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Volume - 4

Mathematics & Logical Reasoning

SIERRA INNOVATIONS

INDEX

S No.	Chapter Title	Page No.
1	Number System	1
2	Simplification	11
3	LCM & HCF	15
4	Average	18
5	Age	22
6	Percentage	23
7	Profit and Loss	28
8	Discount and Dishonest shopkeeper	32
9	Ratio, proportion and variation	35
10	Time and Work	39
11	Time Speed and Distance	43
12	Simple Interest	48
13	Compound Interest	52
14	Algebra	56
15	Trigonometry	62
16	Geometry	70
17	Co–Ordinate Geometry	91
18	Mensuration 2D	98
19	Mensuration 3D	110
20	Data Interpretation	119
21	English Alphabet Test	135
22	Number and Alphabet Series Test	137
23	Analogy Test	143

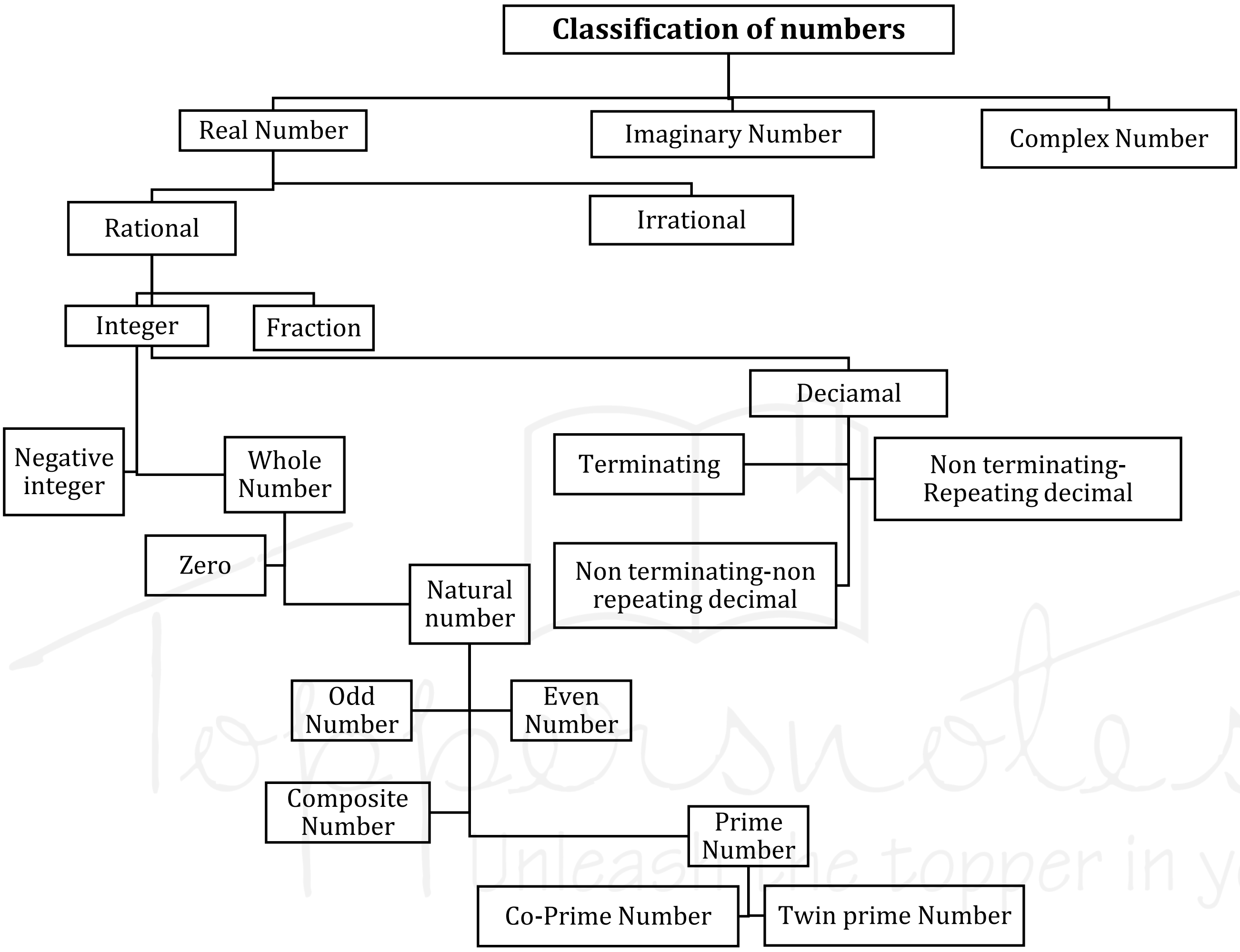
INDEX

S No.	Chapter Title	Page No.
24	Coding and Decoding	148
25	Classification Test	153
26	Logical Order of Words	157
27	Direction and Distance	160
28	Seating Arrangement	165
29	Clock	173
30	Calendar	176
31	Blood Relation	179
32	Mathematical Operations	183
33	Matrix	186
34	Venn Diagram	189
35	Syllogism	193
36	Dice	199
37	Statement and Conclusion	203
38	Mirror Image and Water Image	207
39	Figure Series	209
40	Figure Classification	214
41	Figure Formation	216
42	Figure Completion	219
43	Counting of Figures	221

1 CHAPTER

Number System

- **Number System**: A number system is a **method of representing and working with numbers** using a defined set of symbols and rules.

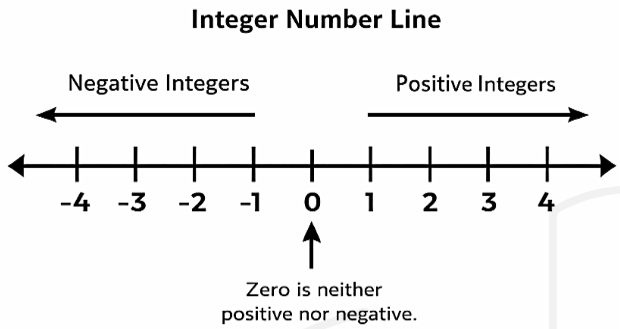


Types	Definition
Real Number	A real number is any number that can be represented on the number line. A real number is a number that includes all rational and irrational numbers and can be expressed as a point on the number line.
Rational Number	A rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Irrational Number	An irrational number is a number that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$
Fraction	A fraction is a number that represents a part of a whole or a ratio of two quantities. It is written in the form $\frac{a}{b}$.
Integer	An integer is a whole number that can be positive, negative, or zero, and does not include any fractional or decimal part

Negative Integer	Negative integers are whole numbers with a negative sign, such as $-1, -2, -3, \ldots$
Whole Number	A whole number is a non-negative integer, including zero.
Natural number	Natural numbers are the numbers starting from 1 and increasing by 1 each time. 1,2,3, 4,…
Odd Number	An odd number is a natural number that is not divisible by 2 or in the form of $2n + 1$
Even Numbers	An even number is a natural number that is completely divisible by 2 or in the form of $2n$.
Prime number	A prime number is a natural number greater than 1 that has exactly two distinct factors: 1 and the number itself. 2,3,5, 7. ➢ 2 is the smallest prime number. ➢ 2 is only even prime number. ➢ All prime number (except 2 and 3) can be written in the form of $6n + 1$ or $6n + 5$ where n is natural number however, the converse is not true. ➢ (3,5,7) is only group of three primes, which are consecutive odds. ➢ 101 is smallest 3-digit prime number. ➢ 997 is largest 3-digit prime number.
Composite Number	A composite number is a natural number greater than 1 that has more than two factors. A composite number can be divided exactly by 1, itself, and at least one other number. 4,6,8,9,10,12, 14… ➢ The smallest composite number is 4. ➢ The smallest odd composite number is 9 If a and b are any two odd primes then $a^2 + b^2$ and $a^2 - b^2$ is composite numbers.

