Episode (C) Epigene

(D) Epitome (E) Epilogue

Ans: In a Dictionary, every word is arranged alphabetically. Also, in each word each and every letter is positioned alphabetically. So here, dictionary the arrangement of words in a Dictionary, every word is arranged alphabetically.

(C) Epigene (E) Epilogue (B) Episode

(A) Epitaxy (D) Epitome

Therefore, the correct answer is (C), (E), (B), (A), (D)

Type-5 Meaningful Logical Order of Words

Ex: How many meaningful words of 5 letters can be made with the alphabets K, E, D, H, I each being used only once in each word?

Ans: Given letters: K, E, D, H, I

Meaningful word can be formed out of the alphabets K, E, D, H, I:

HIKED: Walk for a long distance, Hence, only one such word is possible.

Ex: If a five-letter meaningful word can be formed by using the first, third, fifth, sixth and eighth letters from the word "HALLOWEEN", then what is the last letter from the left end of the newly formed word?

Ans: The 1st, 3rd, 5th, 6th, and 8th letters from the left end of the word "HALLOWEEN" are H, L, O, W, and E respectively. The only five-letter meaningful word that can be formed using the letters is WHOLE.



Number and Alphabet Series Test



- A series is a systematic arrangement of numbers or letters within a defined group. In competitive exams, sequences of numbers, letters, or a mix of both are presented. One position in the sequence is either left blank or contains an incorrect number or letter. Candidates are tasked with completing the series by selecting the correct option to fill the blank or identify the incorrect element.

Here's a clearer and concise way to solve number series problems:

- **Look for Patterns:** Check if the numbers are increasing or decreasing by a constant (addition/subtraction) or multiplied/divided by a constant (multiplication/division).
- **Find Differences:**
- ✓ If the difference between consecutive numbers is the same, it's an arithmetic series (e.g., 2, 5, 8, 11).
 - ✓ If the second difference (difference of differences) is constant, it's a quadratic series.
- **Check for Multiplication/Division:** Look if each number is multiplied or divided by a constant to get the next number (e.g., 3, 6, 12, 24 where each number is doubled).
- **Recognize Special Sequences:**
- ✓ **Squares:** 1, 4, 9, 16, ...
 - ✓ **Cubes:** 1, 8, 27, ...
 - ✓ **Fibonacci:** Each number is the sum of the two preceding ones (e.g., 0, 1, 1, 2, 3, 5).
- **Check for Ratios:** If the numbers increase by a constant ratio, it's a geometric progression (e.g., 2, 4, 8, 16).
- **Test the Options:** If options are provided, check which one follows the identified pattern.

Type-1 Series in Increasing Order

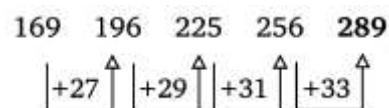
Ex: In the following question, select the missing number from the given series.



169, 196, 225, 256, _?

1. 289 2. 324
3. 441 4. 361

Ans:

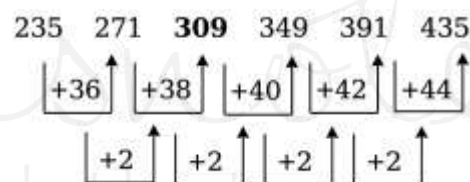


Ex: Which of the following numbers will replace the question mark (?) in the given series?

235, 271, ?, 349, 391, 435

1. 311 2. 307
3. 313 4. 309

Ans:

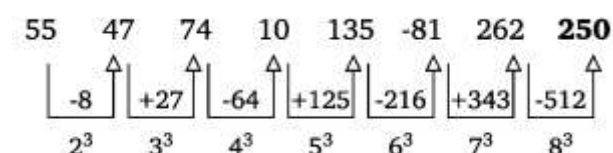


Ex: What number should replace question Mark (?) in the series given below.

55, 47, 74, 10, 135, -81, 262, ?

1. 774 2. -250
3. 343 4. -343

Ans:

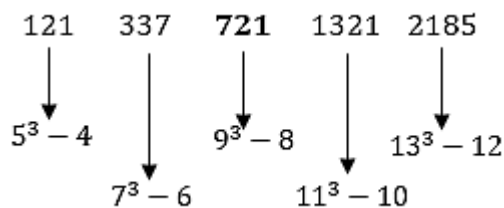


Ex: Find the missing number in the following series:

121, 337, ?, 1321, 2185

1. 713 2. 720
3. 721 4. 737

Ans:



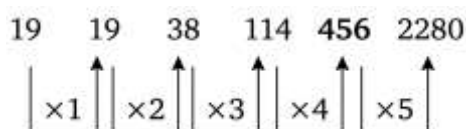
Type-2 Multiplication Based Increasing Series

Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

19, 19, 38, 114, ?, 2280

1. 344 2. 1140
3. 456 4. 224

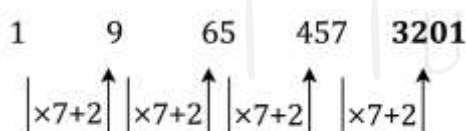
Ans:



Ex: Which of the following numbers will replace the question mark (?) in the given series?

- 1, 9, 65, 457, ?
1. 4258 2. 3125
3. 3201 4. 5289

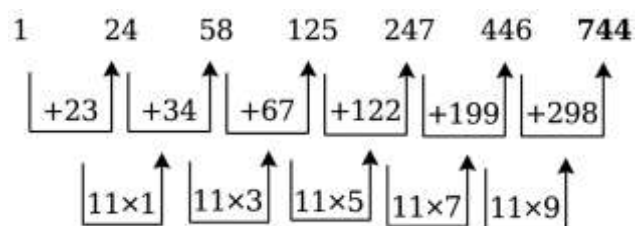
Ans:



Ex: Which of the following numbers will replace the question mark (?) in the given series?

- 1, 24, 58, 125, 247, 446, ?
1. 774 2. 747
3. 744 4. 777

Ans:

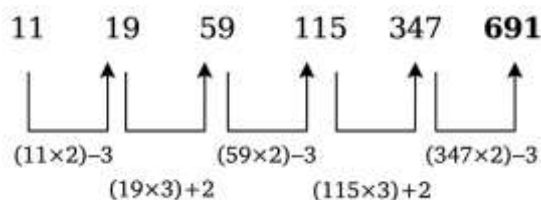


Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

11, 19, 59, 115, 347, ?