Co-Prime Numbers	Two or more numbers are called co-prime (or relatively prime) if their only common factor (HCF) is 1. ➢ 1 is neither prime or composite.
Twin Prime Numbers	Twin numbers (twin primes) are a pair of prime numbers whose difference is exactly 2. ➢ 5 is only prime which is in 2 twin prime pairs. (3, 5) (5, 7) ➢ Sum of twin prime number (expect 3 and 5) is always divisible by 12.
Decimal Number	A decimal number is a number that has a decimal point (.) and consists of a whole part and a fractional part. Ex- 3.5,12.75 etc
Terminating Decimal	A terminating decimal is a decimal number that ends after a finite number of digits after the decimal point. Ex- 2.5,0.75
Non-Terminating Repeating Decimal	A non-terminating repeating decimal is a decimal number that does not end and in which one or more digits repeat continuously after the decimal point. Ex- 0. 333..,0. 1212
Non-Terminating Non-Repeating Decimal	A non-terminating non-repeating decimal is a decimal number that does not end and does not repeat any digit or pattern after the decimal point. Ex- 1.1412,3.14
Imaginary Number	An imaginary number is a number that can be written in the form $= bi$ b is a real number i is the imaginary unit, defined by

Complex Number	A complex number is a number that consists of two parts—a real part and an imaginary part—and can be expressed in the standard form $z = a + ib$ a is a real number, called the real part of z b is a real number, called the imaginary part of z i is the imaginary unit, defined by the property



How to check the given number is prime or not?

- To determine whether a number is prime, first find its square root and round it down to the nearest whole number. Then check whether the number is divisible by any prime number up to this value. If it is not divisible by any of them, the number is a prime number.

Between	Number or prime number
1-50	15
1-100	25
1-200	46

Ramanujan number

A Ramanujan number is a number that can be expressed as the sum of two positive cubes in two different ways. It is also known as the Hardy–Ramanujan number or taxi-cab number.

Smallest Ramanujan number = 1729

$1729 = 1^3 + 12^3 = 9^3 + 10^3$

Perfect number

A perfect number is a natural number that is equal to the sum of its proper divisors (that is, all its positive divisors excluding the number itself).

Ex: 4 is divisible by 1 and 2, so $1 + 2 = 3 \neq 4$; therefore,4 is not a perfect number.

6 is divisible by 1,2 and 3, so, $1 + 2 + 3 = 6 = 6$; therefore, 6 is a perfect number.

Key points

Even + Even = Even

Even × Even = Even

Even + Odd = odd

Even × Odd = Even

Odd + odd = Even

Odd × Odd = Odd

Sum Of 'n' odd number is odd, if n is odd.

Sum of 'n' odd number is even, if n is even.

Type 1: Question based on definitions

Ex: 173 is prime number of not?

Sol: The Square root of 173 is approximately 13.

The Prime number less than or equal to 13 are 2,3,5,7,11 and 13,

Since 173 is not divisible by any number, it is a prime number.

Ex: x, y and z are distinct prime number where $x < y < z$ if $x + y + z = 70$, then what is the value of z?

Sol: Now, sum is 70, means at least one of the numbers is even. A we know, only one even prime number exists, and that is 2.

2 is also the smallest prime number.

Means $x = 2$

Now, $70 - 2 = 68 = y + z$

By substituting the values of different prime numbers, we obtain the result.

$y = 31$ and $x = 37$

Ex: How many composite numbers are there from 53 to 97?

Sol: If we find the total number of integers between 53 and 97 and then subtract the number of prime numbers between them, we obtain the number of composite numbers.

Total number = $97 - 53 + 1 = 45$ (+1 when both numbers are included)

Total prime number from 53 to 97 is 10

So, composite number = 45 – 10 = 35

Ex: Which one of the following is TRUE?

(A) All irrational numbers are real

(B) All real numbers are irrational.

(C) Rational numbers are not real.

(D) Integers are not rational.

Sol: Irrational numbers form a subset of real numbers, so all irrational numbers are real.
Not all real numbers are irrational;
rational numbers are also real.
Rational numbers are real numbers.
Integers are a subset of rational numbers since any integer can be written as $\frac{p}{q}$(e.g., $5 = \frac{5}{1}$).
So, Answer is (A).

Special concept: based on sum of digits for special types of numbers

Number	Square	Sum of digits
11^2	121	3
111^2	12321	9
So, on		
111111111^2	12345678987654321	81

Ex: Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of digit of the resulting number.

Sol- using the concept
$111111111^2 \ggg 81$

Type 2: Unit digit based

To find the units digit of an expression, consider only the unit digits of the numbers instead of evaluating the entire expression.
Unit digit of (a + b) = unit digit of a + unit digit of b
Unit digit of (a − b) = unit digit of a − unit digit of b
Unit digit of a × b = unit digit of a × unit digit of b

Ex: What are the unit digit of 435×433 .

Sol:Unit digit of a × b = unit digit of a × unit digit of b
$5 \times 3 = 15$ so, unit digit is 5

Cyclicity

Cyclicity in the number system means the repeating pattern of digits or remainders when a number is raised to higher powers. The unit digit remains unchanged for all powers.

$0 \to 0$ $1 \to 1$
$5 \to 5$ $6 \to 6$

Cycle of Length 2-The unit digit alternates between two values.
$4 \to 4, 6$
When the power is odd, the unit digit is 4, and when the power is even, the unit digit is 6.
$9 \to 9, 1$
When the power is odd, the unit digit is 9, and when the power is even, the unit digit is 1.
Cycle of Length 4- The unit digit repeats after four powers.
$2 \to 2, 4, 8, 6$ $3 \to 3, 9, 7, 1$
$7 \to 7, 9, 3, 1$ $8 \to 8, 4, 2, 6$
Let $N = x^y$
To find the unit digit of (N), we only need to consider the unit digit of the base number (x).
The unit digit of an exponential expression can be determined by finding the remainder when the power is divided by 4.

Type 3: Cyclicity - Unit Digit Based on Arithmetic Equations

Ex: If $x = (164)^{169} + (333)^{337} - (727)^{726}$ then what is the unit digit of x?

Sol: In the expression, the **first term** has an odd power of 4, so the unit digit of the first term is 4. For the **second term**, dividing 337 by 4 gives a remainder of 1 so the unit digit of the second term is 3. For the **third term**, dividing 726 by 4 gives a remainder of 2; hence, the unit digit of the third term is 9.
So, unit digit of the expression $4 + 3 - 9 = -2$
If the unit digit comes out negative, add 10 to obtain the correct unit digit.Unit digit is $10 - 2 = 8$

Ex: What is the unit digit of $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 20^5$

Sol: In each term, the cyclicity is 1. Therefore, for every term, the units digit is the same as the number itself.
The Unit digit for 1 to 10 is zero.

$$= (1 + 2 + 2..+9 + 0) + (1 + 2 + 3..+9 + 0) = 0$$

Ex: Find the unit digit $x = 187^{280} \times 529^{320} \times 343^{236}$

Sol: If remainder is 0 then put power equal to the 4.

For the first terms – 7^4 cyclicity is 1

For the second terms – power of 9 is even so the unit digit is 1

For the third terms - 3^4 cyclicity is 1

$Unit\ digit\ is\ = 1 \times 1 \times 1 = 1$

Ex: What is the unit digit in the expansion of $(57242)^{9\times7\times5\times3\times1}$?

Sol: We only check only for digit 2. Power will be divided by 4.

$= 2^{1\times(-1)\times1\times(-1)\times1} = 2^1$

So, the unit digit is 2

Type 4: Counting based Questions (a digit or page counting or key stokes)

1 to 9 → required digit is = 9 10 to 99 → 90×2 = 180

Ex: What is the number of digits required for numbering a book with 428 pages?

Sol: 1 to 9 → required digit is = 9

10 to 99 → 90×2 = 180

100 to 428 = (428-100+1) = 329→329×3=987

Total number of digits required = 9 +180 + 987 =1176

Type 5: Perfect Square Based Questions

How to check if a number is a perfect square (it shows the possibility)

1. The last two digits of any perfect square number must be among the squares from 1 to 24.
2. The units digit should not be 2, 3, 7, or 8.
3. The number of zeros in the number and the denominator should both be even.
4. After dividing a perfect square by 9, the remainder must be 0, 1, 4, or 7.

Ex: Is it possible that 562576 is a perfect square or not?

Sol: The number ends in 76, which is possible for a perfect square. The unit digit is 6, so it is possible for the number to be a perfect square.

Sum of digit of 562576 = 21

after dividing remainder is 3.

So, this number is not perfect.

Type 6 Decimal to fraction conversion

- The number of zeros in the denominator is equal to the number of digits after the decimal point.

  $0.abc = \frac{abc}{1000}$

- The number of 9 in the denominator is equal to the number of digits after the decimal point.

  $0.\overline{abc} = \frac{abc}{999}$

- When some digits are not covered by the overline

  $0.a\overline{bc} = \frac{abc - a}{990}$

  $0.ab\overline{cd} = \frac{abcd - ab}{9900}$

- Mix concept

  $a.b\overline{cd} = a + \frac{bcd - b}{990} = \frac{abcd - ab}{990}$

Ex: If $A = 0.3\overline{12}$, $B = 0.4\overline{15}$ and $C = 0.30\overline{9}$ what is the value of $A + B + C$.

Sol:

$$A + B + C = \frac{312 - 3}{990} + \frac{415 - 4}{990} + \frac{309 - 30}{900}$$

$$A + B + C = \frac{720}{999} + \frac{279}{900}$$

$$A + B + C = \frac{10269}{9900} = \frac{1141}{1100}$$

Type 7: Number of zeros

- Zeros at the end of a number are determined by the number of 10s and in its factorization, primarily based on the pairs of 5s and 2s.
- **Factorial** is a mathematical operation defined for non-negative integers.

- **For** a positive integer n, the factorial of n, denoted by $n!$, is the product of all positive integers from 1 to n.
  $n! = n \times (n-1) \times (n-2) \times \cdots \times 1$
  $0! = 1,\ 1! = 1$
  - ✓ The unit digit of numbers after 4! is zero.
  - ✓ 4! and all factorials after it are divisible by 4.
  - ✓ Product of 'n' consecutive natural number is divisible by n
- **Power of number contained in a factorial:** Highest power of a prime number 'p' contained in n! is given by
  $= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$
- The product of n consecutive natural number is always divisible by n!

Ex: The three numbers 24, 25, and 26 are divisible by.

Sol: The product of n consecutive natural number is always divisible by n!
Means 24,25,26 is divisible by 3!

Ex: Find the number of trailing zeros in 100!

Sol: In 100! factors of 2 are abundant, we only count factors of 5.
Each multiple of 5 contributes at least one factor of 5. Numbers like 25, 50, 75, 100 contain an extra factor of 5 because $25 = 5^2$.

$$\left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] = 20 + 4 + 0 = 24$$

Number of trailing zeros = 24

Ex: **Find the number of trailing zeros in 2×4×6...×250.**

Sol: factors of 2 are abundant, we only count factors of 5.

$2 \times 4 \times 6 \ldots \times 250$
$= (2 \times 1) \times (2 \times 2) \ldots . (2 \times 125)$
$(2 \times 1) \times (2 \times 2) \ldots . (2 \times 125)$
$= 2^{125}(1 \times 2 \times \ldots 125)$
$2^{125}(1 \times 2 \times \ldots 125) = 2^{125} \times 125!$

$$\left[\frac{125}{5}\right] + \left[\frac{125}{25}\right] + \left[\frac{125}{125}\right] = 25 + 5 + 1 = 31$$

Number of trailing zeros = 31

Divisibility

Number	Divisibility Rule
2	Last digit is 0, 2, 4, 6, 8
3	Sum of digits divisible by 3
4	Last two digits form a number divisible by 4
5	Last digit is 0 or 5
25	Last two digit is 00 or divisibly by 25
6	Number divisible by both 2 and 3
7	Subtract twice the last digit from the remaining; result divisible by 7
8	Last three digits divisible by 8
9	Sum of digits divisible by 9
11	Difference between sum of digits at even and odd places is 0 or divisible by 11

Special cases

1. 1001 = 7×11×13
   $1001 \times abc\ =\ abcabc$
2. 10101 = 3×7×13×37
   $10101 \times ab\ =\ ababab$

Type 8: Based on Divisibility rules

Ex: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

Sol: When any number is repeated n times, the combination of the 6-digit number is divisible by 7, 11, and 13.
96 times 9 will be divisible by 13, and only three 9s will remain.

$= \frac{999}{13} \Rightarrow R \to 11$

Ex: Find the greatest possible value of $(a + b)$ for which the 8-digit number 143b203a is divisible by 15.