1. 697 2. 619
3. 679 4. 691

Ans:



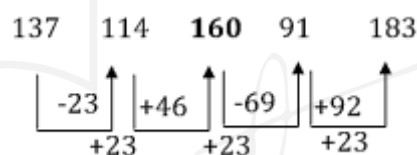
Type-3 Addition and Subtraction Series

Ex: Which of the following number will replace the question mark (?) in the given series?

137, 114, ?, 91, 183

1. 145 2. 160
3. 125 4. 112

Ans:

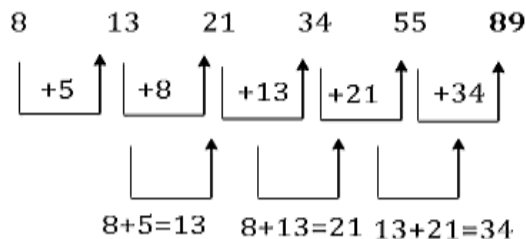


Ex: Which of the following numbers will replace the question mark (?) in the given series?

8, 13, 21, 34, 55, ?

1. 74 2. 68 3. 72 4. 89

Ans:



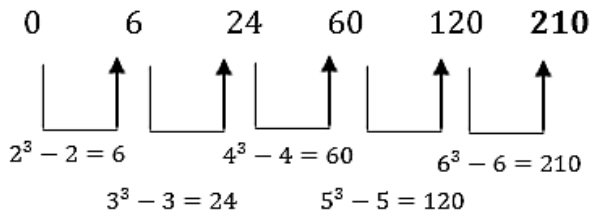
Type-4 Square and Cube Based Series

Ex: Find the next term of the series:

0, 6, 24, 60, 120, ?

1. 180 2. 210
3. 216 4. 240

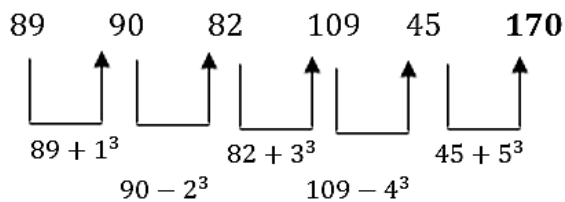
Ans:



Ex: Which number will replace the question mark (?) in the following series?

89, 90, 82, 109, 45, ?

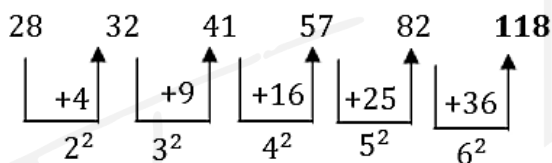
Ans:



Ex: Which number will replace the question mark (?) in the following series?

28, 32, 41, 57, 82, ?

Ans:

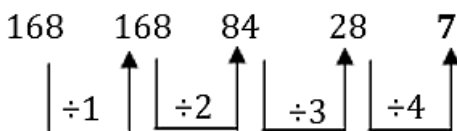


Type-5 Division Based Decreasing Series

Ex: Which number will replace the question mark (?) in the following number series?

168, 168, 84, 28, ?

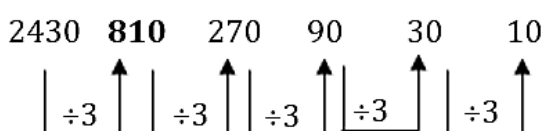
Ans:



Ex: Which number will replace the question mark (?) in the following series?

2430, ?, 270, 90, 30, 10

Ans:



Type-6 Mixed Number Series

Ex: Find the next terms 11, 13, 17, 19, 23, ?

1. 27 2. 29
3. 31 4. 33

Ans:

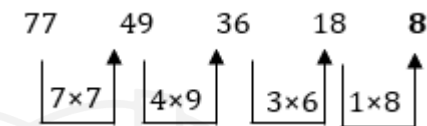
11 → Prime number
13 → Prime number
17 → Prime number
19 → Prime number
23 → Prime number
Next prime number = 29

Ex: What will come in place of the question mark (?) in the following series?

77, 49, 36, 18, ?

1. 10 2. 12
3. 8 4. 16

Ans:

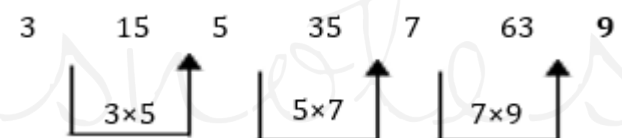


Ex: What will come in place of the question mark (?) in the following series?

3, 15, 5, 35, 7, 63, ?

1. 10 2. 126
3. 9 4. 84

Ans:

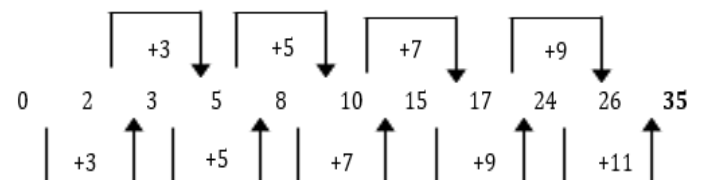


Ex: In the following series, what comes in place of the question mark (?)?

0, 2, 3, 5, 8, 10, 15, 17, 24, 26, ?

1. 28 2. 30
3. 32 4. 35

Ans:



Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

62, 74, 80, 86, 95, ?, 158

1. 113 2. 100
3. 108 4. 122

62 74 80 86 95 113 158

$\uparrow +12$ $\uparrow +6$ $\uparrow +6$ $\uparrow +9$ $\uparrow +18$ $\uparrow +45$

$\uparrow \times 0.5$ $\uparrow \times 1$ $\uparrow \times 1.5$ $\uparrow \times 2$ $\uparrow \times 2.5$

18 37 60 99 120 157

└─┬─┘ └─┬─┘ └─┬─┘ └─┬─┘ └─┬─┘

+19 +23 +39 +21 +37

6

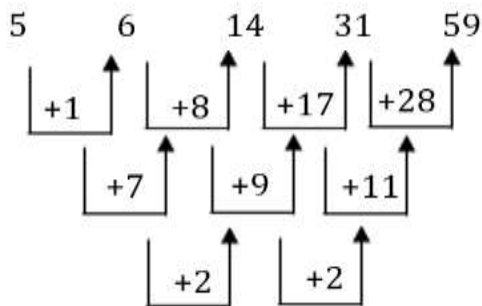
Ex: Find the wrong term from the given number series.