Sol: Divisibility of 3 - if sum of its digits is divisible by 3.
Divisibility of 5- if the last number is either 0 or 5, the entire no. is divisible by 5.
Factors of 15 = (3 × 5) Hence, the number should be divisible by both 3 and 5.
So, a can be 0 or 5. But since the question asks for greatest value, a must be 5.
After that sum of the number is = 18+b
For greatest = b=9
So, $(a + b)\ =\ (9 + 5)\ =\ 14$

Ex: If the 9-digit number $72x8431y4$is divisible by 36, find the value of $\left(\frac{x}{y}-\frac{y}{x}\right)$ for the smallest possible value of y, where $x$and $y$are natural numbers.

Sol: The divisibility rule of 36 is that the number must be divisible by 4 and 9.

Last two-digit number = $y4$, on putting $y = 2$, the last two-digit number becomes 24,

Therefore, the value of $y = 2$. (y is the smallest possible value)

For divisibility rule of 9,

sum of number = $31 + x$

So, $x = 5$

Now the value

$\left(\frac{x}{y}-\frac{y}{x}\right) = \left(\frac{5}{2}-\frac{2}{5}\right) = 2\frac{1}{10}$

Type 9: Divisibility of expressions

Expression	'n' is odd	'n' is even
$x^n - y^n$	$(x-y)$	$(x+y)(x-y)$
$x^n + y^n$	$(x+y)$	$Can't\ say$

Ex: What is the remainder if $(17^{26} - 11^{26})$ is divided by 42?

Sol: n is even. So, $(x+y)(x-y)$

$= \frac{(17+11)(17-11)}{42} = \frac{28\times 6}{42}$,

$Remainder = 0$

Key Point: If 'n' is odd and $a, b, c \ldots z$ are consecutive natural number then $(a^n + b^n + \cdots + z^n)$ is divisible by $(a + b + c \ldots z)$

Ex: $11^5 + 12^5 + 13^5$ is divisible by?

Sol- using concept

11+12+13 = 36

So, the expression is divisible by 36

Type 10: Digit and its Reverse Form 2-digit and 3 digit and its reverse form

1. **For 2-digit**

   Let, two-digit original number = $10x + y$, Reversed number = $10y + x$

   Sum of original and reserved number $(10x + y) + (10y + x) = 11(x + y)$

   Difference of original and reversed number $(10x + y) - (10y + x) = 9(x - y)$

2. **For 3-digit**

   Let, Hundred digit = x, Tenth digit = y, Unit digit = z So,

   original number = $100x + 10y + z$

   Hundred and unit digits interchange, new number = $100z + 10y + x$

   Difference of original and reverse number = $99(x - z)$

Ex: The sum of the two-digit number and the number obtained by inter-changing the digit is 99. If the difference of digits is 1, then the number is;

Sol: Let, number = $10x + y$

$(10x + y) + (10y + x) = 99$

$x + y = 9$

$x - y = 1$

After the equation $x = 5, y = 4$

So, the number is

$= 10 \times 5 + 4 = 54$

Ex: What will be the smallest natural number to be filled in the blank for the number 23x45678 to be divisible by 22.

Sol: The divisibility rule for 22 is based on divisibility by both 2 and 11. A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is divisible by 11.

Odd positions sum: $3 + 4 + 6 + 8 = 21$

Even positions sum: $2 + x + 5 + 7 = 14 + x$

$21 - (14 + x) = 7 - x$

$7 - x = 11k$, for some integer k

For the smallest x, let's try k = 0

$7 - x = 0 \Rightarrow x = 7$

Thus, the smallest value of $x$that makes the number $23x45678$ divisible by 22 is 5.

Ex: How many numbers are there from 700 to 950 (including both) which are neither divisible by 3 nor by 7?

Sol:

Neither divisible by 3 nor by 7

= Total − (Divisible by 3 or 7) = Total − [N(3) + N(7) − N(21)]

Total number from 700 to 950 = 251

Divisible by 3 = $\frac{251}{3} \approx 83$

Divisible by 7 = $\frac{251}{7} \approx 35$

Divisible by 21= $\frac{251}{21} \approx 11$

Required number

$= 251 - (83 + 35 - 11) = 144$

Factorization

A factor of a number is a whole number that can be multiplied by another whole number to result in the original number. In other words, a factor divides the number evenly without leaving any remainder.

Let $N = a^p \times b^q \times c^r$

Total number of factors $= (p+1)(q+1)(r+1)$

Total number of even factors $= p(q+1)(r+1)$

Total number of odd factors $= (q+1)(r+1)$

Sum of all factors $= (a^0 + a^1 + \cdots a^p)(b^0 + b^1 + \cdots b^q)(c^0 + c^1 + \cdots + c^q)$

Sum of even factors $= (2^1 + 2^2 + \cdots + 2^p)(b^0 + b^1 + \cdots + b^q)(c^0 + c^1 + \cdots + c^r)$

Sum of odd factors $= 2^0(b^1 + b^2 + \cdots + b^q)(c^1 + c^2 + \cdots + c^r)$

Average of factor $= \frac{\text{sum of factor}}{\text{number of factor}}$

Sum of reciprocal of factors $= \frac{sum\ of\ factor}{given\ number}$

Number of prime factors $= p + q + r$

Number of distinct prime factor $= no\ of\ prime\ number\ in\ factor$

Number of composite numbers $= total - distinct\ prime\ number - 1$

- Every number has positive and negative factors.
- All numbers in the form $(prime)^2$ have 3 factors.

Type 11: Find the Number of Factors Based

Ex: Find all types of factors and the sum of all types of factors for $N = 3600$.

Sol: $N = 3600 = 2^4 \times 3^2 \times 5^2$

- Total number of factors $= 5 \times 3 \times 3 = 45$
- Number of even factors
  $2 \times (1800) = 2(2^3 \times 3^2 \times 5^2)$
  $= 4 \times 3 \times 3 = 36$
- Number of odd factors $= 3 \times 3 = 9$
- Number of prime factors $= 4 + 2 + 2 = 8$
- Number of distinct Prime factor (visible in factorization) $= 1 + 1 + 1 = 3$
- Number of Composite numbers
  $= 45 - 3 - 1 = 41$
- Perfect square $= (2^2)^2(3^2)^1(5^2)^1$
  $= 3 \times 2 \times 2 = 12$
- Perfect cube $= (2^3)^1 = (1+1) = 2$
- Sum of all factor
  $= (2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$
  $= 12493$
- Sum of even factor
  $= (2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 12090$
- Sum of odd factor
  $= (3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2) = 403$
- Sum of prime $= 2 + 3 + 5 = 10$
- Sum of composite number
  $= total\ sum - (sum\ of\ prime + 1) =$
  $12493 - 10 - 1 = 12482$
- Perfect square
  $= (2^0 + 2^2 + 2^4)(3^0 + 3^2)(5^0 + 5^1) = 5460$

Ex: Calculate the total number of prime factors in the expression $(4^{11} \times 5^5 \times 3^2 \times 13^2)$

Sol:

$(4^{11} \times 5^5 \times 3^2 \times 13^2) = 2^{22} \times 3^2 \times 5^5 \times 13^2$

Total number of prime factors

$= 22 + 2 + 5 + 2 = 31$

Ex: How many factors of $(30^{16} \times 16^{18} \times 20^{21})$ are perfect square as well as perfect cube?

Sol: $30^{16} \times 16^{18} \times 20^{21} = 2^{130} \times 3^{16} \times 5^{37}$

When asked to check for a perfect square or perfect cube, check if the power is a multiple of 6.

$2^{130} \times 3^{16} \times 5^{37}$
$= 2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$

$2^{126} \times 2^4 \times 3^{12} \times 3^4 \times 5^{36} \times 5^1$
$= (2^6)^{21} \times 2^4 \times (3^6)^2 \times 3^4 \times (5^6)^6 \times 5^1$

The total number of factors that are both perfect squares and perfect cubes

$= (21+1)(2+1)(6+1) = 462$

Ex: How many factors of the number $2^8 \times 3^6 \times 5^4 \times 10^5$ are multiple of 120.

Sol: $N = 2^8 \times 3^6 \times 5^4 \times 10^5 = 2^{13} \times 3^6 \times 5^9$

$\frac{N}{120} = \frac{2^{13} \times 3^6 \times 5^9}{2^3 \times 3^1 \times 5^1} = 2^{10} \times 3^5 \times 5^8$

Number of factors $= 11 \times 6 \times 9 = 594$

Remainder

Let suppose number N when divided by divisor D, leaves the remainder as R and quotient as Q.

$N = D \times Q + R$

Dividend = Divisor × quotient + Remainder

If a gives r remainder when divided by n, then ka gives kr remainder when divided by n.

If a and b give r_1 and r_2 remainders respectively when divided by n then.

$a + b$ divided by n, gives remainder $r_1 + r_2$

$a \times b$ divided by n, gives remainder $r_1 + r_2$

Type 12: Question based on remainder theorem

Ex: A number being divided by 52 gives a remainder 45. If the number is divided by 13, the remainder will be?

Sol: Since 13 is a multiple of 52, we can directly divide the remainder of one by the divisor.

$$= \frac{45}{13} \Rightarrow R \to 6$$

Ex: A number when divided by 12 leaves 5 remainder. What is the remainder if square of this number is divided by 8.

Sol: When a mathematical operation is performed on a number, the same operation can also be performed on the remainder.

$$= \frac{5^2}{8} = \frac{25}{8} \Rightarrow R \to 1$$

Ex: If the dividend is 45, the divisor is 8, and the quotient is 5, find the remainder.

Sol- $N = D \times Q + R$

$45 = 8 \times 5 + Remainder$

Remainder $= 45 - 40 = 5$

Type 13: Important concept of remainder

$$\frac{(x+a)^n}{x} \Rightarrow R \to a^n$$

$$\frac{(x+1)^n}{x} \Rightarrow R \to 1$$

$$\frac{(x-1)^n}{x} \Rightarrow R \to (1)^n$$

$R = 1 \; when \; n \; is \; even$

$R = -1 \; when \; n \; is \; odd$

Special case:

$$\frac{4}{6} \Rightarrow R \to 4$$

$$\frac{4^n}{6} \Rightarrow R \to 4$$

Ex: What is the remainder when 2^{75} is divided by 14?

Sol:

$$\frac{2^{75}}{15} = \frac{(2^4)^{18}.2^3}{15} = \frac{(16-1)^{18} \times 8}{15} \Rightarrow R \to 8$$

Ex: What is the remainder if 2^{192} is divided by 6?

Sol: Using the concept

$$\frac{2^{192}}{6} = \frac{4^{96}}{6} \Rightarrow R \to 4$$

Ex: What is the remainder when 37^{47} is divided by 19.

Sol:

$$\frac{37^{47}}{19} = \frac{(38-1)^{47}}{19} \Rightarrow R \to -1$$

Remainder cannot be negative, so the remainder is subtracted from the divisor.

$R \to -1 \Rightarrow R \to 19 - 1 = 18$

Type 14: Fermat's little theorem

$If \; \frac{a^{P-1}}{P}, \quad \Rightarrow R \to 1$

Conditions

1. P is prime number.
2. a, P are co-prime.

Ex: What is the remainder when 21^{47} is divided by 47?

Sol: 21 and 47 are coprime number so we can use Fermat's little theorem.

$$\frac{21^{47}}{47} = \frac{21^{46} \times 21}{47} \Rightarrow R \to 21$$

Type 15: Euler's (totient) theorem

$$\frac{A^X}{d} \Rightarrow R \to 1$$

Where A and d are co-prime.

X – number of numbers less than d and co-prime to d.

$$X = d\left(1 - \frac{1}{P^1}\right)\left(1 - \frac{1}{P^2}\right) \ldots \ldots \ldots$$

Where P^1 and P^2 are distinct prime factors of d.

Ex: If φ is the Euler's totient functions, then $\varphi(92)$ is.

Sol: $92 = 2^2 \times 23$

$$\varphi(92) = 92\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{23}\right) = 44$$

Ex: 7^{82} is divided by 11, R=?

Sol: To find the remainder when $7^{82}$is divided by 11, we can use Fermat's Little Theorem, which states that if p is a prime number and $a$is an integer not divisible by p, then

$$\varphi(11) = 11\left(1-\frac{1}{11}\right) = 10$$

82 is divided by 10, and the remainder is

$2.\frac{7^2}{11} \Rightarrow R \rightarrow 5$

Type 16: Wilson's theorem

If P is prime number, then,

$\frac{(P-1)!}{P} \quad \Rightarrow R \rightarrow P-1$

$\frac{(P-2)!}{P} \quad \Rightarrow R \rightarrow 1$

Ex: What will be the remainder when 568! is divided by 569.

Sol- Using the Wilson theorem

$$\frac{568!}{596} = \frac{(569-1)!}{P} \quad \Rightarrow R \rightarrow 569-1$$

$$= 568$$

21 CHAPTER

English Alphabet Test

- This chapter includes questions based on the English alphabet (A–Z). Candidates should know the positions of all 26 letters and basic related concepts clearly.
- **Letters are of two types:**
  - ✓ Vowels – A, E, I, O, U (There are 5 vowels in the English alphabet.)
  - ✓ Consonants – B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z (There are 21 consonants in the English alphabet.)
- **The alphabet is divided into two halves:**
  - ✓ First Half – A to M (The first half contains 13 letters, i.e., positions 1 to 13.)
  - ✓ Second Half – N to Z (The second half contains 13 letters, i.e., positions 14 to 26.)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Short trick:

- **EJOTY (इजोटी)**

5	10	15	20	25
E	J	O	T	Y

- **CFILORUX (सिफिलोरक्स)**

3	6	9	12	15	18	21	24
C	F	I	L	O	R	U	X

- **Finding letters from the right side can be simplified using the formula:**
  - ✓ Position from left = 27 − Position from right
- **Trick to Remember Opposite Letters**

Pair	**AZ**	**BY**	**CX**	**DW**	**EV**	**FU**	**GT**
Trick	**AZ**	**BY**e	Cracks	**D**e**W**	**EV**ening	Few / Uff	**G.T**. Road

Pair	**HS**	**IR**	**JQ**	**KP**	**LO**	**MN**
Trick	**H**igh **S**chool	**I**ndian **R**ailway	**J**aipur **Q**ueen	**K**an**P**ur	**L**ife **O**K	**M**a**N**

Type-1 Position of a Letter in the English Alphabet

- If counting in the **same direction** (left to left or right to right), **subtract** both positions.
- If counting in **opposite directions** (left to right or right to left), **add** both positions.