5, 7, 14, 31, 59

1. 31 2. 5

3. 59 4. 7

Ans: Correct answer in 7



In the place of 6, it is written 7, so the answer is 7.

Ex: Find the Wrong number in the given number series:

6, 7, 10, 13, 21, 37

1. 10 2. 37

3. 6 4. 13

5. 21

Ans:

The given number series follows the pattern:

$$6 + 2^0 = 7 \qquad 7 + 2^1 = 10$$

$$10 + 2^2 = 14 \qquad 14 + 2^3 = 21$$

$$21 + 2^4 = 37$$

Wrong Number in the given number series is 13.

Type-8 Alphabet Series

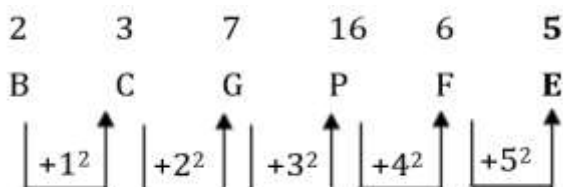
Ex: Which of the following letters will replace the question mark (?) and complete the following letter series?

B, C, G, P, F, ?

1. F 2. E

3. C 4. D

Ans:



Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.

ABCD, CUKA, ENSX, GGAU, ?

1. IQRT 2. MNOQ

3. IRQT 4. IZIR

Ans:

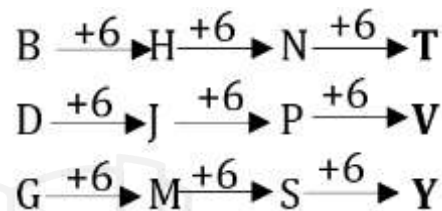
Logic:

- In the first letter, add 2 to its position in the English alphabet.
 - In the second letter, subtract 7 from its position.
 - In the third letter, add 8 to its position.
 - In the fourth letter, subtract 3 from its position.
- Hence, "IZIR" is the correct answer.

Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.

BDG, HJM, NPS, ?

Ans:



Ex: A series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.

KIMnO, qRs Tu, WxYzA, cDeFg, ?

1. iJkLm

2. HiJkL

3. IjKIM

4. hijkl

Ans:

The pattern consists of groups of five letters arranged in the English alphabetical order. After each term, one letter is skipped. Additionally, letters follow an alternating pattern of uppercase and lowercase.

After 'g', the next omitted letter is 'h'. Also, the first letter and every alternate letter must be in uppercase.

Hence, the missing term is IjKIM.

Type-9 Mixed Series

Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.

FK27, LQ64, RW125, ?

1. CX216

2. XB216

3. XC216

4. YB343

Ans:

Letters follow +6 steps in the alphabet

F → L → R → X and K → Q → W → C (after Z it starts from A)

Numbers are cubes of consecutive numbers

27 = 3³, 64 = 4³, 125 = 5³, 216 = 6³

Final answer XC216

Ex: Find the missing term. C4X, F9U, 116R,

1. L25P 2. L250

3. L270 4. L27P

Ans:

First part (letters):

C → F → I → L (each +3)

Second part (numbers):

4 → 9 → 16 → 25 (perfect squares 2², 3², 4², 5²)

Third part (letters):

X → U → R → O (each -3)

So, the missing term is L250

Ex: What will be the next term in the following category?

G7Z26, H8X24, I9V22, _____?

1. J10T20 2. W23J10

3. J10W23 4. W23T20

Ans:

First letter: G → H → I → J (+1 each step)

Number: 7 → 8 → 9 → 10 (+1 each step)

Second letter: Z → X → V → T (-2 each step)

Last number: 26 → 24 → 22 → 20 (-2 each step)

Final answer: J10T20

Type-10 Consecutive Series

In such questions, a sequence of letters and/or numbers is given, formed using uppercase and/or lowercase English alphabets along with numerical values, following a specific pattern from left to right. Certain positions in the sequence are left blank. The task is to identify the correct letters or numbers that should fill these blanks, assuming the pattern continues consistently.

Ex: Select the option that represents the letters that, when sequentially placed from left to right in the blanks below, will complete the letter series.

A _ JL DG LAD J AD _ L

1. DGALGLGJ 2. DGAJGLGJ

3. DAGJGLGL 4. DGALGJGJ

Ans:

Observe carefully:

➤ The series follows a repeating pattern of ADGLJ (with slight shifts).

➤ Letters appear in a cyclic order:

A → D → G → J → L → A...

➤ Fill the blanks to maintain this consistent progression.

Filling accordingly:

A D G J L A D G L L A D G J L A D G J L

So, the missing letters are:

D G A L G J G J

Ex: Which one set of letters when sequentially placed at the gaps in the given letter series shall complete it?

mc_bcm_cbbc_m_bbcm

1. mccb 2. bcmb

3. cbmb 4. Bmmc

Ans:

➤ mccb → mc**mb**cm/c**cb**bc/m**bb**bcm

➤ bcmb → mc**bb**cm/c**cb**bc/m**bb**bcm

➤ cbmb → mc**cb**cm/**cb**bc/m**bb**bcm

➤ bmmc → mc**bb**cm/**mc**bbcm/m**cb**bcm

Hence, 'bmmc' is the correct answer.

Ex: Select the option that represents the letters that, when sequentially placed from left to right in the blanks, will complete the letter series.

g_c_p_c_bc_ps_g_cp_sc

1. bpsppsbp 2. bpcgpsbp

3. bpsgpcbp 4. Bpspgcbp

Ans:

➤ bpsppsbp → g**bc**ppsc**pb**cpssg**bc**ppsc

➤ bpcgpsbp → g**bc**ppccg**bc**ppssg**bc**ppsc

➤ bpsgpcbp → g**bc**ppscg**bc**ppscg**bc**ppsc

➤ bpspgcbp → g**bc**ppsc**pb**cgpscg**bc**ppsc

Hence, 'bpsgpcbp' is the correct answer.

Ex: Select the option that represents the letters that, when placed from left to right in the same sequence in the blanks below, will complete the letter series.