Ex: In the English alphabet, what is the 10th letter to the left of the 21st letter from the left?

Ans: Using the trick**:** If counting in the same direction (left to left or right to right), subtract both positions.

English alphabet = 21-10= 11^{th} alphabet = K

Ex: In the English alphabet, what is the 9th letter to the left of the 11th letter from the right?

Ans: In these types of question first we calculate from the left and then subtract from the 27.

Alphabet= 11+9=20th from the left

Original alphabet = 27-20 = 7th alphabet = G

Type-2 Forming Letter Pairs

- The letter pair can be formed in both forward and backward directions.
- Multiple pairs can be created from a single word.
- After forming a pair with a letter, you can form another pair with the same letter if they are at the same distance according to the English alphabet.

Ex: How many such pairs of letters are there in the word 'COMBINE', each of which has as many letters between them in the word (both forward and backward direction) as they have between them in the English Alphabet?

Ans: First of all, we will write number of positions of 'COMBINE' word according to English alphabet i.e. C is written as 3 and O will be written as 15 and so on.

C	O	M	B	I	N	E
3	15	13	2	9	14	5

Here we can see that only B and E is making one pair.

Type-3 Letter Problems

Ex: If the first and eighth letters of the word 'REPRESENTATIVE' are swapped, then the second and ninth letters, and so on, are also swapped, what will be the fourth letter to the left of the 6th letter from the left in the new arrangement?

Ans:

Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Original Letter	R	E	P	R	E	S	E	N	T	A	T	I	V	E
New Letter	N	T	A	T	I	V	E	R	E	P	R	E	S	E

Fourth letter to the left of the 6th letter from the left= 6-2=4th = T

Type-4 Arrangement of English Words

- Arranging English words in alphabetical or dictionary order is called arrangement of words.

Ex: Arrange the following words according to English dictionary arrangement:

(A) Epitaxy (B) Episode (C) Epigene

(D) Epitome (E) Epilogue

Ans: In a Dictionary, every word is arranged alphabetically. Also, in each word each and every letter is positioned alphabetically. So here, dictionary the arrangement of words in a Dictionary, every word is arranged alphabetically.

(C) Epigene (E) Epilogue (B) Episode

(A) Epitaxy (D) Epitome

Therefore, the correct answer is (C), (E), (B), (A), (D)

Type-5 Meaningful Logical Order of Words

Ex: How many meaningful words of 5 letters can be made with the alphabets K, E, D, H, I each being used only once in each word?

Ans: Given letters: K, E, D, H, I

Meaningful word can be formed out of the alphabets K, E, D, H, I:

HIKED: Walk for a long distance, Hence, only one such word is possible.

Ex: If a five-letter meaningful word can be formed by using the first, third, fifth, sixth and eighth letters from the word "HALLOWEEN", then what is the last letter from the left end of the newly formed word?

Ans: The 1st, 3rd, 5th, 6th, and 8th letters from the left end of the word "HALLOWEEN" are H, L, O, W, and E respectively. The only five-letter meaningful word that can be formed using the letters is WHOLE.

22 CHAPTER

Number and Alphabet Series Test

- A series is a systematic arrangement of numbers or letters within a defined group. In competitive exams, sequences of numbers, letters, or a mix of both are presented. One position in the sequence is either left blank or contains an incorrect number or letter. Candidates are tasked with completing the series by selecting the correct option to fill the blank or identify the incorrect element.

<u>Here's a clearer and concise way to solve number series problems:</u>

- **Look for Patterns**: Check if the numbers are increasing or decreasing by a constant (addition/subtraction) or multiplied/divided by a constant (multiplication/division).
- **Find Differences**:
  - ✓ If the difference between consecutive numbers is the same, it's an arithmetic series (e.g., 2, 5, 8, 11).
  - ✓ If the second difference (difference of differences) is constant, it's a quadratic series.
- **Check for Multiplication/Division**: Look if each number is multiplied or divided by a constant to get the next number (e.g., 3, 6, 12, 24 where each number is doubled).
- **Recognize Special Sequences**:
  - ✓ **Squares**: 1, 4, 9, 16, ...
  - ✓ **Cubes**: 1, 8, 27, ...
  - ✓ **Fibonacci**: Each number is the sum of the two preceding ones (e.g., 0, 1, 1, 2, 3, 5).
- **Check for Ratios**: If the numbers increase by a constant ratio, it's a geometric progression (e.g., 2, 4, 8, 16).
- **Test the Options**: If options are provided, check which one follows the identified pattern.

<u>Type-1 Series in Increasing Order</u>

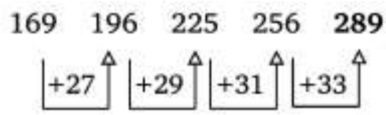
Ex: In the following question, select the missing number from the given series.

169, 196, 225, 256,_?

1. 289 2. 324

3. 441 4. 361

Ans:



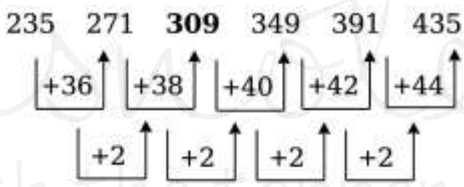
Ex: Which of the following numbers will replace the question mark (?) in the given series?

235, 271, ?, 349, 391, 435

1. 311 2. 307

3. 313 4. 309

Ans:



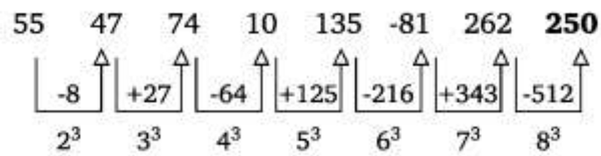
Ex: What number should replace question Mark (?) in the series given below.

55, 47, 74, 10, 135, -81, 262, ?