D _ B _ A _ C _ _ G F _ E _ _ H

1. BACDEFGH 2. CABDHEFG

3. BACDHEFG 4. CABDEFGH

Ans:

D _ B _ A _ C _ _ G F _ E _ _ H

Reverse

Order

Forward

Order

Reverse

Order

Forward

Order

CABDHEFG → DCBA/ABCD/HGFE/EFGH

Logical Order of Words



- "Logical sequence of words" involves arranging words in a specific order, as shown in the options based on different number sequences. Usually, the words are listed as 1, 2, 3... or (i), (ii), (iii)... or (a), (b), (c)... etc.
- In this chapter, at least four or five words should be included to determine their correct order. There's no limit to how many numbers can be in the sequence. You might get four, five, or six words that are connected in some way, but they will be given in a random order. You need to arrange them logically in either increasing or decreasing order.
- The questions are usually about everyday topics, cooking recipes, office schedules, timetables, administrative tasks, oceans and continents, animals and birds, different regions of the world, and our neighbouring countries.

Type-1 Arrangement of English Words According to The Dictionary

Ex: In the following question, arrange the given words in the correct sequence and select the correct category from the options provided below the question.

- (i) Fibre (ii) Plant (iii) Saree
(iv) Cotton (v) Cloth

Ans: To arrange the given words in the correct sequence, let's analyse the relationship between them:

- **Plant** – The source of the raw material.
- **Cotton** – A natural fibre that comes from a plant.
- **Fibre** – The cotton fibre is extracted.
- **Cloth** – The fibres are woven or knitted into cloth.
- **Saree** – A type of garment made from the cloth.

Correct sequence: **Plant** → **Cotton** → **Fibre** → **Cloth** → **Saree**.

Ex: Arrange the following in a logical order:

1. Frog 2. Hawk
3. Grasshopper 4. Snake 5. Grass

Ans: To arrange the given words in a logical order, we need to follow the natural food chain or ecological sequence:

1. **Grass** – The primary producer (plants).
2. **Grasshopper** – An herbivore that feeds on grass.
3. **Frog** – A predator that feeds on grasshoppers.
4. **Snake** – A predator that may feed on frogs.
5. **Hawk** – A top predator that may feed on snakes.

The correct order is: **Grass** → **Grasshopper** → **Frog** → **Snake** → **Hawk**.

Ex: Arrange the following in a logical and meaningful order.

1. Print the application and appointment letter.
2. Register on the Passport Seva Online Portal.
3. Log in using your login credentials.
4. Apply.
5. Fill out and submit the application.
6. Make the payment and schedule an appointment.
7. Visit the appointment centre for verification.

Ans: The logical and meaningful order for the given steps is:

- 2-Register on the Passport Seva Online Portal.
- 3-Log in using your login credentials.
- 5-Fill out and submit the application.
- 6-Make the payment and schedule an appointment.
- 1-Print the application and appointment letter.
- 7-Visit the appointment centre for verification.
- 4-Apply.

Correct order: 2 → 3 → 5 → 6 → 1 → 7 → 4.

Ex: Choose the correct sequence for the stages of process writing approach

- a. drafting b. proof reading
c. brain storming d. revising
e. outlining f. finalising

Ans: The correct sequence for the stages of the process writing approach is:

1. Brainstorming (c) – Generating ideas before writing.
2. Outlining (e) – Organizing ideas and creating a structure.
3. Drafting (a) – Writing the first version of the content.
4. Revising (d) – Making changes to improve content and clarity.
5. Proof reading (b) – Checking for grammar, spelling, and punctuation errors.
6. Finalising (f) – Completing the final version.

Correct order: c → e → a → d → b → f.

Ex: Arrange the following processes in the sequence as per the selection process in a company?

1. Employment Interviews
2. Preliminary Interviews
3. Sourcing
4. Appointment letter
5. Job Analysis

Ans: The points given above are steps of the selection process:

5. Job Analysis → 3. Sourcing → 2. Preliminary Interviews → 1. Employment Interviews → 4. Appointment letter

Thus, 53214 is the answer.

Ex: Arrange the following steps in the sequence as per the film making process.

1. Filming 2. Editing 3. Planning
4. Distribution 5. Casting

Ans:

Steps in the film making process are as follows:

3. Planning → 5. Casting → 1. Filming → 2. Editing → 4. Distribution

Thus, '35124' is the correct sequence.

Type-2 Sequence of Occurrence of Events

Ex: Which of the following options shows the logical and meaningful sequence of the given words?

- (1) Cloud (2) Shelter (3) Monsoon
(4) Rain (5) Flood (6) Relief

Ans: Explanation:

1. The Monsoon season leads to Rain.
2. The Rain is often associated with Clouds.
3. The rain can cause a Flood.
4. People seek Shelter during the flood.
5. After the flood, Relief efforts are made.

(3) Monsoon → (4) Rain → (1) Cloud → (5) Flood → (2) Shelter → (6) Relief

Ex: Which of the following options represents the logical and meaningful sequence of the words given below?

1. Verdict 2. Arrest 3. Imprisonment
4. Court 5. Crime 6. Punishment

Ans: If a crime like murder of a person is committed, the police would arrest the suspect and take him to the court where the judgement is made and the punishment, if proved guilty, would include staying in prison for as many as 14 years.

Crime → arrest → court → judgement → punishment → prison

Ex: Arrange the following activities in a logical and meaningful order.

1. Television audio defect
2. Restart enjoying
3. Contact mechanic
4. Television repair done
5. Switch on television

Ans: The correct meaningful logical order is

5. Switch on television
1. Television audio defect
3. Contact mechanic
4. Television repair done
2. Restart enjoying

Hence, the correct order is "5, 1, 3, 4, 2".

Ex: Arrange the following activities in a logical and meaningful order.

1. Prescribing medications
2. Feeling pain in stomach
3. Meeting doctor
4. Diagnosis by doctor
5. Following medication schedule
6. Recovery from pain

Ans: The sequence in which a patient seeks doctor's treatment and recovers is as follows:

2. Feeling pain in stomach
3. Meeting doctor
4. Diagnosis by doctor
1. Prescribing medications
5. Following medication schedule
6. Recovery from pain

Hence, the correct answer is 2, 3, 4, 1, 5, 6.

Type-3 Sequence in Increasing or Decreasing Order

Ex: Arrange the words in ascending order of their area

1. District
2. City
3. State
4. Panchayat
5. Country

Ans: The words arranged in ascending order of their area are:

- 4. Panchayat → 2. City → 1. District → 3. State → 5. Country**

Ex: Which of the given options would be a logical sequence of the following words?

1. Decimeter
2. Millimeter
3. Hectometer
4. Centimeter
5. Decimeter

Ans: The highest unit of measurement among the given units is Hectometre

After that comes Decametre,

After Decametre it is Decimetre,

After Decimetre it is Centimetre,

And the smallest is Millimetre.