1. 774 2. -250

3. 343 4. -343

Ans:



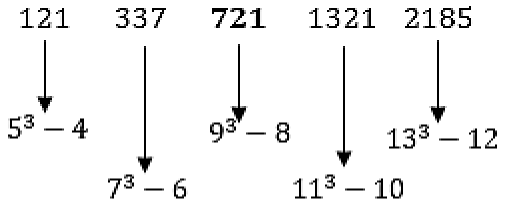
Ex: Find the missing number in the following series:

121, 337,?, 1321, 2185

1. 713 2. 720

3. 721 4. 737

Ans:



Type-2 Multiplication Based Increasing Series

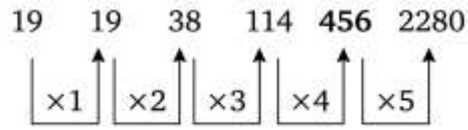
Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

19, 19, 38, 114, ?, 2280

1. 344 2. 1140

3. 456 4. 224

Ans:



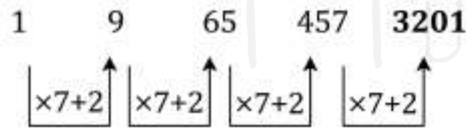
Ex: Which of the following numbers will replace the question mark (?) in the given series?

1, 9, 65, 457, ?

1. 4258 2. 3125

3. 3201 4. 5289

Ans:



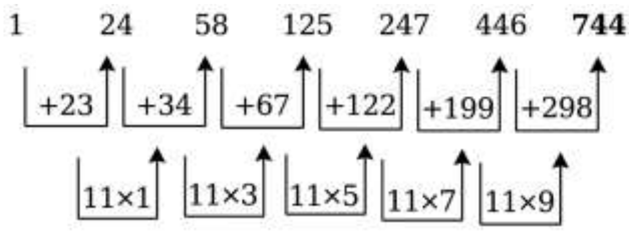
Ex: Which of the following numbers will replace the question mark (?) in the given series?

1, 24, 58, 125, 247, 446, ?

1. 774 2. 747

3. 744 4. 777

Ans:



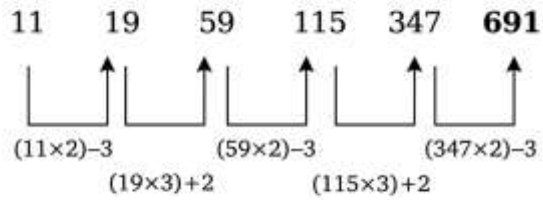
Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

11, 19, 59, 115, 347, ?

1. 697 2. 619

3. 679 4. 691

Ans:



Type-3 Addition and Subtraction Series

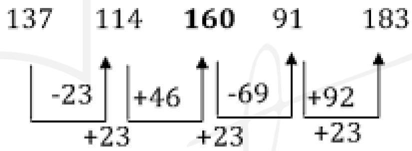
Ex: Which of the following number will replace the question mark (?) in the given series?

137, 114, ?, 91, 183

1. 145 2. 160

3. 125 4. 112

Ans:

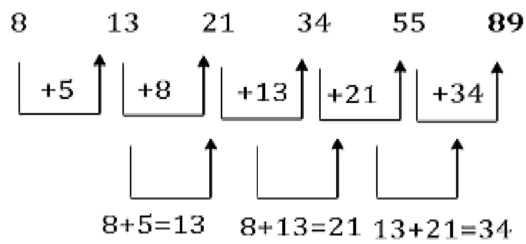


Ex: Which of the following numbers will replace the question mark (?) in the given series?

8, 13, 21, 34, 55, ?

1. 74 2. 68 3. 72 4. 89

Ans:



Type-4 Square and Cube Based Series

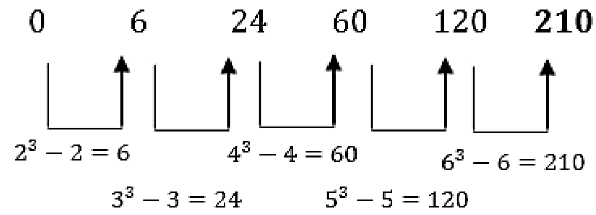
Ex: Find the next term of the series:

0, 6, 24, 60, 120, ?

1. 180 2. 210

3. 216 4. 240

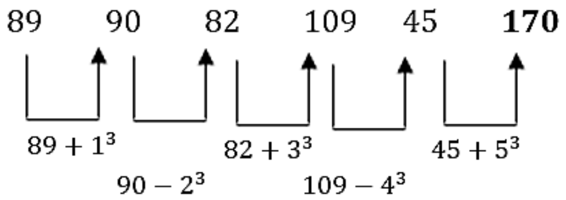
Ans:



Ex: Which number will replace the question mark (?) in the following series?

89, 90, 82, 109, 45, ?

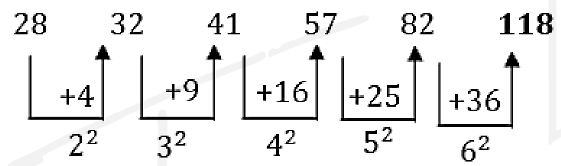
Ans:



Ex: Which number will replace the question mark (?) in the following series?

28, 32, 41, 57, 82, ?

Ans:

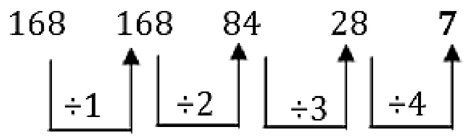


Type-5 Division Based Decreasing Series

Ex: Which number will replace the question mark (?) in the following number series?

168, 168, 84, 28, ?

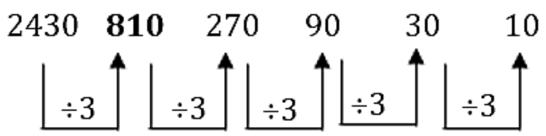
Ans:



Ex: Which number will replace the question mark (?) in the following series?

2430, ?, 270, 90, 30, 10

Ans:



Type-6 Mixed Number Series

Ex: Find the next terms 11,13,17,19, 23,_?

1. 27 2. 29

3. 31 4. 33

Ans:

11→ Prime number
13→Prime number
17→ Prime number
19→ Prime number
23→ Prime number
Next prime number = 29

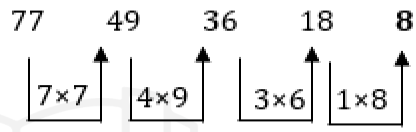
Ex: What will come in place of the question mark (?) in the following series?

77, 49, 36, 18, ?

1. 10 2. 12

3. 8 4. 16

Ans:



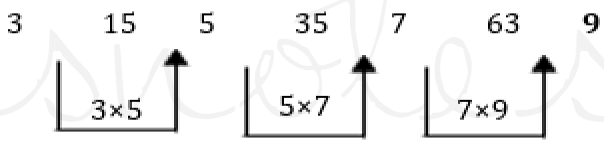
Ex: What will come in place of the question mark (?) in the following series?

3, 15, 5, 35, 7, 63, ?

1. 10 2. 126

3. 9 4. 84

Ans:



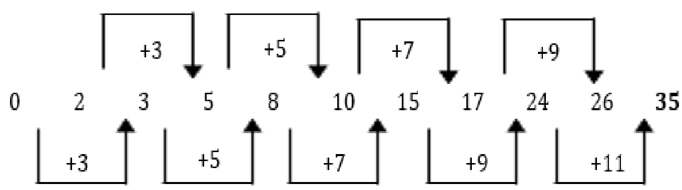
Ex: In the following series, what comes in place of the question mark (?)?