- 3. Hectometre → 1. Decametre → 5. Decimeter → 4. Centimetre → 2. Millimetre**

Ex: Arrange the following words in a logical and meaningful order (according to population).

1. Andhra Pradesh
2. Madhya Pradesh
3. Arunachal Pradesh
4. Uttar Pradesh
5. Himachal Pradesh

Ans: Arranging the states according to the population,

Uttar Pradesh has the highest population.

Uttar Pradesh is followed by Madhya Pradesh.

Madhya Pradesh is followed by Andhra Pradesh.

Andhra Pradesh is followed by Himachal Pradesh.

Himachal Pradesh is followed by Arunachal Pradesh.

The correct logical order is,

- Uttar Pradesh (4) → Madhya Pradesh (2) → Andhra Pradesh (1) → Himachal Pradesh (5) → Arunachal Pradesh (3)**

Ex: Arrange the following in a logical and meaningful order. (decreasing order with their heights)

- I. Eiffel Tower
- II. Empire State Building
- III. Qutub Minar
- IV. Burj Khalifa

Ans:

According to height	Height	Height to Tip
Burj Khalifa	828 m	829.8 m (830 m)
Empire State Building	381 m	443 m
Eiffel Tower	300 m	330 m
Qutub Minar	73 m	-

Logical order is,

- Burj Khalifa → Empire State Building → Eiffel Tower → Qutub Minar**

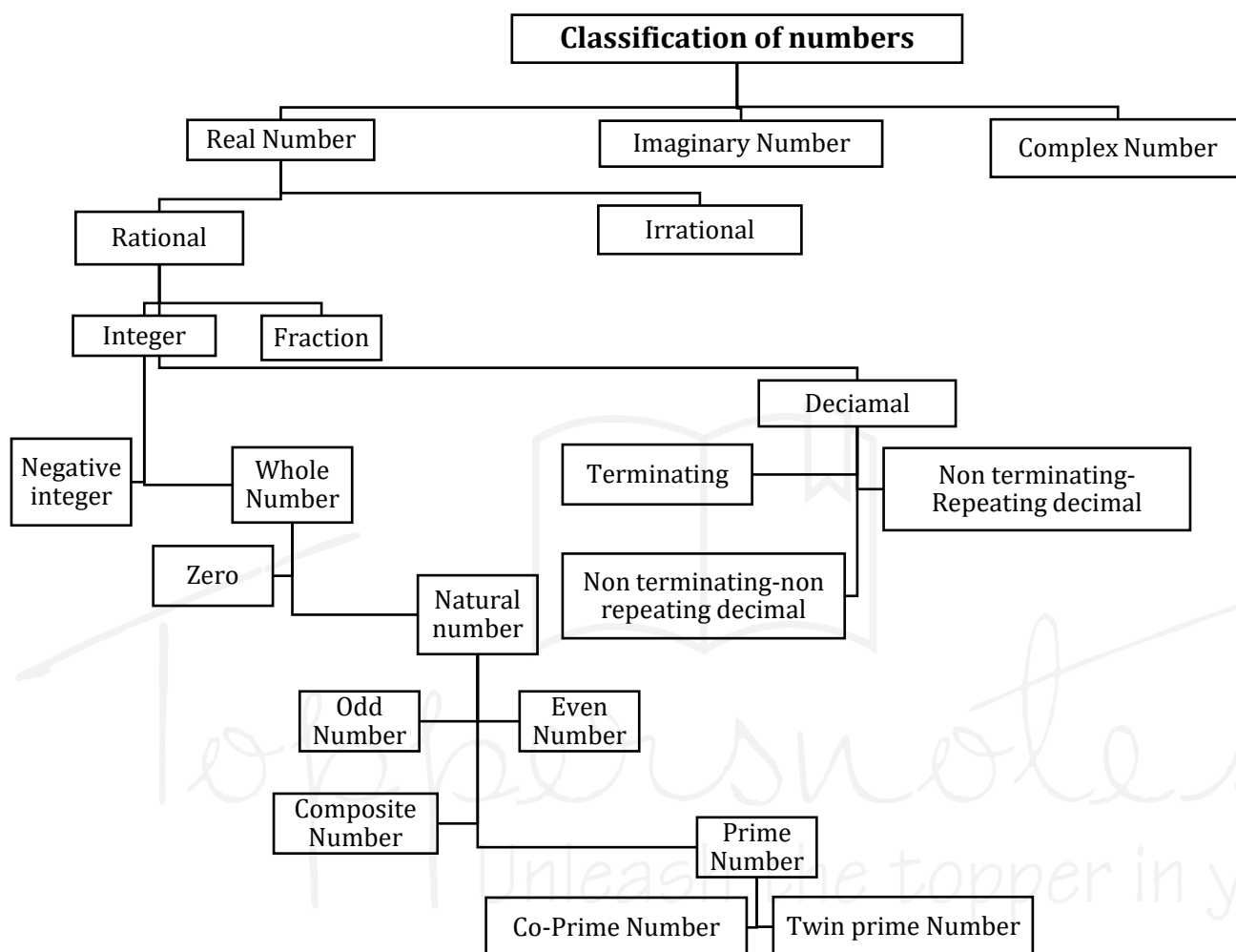
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CHAPTER

Number System



- **Number System:** A number system is a **method of representing and working with numbers** using a defined set of symbols and rules.



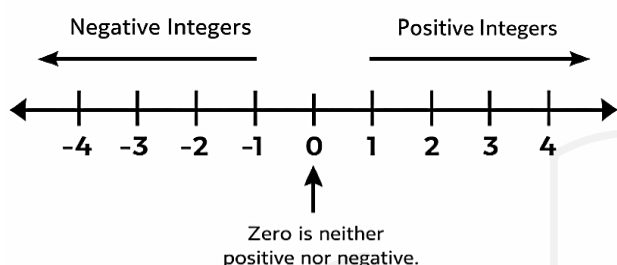
Types	Definition
Real Number	A real number is any number that can be represented on the number line. A real number is a number that includes all rational and irrational numbers and can be expressed as a point on the number line.
Rational Number	A rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Irrational Number	An irrational number is a number that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$
Fraction	A fraction is a number that represents a part of a whole or a ratio of two quantities. It is written in the form $\frac{a}{b}$.
Integer	An integer is a whole number that can be positive, negative, or zero, and does not include any fractional or decimal part

Negative Integer	Negative integers are whole numbers with a negative sign, such as $-1, -2, -3, \dots$	Co-Prime Numbers	Two or more numbers are called co-prime (or relatively prime) if their only common factor (HCF) is 1. ➤ 1 is neither prime or composite.
Whole Number	A whole number is a non-negative integer, including zero.	Twin Prime Numbers	Twin numbers (twin primes) are a pair of prime numbers whose difference is exactly 2. ➤ 5 is only prime which is in 2 twin prime pairs. (3, 5) (5, 7) ➤ Sum of twin prime number (except 3 and 5) is always divisible by 12.
Natural number	Natural numbers are the numbers starting from 1 and increasing by 1 each time. 1, 2, 3, 4, ...	Decimal Number	A decimal number is a number that has a decimal point (.) and consists of a whole part and a fractional part. Ex- 3.5, 12.75 etc
Odd Number	An odd number is a natural number that is not divisible by 2 or in the form of $2n + 1$	Terminating Decimal	A terminating decimal is a decimal number that ends after a finite number of digits after the decimal point. Ex- 2.5, 0.75
Even Numbers	An even number is a natural number that is completely divisible by 2 or in the form of $2n$.	Non-Terminating Repeating Decimal	A non-terminating repeating decimal is a decimal number that does not end and in which one or more digits repeat continuously after the decimal point. Ex- 0. 333., 0. 1212
Prime number	A prime number is a natural number greater than 1 that has exactly two distinct factors: 1 and the number itself. 2, 3, 5, 7. ➤ 2 is the smallest prime number. ➤ 2 is only even prime number. ➤ All prime number (except 2 and 3) can be written in the form of $6n + 1$ or $6n + 5$ where n is natural number however, the converse is not true. ➤ (3, 5, 7) is only group of three primes, which are consecutive odds. ➤ 101 is smallest 3-digit prime number. ➤ 997 is largest 3-digit prime number.	Non-Terminating Non-Repeating Decimal	A non-terminating non-repeating decimal is a decimal number that does not end and does not repeat any digit or pattern after the decimal point. Ex- 1.1412, 3.14
Composite Number	A composite number is a natural number greater than 1 that has more than two factors. A composite number can be divided exactly by 1, itself, and at least one other number. 4, 6, 8, 9, 10, 12, 14... ➤ The smallest composite number is 4. ➤ The smallest odd composite number is 9 If a and b are any two odd primes then $a^2 + b^2$ and $a^2 - b^2$ is composite numbers.	Imaginary Number	An imaginary number is a number that can be written in the form $= bi$ b is a real number i is the imaginary unit, defined by

Complex Number	A complex number is a number that consists of two parts—a real part and an imaginary part—and can be expressed in the standard form $z = a + ib$ a is a real number, called the real part of z b is a real number, called the imaginary part of z i is the imaginary unit, defined by the property
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Integer Number Line



How to check the given number is prime or not?

- To determine whether a number is prime, first find its square root and round it down to the nearest whole number. Then check whether the number is divisible by any prime number up to this value. If it is not divisible by any of them, the number is a prime number.

Between	Number or prime number
1-50	15
1-100	25
1-200	46

Ramanujan number

A Ramanujan number is a number that can be expressed as the sum of two positive cubes in two different ways. It is also known as the Hardy-Ramanujan number or taxi-cab number.

Smallest Ramanujan number = 1729

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Perfect number

A perfect number is a natural number that is equal to the sum of its proper divisors (that is, all its positive divisors excluding the number itself).

Ex: 4 is divisible by 1 and 2, so $1 + 2 = 3 \neq 4$; therefore, 4 is not a perfect number.

6 is divisible by 1, 2 and 3, so, $1 + 2 + 3 = 6 = 6$; therefore, 6 is a perfect number.

Key points

Even + Even = Even

Even $\times$ Even = Even

Even + Odd = odd

Even $\times$ Odd = Even

Odd + odd = Even

Odd $\times$ Odd = Odd

Sum Of ' n ' odd number is odd, if n is odd.

Sum of ' n ' odd number is even, if n is even.

Type 1: Question based on definitions



Ex: 173 is prime number or not?

Sol: The Square root of 173 is approximately 13.

The Prime number less than or equal to 13 are 2, 3, 5, 7, 11 and 13,

Since 173 is not divisible by any number, it is a prime number.

Ex: x, y and z are distinct prime number where $x < y < z$ if $x + y + z = 70$, then what is the value of z ?

Sol: Now, sum is 70, means at least one of the numbers is even. As we know, only one even prime number exists, and that is 2.

2 is also the smallest prime number.

Means $x = 2$

Now, $70 - 2 = 68 = y + z$

By substituting the values of different prime numbers, we obtain the result.

$y = 31$ and $x = 37$

Ex: How many composite numbers are there from 53 to 97?

Sol: If we find the total number of integers between 53 and 97 and then subtract the number of prime numbers between them, we obtain the number of composite numbers.

Total number = $97 - 53 + 1 = 45$ (+1 when both numbers are included)

Total prime number from 53 to 97 is 10

So, composite number = $45 - 10 = 35$

Ex: Which one of the following is TRUE?

- (A) All irrational numbers are real
- (B) All real numbers are irrational.
- (C) Rational numbers are not real.
- (D) Integers are not rational.

Sol: Irrational numbers form a subset of real numbers, so all irrational numbers are real.

Not all real numbers are irrational; rational numbers are also real.

Rational numbers are real numbers.

Integers are a subset of rational numbers since any integer can be written as $\frac{p}{q}$ (e.g., $5 = \frac{5}{1}$).

So, Answer is (A).

Special concept: based on sum of digits for special types of numbers

Number	Square	Sum of digits
11^2	121	3
111^2	12321	9
So, on		
111111111^2	12345678987654321	81

Ex: Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of digit of the resulting number.

Sol- using the concept

$$111111111^2 = \gg 81$$

Type 2: Unit digit based

To find the units digit of an expression, consider only the unit digits of the numbers instead of evaluating the entire expression.

Unit digit of $(a + b)$ = unit digit of a + unit digit of b

Unit digit of $(a - b)$ = unit digit of a - unit digit of b

Unit digit of $a \times b$ = unit digit of $a \times$ unit digit of b

Ex: What are the unit digit of 435×433 .