0, 2, 3, 5, 8, 10, 15, 17, 24, 26, ?

1. 28 2. 30

3. 32 4. 35

Ans:



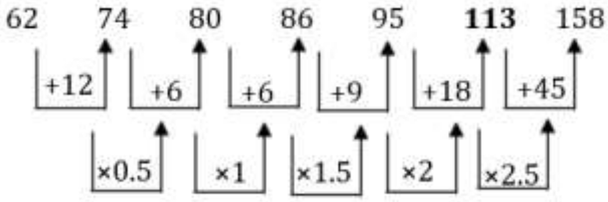
Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

62, 74, 80, 86, 95, ?, 158

1. 113 2. 100

3. 108 4. 122

Ans: The correct answer is 113



Ex: Find the wrong number in the FIRST series and then find the value of the following question.

(i) 455, 212, 131, 104, 94

(ii) Analyze the series and find the missing number (The exact place of the missing number is not known it can be on 1st, 2nd... 6th position) 57, 82, 167, 231, 312

1. Wrong term's position from left.

2. Find the position of the missing term from the right side.

Ans:

First Series:

455-212=243 = 3^5

212-131=81 = 3^4

131-104=27 = 3^3

104-95 = 3^2

95-92 = 3^1 (Wrong term 5^{th} from left)

Second Series:

57 + 5^2 = 82

82 + 6^2 = 118 (Missing term 4^{th} from right)

118 + 7^2 = 167

167 + 8^2 = 231

231 + 9^2 = 312

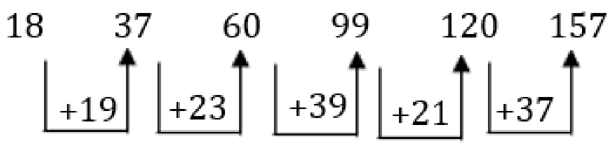
Type-7 Finding the Wrong Term in a Series

Ex: Find the wrong number in the following series

18, 37, 60, 99, 120, 157

1. 18 2. 37
3. 60 4. 99

Ans:



Logic used: (+) Increasing prime

18+19=37 37 +23=60

60+29=89 89+31=120

120+37=157

But given number is 99 instead of 89.

So, wrong number is 99.

Ex: Find the wrong number in the following series

2, 5, 14, 41, 124, 371

1. 14 2. 41
3. 371 4. 2
5. 124

Ans:

2 × 3 -1 = 5 5 ×3 - 1 = 14

14 × 3 - 1 = 4141 × 3 - 1 = 122 (Not 124)

122 × 3 - 1 = 365

Wrong number = 124

Ex: In the following number series, a wrong number is given. Find out the wrong number.

781, 783, 793, 791, 805, 800

1. 783 2. 793
3. 791 4. 805
5. 800

Ans:

156 × 5 + 1 = 781 157 × 5 - 2 = 783

158 ×5 + 3 = 793 159 × 5 - 4 = 791

160 × 5 + 5 = 805 161× 5 - 6 = 799

Wrong number in the given series is 800

Ex: Find the wrong number in the following series

2, 12, 66, 404, 2828, 22716

1. 2 2. 404
3. 2828 4. 22716
5. 12

Ans:

Each term is obtained by multiplying the previous term with successive numbers (4, 5, 6, 7, 8) and then adding successive even numbers (4, 6, 8, 10, 12).

2 × 4 + 4 = 8 + 4 = 12

12 × 5 + 6 = 60 + 6 = 66

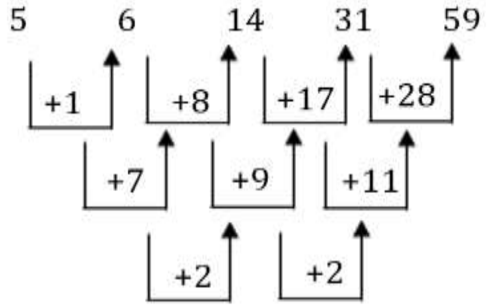
66 × 6 + 8 = 396 + 8 = 404

404 × 7 + 10 = 2828 + 10 = 2838

2838 × 8 + 12 = 22704 + 12 = 22716

The wrong number is 2828.

Ex: Find the wrong term from the given number series.
5, 7, 14, 31, 59
1. 31 **2.5**
3. 59 **4.7**
Ans: Correct answer in 7

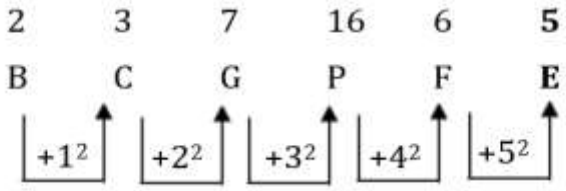


In the place of 6, it is written 7, so the answer is 7.

Ex: Find the Wrong number in the given number series:
6, 7, 10, 13, 21, 37
1. 10 2. 37
3. 6 4. 13
5. 21
Ans:
The given number series follows the pattern:
$6 + 2^0 = 7$ $7 + 2^1 = 9$
$9 + 2^2 = 13$ $13 + 2^3 = 21$
$21 + 2^4 = 37$
Wrong Number in the given number series is 10.

Type-8 Alphabet Series

Ex: Which of the following letters will replace the question mark (?) and complete the following letter series?
B, C, G, P, F, ?
1. F **2. E**
3. C **4. D**
Ans:

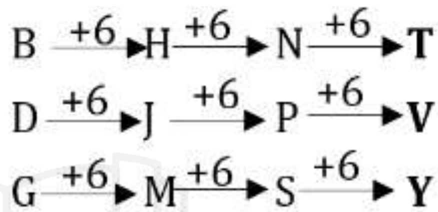


Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.
ABCD, CUKA, ENSX, GGAU, ?
1. IQRT 2. MNOQ
3. IRQT 4. IZIR

Ans:
Logic:
- In the first letter, add 2 to its position in the English alphabet.
- In the second letter, subtract 7 from its position.
- In the third letter, add 8 to its position.
- In the fourth letter, subtract 3 from its position.

Hence, "IZIR" is the correct answer.

Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.
BDG, HJM, NPS, ?
Ans:



Ex: A series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.
KIMnO, qRs Tu, WxYzA, cDeFg, ?
1. iJkLm 2. HiJkL
3. IjKIM 4. hijkl
Ans:
The pattern consists of groups of five letters arranged in the English alphabetical order. After each term, one letter is skipped. Additionally, letters follow an alternating pattern of uppercase and lowercase.
After 'g', the next omitted letter is 'h'. Also, the first letter and every alternate letter must be in uppercase.
Hence, the missing term is IjKIM.

Type-9 Mixed Series

Ex: A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.
FK27, LQ64, RW125, ?
1. CX216 2. XB216
3. XC216 4. YB343