Sol: Unit digit of $a \times b$ = unit digit of $a \times$ unit digit of b

$$5 \times 3 = 15 \text{ so, unit digit is } 5$$

Cyclicity

Cyclicity in the number system means the repeating pattern of digits or remainders when a number is raised to higher powers. The unit digit remains unchanged for all powers.

$$0 \rightarrow 0 \quad 1 \rightarrow 1$$

$$5 \rightarrow 5 \quad 6 \rightarrow 6$$

Cycle of Length 2-The unit digit alternates between two values.

$$4 \rightarrow 4, 6$$

When the power is odd, the unit digit is 4, and when the power is even, the unit digit is 6.

$$9 \rightarrow 9, 1$$

When the power is odd, the unit digit is 9, and when the power is even, the unit digit is 1.

Cycle of Length 4- The unit digit repeats after four powers.

$$2 \rightarrow 2, 4, 8, 6$$

$$3 \rightarrow 3, 9, 7, 1$$

$$7 \rightarrow 7, 9, 3, 1$$

$$8 \rightarrow 8, 4, 2, 6$$

$$\text{Let } N = x^y$$

To find the unit digit of (N) , we only need to consider the unit digit of the base number (x) .

The unit digit of an exponential expression can be determined by finding the remainder when the power is divided by 4.

Type 3: Cyclicity - Unit Digit Based on Arithmetic Equations



Ex: If $x = (164)^{169} + (333)^{337} - (727)^{726}$ then what is the unit digit of x ?

Sol: In the expression, the **first term** has an odd power of 4, so the unit digit of the first term is 4. For the **second term**, dividing 337 by 4 gives a remainder of 1 so the unit digit of the second term is 3. For the **third term**, dividing 726 by 4 gives a remainder of 2; hence, the unit digit of the third term is 9.

So, unit digit of the expression $4 + 3 - 9 = -2$. If the unit digit comes out negative, add 10 to obtain the correct unit digit. Unit digit is $10 - 2 = 8$.

Ex: What is the unit digit of $1^5 + 2^5 + 3^5 + 4^5 + \dots + 20^5$

Sol: In each term, the cyclicity is 1. Therefore, for every term, the units digit is the same as the number itself.

The Unit digit for 1 to 10 is zero.

$$= (1 + 2 + 2.. + 9 + 0)$$

$$+ (1 + 2 + 3.. + 9 + 0) = 0$$

Ex: Find the unit digit $x = 187^{280} \times 529^{320} \times 343^{236}$

Sol: If remainder is 0 then put power equal to the 4.

For the first terms - 7^4 cyclicity is 1

For the second terms - power of 9 is even so the unit digit is 1

For the third terms - 3^4 cyclicity is 1

Unit digit is $= 1 \times 1 \times 1 = 1$

Ex: What is the unit digit in the expansion of $(57242)^{9 \times 7 \times 5 \times 3 \times 1}$?

Sol: We only check only for digit 2. Power will be divided by 4.

$$= 2^{1 \times (-1) \times 1 \times (-1) \times 1} = 2^1$$

So, the unit digit is 2

Type 4: Counting based Questions (a digit or page counting or key strokes)



1 to 9 $\rightarrow$ required digit is = 9

10 to 99 $\rightarrow 90 \times 2 = 180$

Ex: What is the number of digits required for numbering a book with 428 pages?

Sol: 1 to 9 $\rightarrow$ required digit is = 9

10 to 99 $\rightarrow 90 \times 2 = 180$

100 to 428 $= (428 - 100 + 1) = 329 \rightarrow 329 \times 3 = 987$

Total number of digits required = $9 + 180 + 987 = 1176$

Type 5: Perfect Square Based Questions



How to check if a number is a perfect square (it shows the possibility)

1. The last two digits of any perfect square number must be among the squares from 1 to 24.
2. The units digit should not be 2, 3, 7, or 8.
3. The number of zeros in the number and the denominator should both be even.
4. After dividing a perfect square by 9, the remainder must be 0, 1, 4, or 7.

Ex: Is it possible that 562576 is a perfect square or not?

Sol: The number ends in 76, which is possible for a perfect square. The unit digit is 6, so it is possible for the number to be a perfect square.

Sum of digit of 562576 = 21

after dividing remainder is 3.

So, this number is not perfect.

Type 6 Decimal to fraction conversion



- The number of zeros in the denominator is equal to the number of digits after the decimal point.

$$0.abc = \frac{abc}{1000}$$

- The number of 9 in the denominator is equal to the number of digits after the decimal point.

$$0.\overline{abc} = \frac{abc}{999}$$

- When some digits are not covered by the overline

$$0.a\overline{bc} = \frac{abc - a}{990}$$

$$0.ab\overline{cd} = \frac{abcd - ab}{9900}$$

- Mix concept

$$a.b\overline{cd} = a + \frac{bcd - b}{990} = \frac{abcd - ab}{990}$$

Ex: If $A = 0.3\overline{12}$, $B = 0.4\overline{15}$ and $C = 0.30\overline{9}$ what is the value of $A + B + C$.

Sol:

$$A + B + C = \frac{312 - 3}{990} + \frac{415 - 4}{990} + \frac{309 - 30}{900}$$

$$A + B + C = \frac{720}{999} + \frac{279}{900}$$

$$A + B + C = \frac{10269}{9900} = \frac{1141}{1100}$$



Type 7: Number of zeros

- Zeros at the end of a number are determined by the number of 10s and in its factorization, primarily based on the pairs of 5s and 2s.
- **Factorial** is a mathematical operation defined for non-negative integers.

- For a positive integer n , the factorial of n , denoted by $n!$, is the product of all positive integers from 1 to n .

$$n! = n \times (n-1) \times (n-2) \times \dots \times 1$$

$$0! = 1, 1! = 1$$

- ✓ The unit digit of numbers after $4!$ is zero.
 - ✓ $4!$ and all factorials after it are divisible by 4.
 - ✓ Product of ' n ' consecutive natural number is divisible by n
- **Power of number contained in a factorial:** Highest power of a prime number ' p ' contained in $n!$ is given by
- $$= \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots$$
- The product of n consecutive natural number is always divisible by $n!$

Ex: The three numbers 24, 25, and 26 are divisible by.

Sol: The product of n consecutive natural number is always divisible by $n!$

Means 24, 25, 26 is divisible by 3!

Ex: Find the number of trailing zeros in 100!

Sol: In $100!$ factors of 2 are abundant, we only count factors of 5.

Each multiple of 5 contributes at least one factor of 5. Numbers like 25, 50, 75, 100 contain an extra factor of 5 because $25 = 5^2$.

$$\left[\frac{100}{5} \right] + \left[\frac{100}{25} \right] + \left[\frac{100}{125} \right] = 20 + 4 + 0 = 24$$

Number of trailing zeros = 24

Ex: Find the number of trailing zeros in $2 \times 4 \times 6 \dots \times 250$.

Sol: factors of 2 are abundant, we only count factors of 5.

$$2 \times 4 \times 6 \dots \times 250$$

$$= (2 \times 1) \times (2 \times 2) \dots (2 \times 125)$$

$$(2 \times 1) \times (2 \times 2) \dots (2 \times 125)$$

$$= 2^{125} (1 \times 2 \times \dots \times 125)$$

$$2^{125} (1 \times 2 \times \dots \times 125) = 2^{125} \times 125!$$

$$\left[\frac{125}{5} \right] + \left[\frac{125}{25} \right] + \left[\frac{125}{125} \right] = 25 + 5 + 1 = 31$$

Number of trailing zeros = 31

Divisibility

Number	Divisibility Rule
2	Last digit is 0, 2, 4, 6, 8
3	Sum of digits divisible by 3
4	Last two digits form a number divisible by 4
5	Last digit is 0 or 5
25	Last two digit is 00 or divisibly by 25
6	Number divisible by both 2 and 3
7	Subtract twice the last digit from the remaining; result divisible by 7
8	Last three digits divisible by 8
9	Sum of digits divisible by 9
11	Difference between sum of digits at even and odd places is 0 or divisible by 11

Special cases

$$1. 1001 = 7 \times 11 \times 13$$

$$1001 \times abc = abcabc$$

$$2. 10101 = 3 \times 7 \times 13 \times 37$$

$$10101 \times ab = ababab$$

Type 8: Based on

Divisibility rules

Ex: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

Sol: When any number is repeated n times, the combination of the 6-digit number is divisible by 7, 11, and 13.

96 times 9 will be divisible by 13, and only three 9s will remain.

$$\frac{999}{13} \Rightarrow R \rightarrow 11$$

Ex: Find the greatest possible value of $(a + b)$ for which the 8-digit number 143b203a is divisible by 15.

Sol: Divisibility of 3 - if sum of its digits is divisible by 3.

Divisibility of 5 - if the last number is either 0 or 5, the entire no. is divisible by 5.

Factors of 15 = (3×5) Hence, the number should be divisible by both 3 and 5.

So, a can be 0 or 5. But since the question asks for greatest value, a must be 5.

After that sum of the number is = $18 + b$

For greatest = $b = 9$

$$\text{So, } (a + b) = (9 + 5) = 14$$



Ex: If the 9-digit number $72x8431y4$ is divisible by 36, find the value of $\left(\frac{x}{y} - \frac{y}{x}\right)$ for the smallest possible value of y , where x and y are natural numbers.

Sol: The divisibility rule of 36 is that the number must be divisible by 4 and 9.

Last two-digit number = $y4$, on putting $y = 2$, the last two-digit number becomes 24, Therefore, the value of $y = 2$. (y is the smallest possible value)

For divisibility rule of 9,
sum of number = $31 + x$

So, $x = 5$

Now the value

$$\left(\frac{x}{y} - \frac{y}{x}\right) = \left(\frac{5}{2} - \frac{2}{5}\right) = 2\frac{1}{10}$$



Type 9: Divisibility of expressions

Expression	'n' is odd	'n' is even
$x^n - y^n$	$(x - y)$	$(x + y)(x - y)$
$x^n + y^n$	$(x + y)$	Can't say

Ex: What is the remainder if $(17^{26} - 11^{26})$ is divided by 42?

Sol: n is even. So, $(x + y)(x - y)$

$$= \frac{(17 + 11)(17 - 11)}{42} = \frac{28 \times 6}{42},$$

Remainder = 0

Key Point: If ' n ' is odd and $a, b, c \dots z$ are consecutive natural number then $(a^n + b^n + \dots + z^n)$ is divisible by $(a + b + c \dots z)$

Ex: $11^5 + 12^5 + 13^5$ is divisible by?

Sol- using concept

$$11 + 12 + 13 = 36$$

So, the expression is divisible by 36

Type 10: Digit and its Reverse Form 2-digit and 3 digit and its reverse form



1. For 2-digit

Let, two-digit original number = $10x + y$, Reversed number = $10y + x$

Sum of original and reserved number
 $(10x + y) + (10y + x) = 11(x + y)$

Difference of original and reversed number
 $(10x + y) - (10y + x) = 9(x - y)$

2. For 3-digit

Let, Hundred digit = x , Tenth digit = y , Unit digit = z So,

original number = $100x + 10y + z$

Hundred and unit digits interchange,

new number = $100z + 10y + x$

Difference of original and reverse number = $99(x - z)$

Ex: The sum of the two-digit number and the number obtained by inter-changing the digit is 99. If the difference of digits is 1, then the number is;

Sol: Let, number = $10x + y$

$$(10x + y) + (10y + x) = 99$$

$$x + y = 9$$

$$x - y = 1$$

After the equation $x = 5, y = 4$

So, the number is

$$= 10 \times 5 + 4 = 54$$

Ex: What will be the smallest natural number to be filled in the blank for the number $23x45678$ to be divisible by 22.

Sol: The divisibility rule for 22 is based on divisibility by both 2 and 11. A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is divisible by 11.

Odd positions sum: $3 + 4 + 6 + 8 = 21$

Even positions sum: $2 + x + 5 + 7 = 14 + x$

$$21 - (14 + x) = 7 - x$$

$$7 - x = 11k, \text{ for some integer } k$$

For the smallest x , let's try $k = 0$

$$7 - x = 0 \Rightarrow x = 7$$

Thus, the smallest value of x that makes the number $23x45678$ divisible by 22 is 5.

Ex: How many numbers are there from 700 to 950 (including both) which are neither divisible by 3 nor by 7?

Sol:

Neither divisible by 3 nor by 7

$$= \text{Total} - (\text{Divisible by 3 or 7}) = \text{Total} - [N(3) + N(7) - N(21)]$$

Total number from 700 to 950 = 251

$$\text{Divisible by 3} = \frac{251}{3} \approx 83$$

$$\text{Divisible by 7} = \frac{251}{7} \approx 35$$

$$\text{Divisible by 21} = \frac{251}{21} \approx 11$$

Required number

$$= 251 - (83 + 35 - 11) = 144$$