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Science and Mathematics
(For Beginners)

(Class 8-10)

Volume - 3

Mathematics (Level 1)

SIERRA INNOVATIONS

INDEX

1

CHAPTER

Negative Numbers & Integers

1. The Need of Negative Numbers

As we know that when a smaller whole number is subtracted from larger whole number, we get a whole number but what about 4- 8, 3 - 9, 8 - 10 etc...?

Clearly there are no whole numbers to represent them. So, there is a need to extend our whole number system to represent the above differences.

Corresponding to natural number 1, 2, 3, 4, 5, 6, …… , we introduce new numbers denoted by -1, -2, -3, -4, -5, -6 ...., respectively such that 1 + (-1) =0, 2 + (-2) = 0, 3 + (-3) = 0, and so on.

The oppositeness of two quantities may be indicated by representing one as a positive and the other as a negative number.

We say that -1 and 1 are the opposites of each other;

-2 and 2 are the opposites of each other;

-3 and 3 are the opposites of each other, and so on.

1.1 Integers

Number greater than 0 are called positive numbers. Extending the number line to the left of 0 allows us to picture negative numbers those are less than 0.

When a single + sign or no sign is in front of a number, the number is a positive number. When a single - sign is in front of a number, the number is a negative number.

-5 indicates "negative five".

5 and +5 indicates "positive five".

The number 0 is neither positive nor negative.

The collection of negative whole numbers along with the collection of whole numbers are known as integers.

2. Representing Integers on Number Line

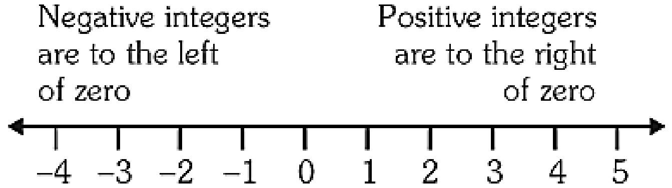
Integers can be represented on a number line.

The number line shows that every integer has an opposite number except '0'. The numbers + 1, + 2, +3 are positive numbers, denoted by +z.

The numbers -1, -2, -3 ..... are negative numbers, denoted by -z.

The positive and negative integers together with O are integers, denoted by Z or I.

thus Z = {.... -3, -2, -1, 0, 1, 2, 3,...}



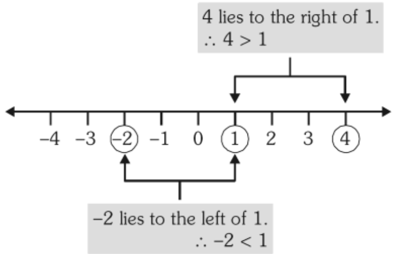
3. Comparing The Values Of Two Integers

Number line can be used to compare the values of two integers.

3.1 Horizontal Numbers Line

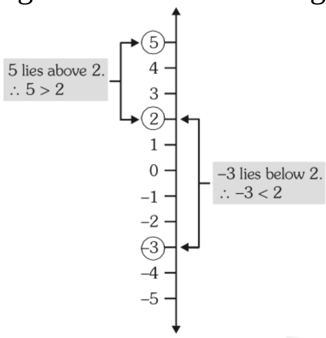
(i) On a horizontal number line, an integer is greater than the integer on its left.

(ii) On a horizontal number line, an integer is less than the integer on its right.



3.2 Vertical Number Line

(i) On a vertical number line, an integer is greater than the integer below it.
(ii) On a vertical number line, an integer is less than the integer above it.



4. Writing Positive And Negative Integers To Represent Word Descriptions

A positive or negative number is used to denote

4.1 An increase or decrease in value

For e.g.,
(i) Rs. 70 withdrawn is denoted by -Rs. 70.
(ii) Rs. 70 deposited is denoted by +Rs. 70.

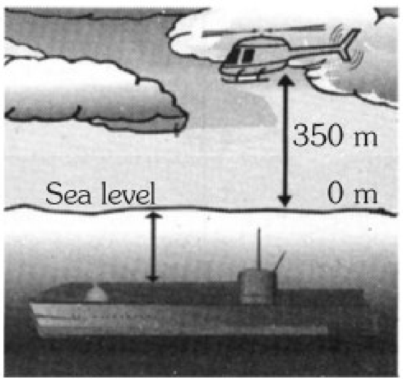
4.2 Values more than zero or less than zero

For e.g.,
(i) -18°C denotes a temperature that is l8°C below 0°C.
(ii) +18°C denotes a temperature that is 18°C above 0°C.

4.3 A positive direction or a negative direction (opposite direction)

For e.g.,
(i) 5 m denotes a direction 5 m to the right.
(ii) -5 m denotes a direction 5 m to the left.

4.4 Position above or below sea level



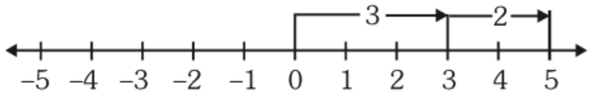
For e.g.,
(i) Sea level is taken as 0 m.
(ii) The helicopter flies 350 m above sea level or +350 m.
(iii) The submarine lies 150 m below sea level or -150 m.
Both the positive and negative integers are called directed numbers since they indicate direction. Another name given to them is signed numbers because of + or - sign which is a part of them.

5. Operations Of Integers

5.1 Addition of Integers

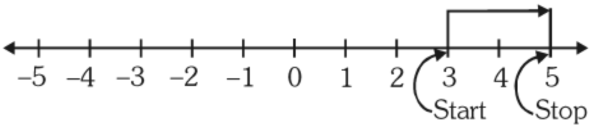
You know how to use the number line to add whole numbers. You can also use the number line in the same way to add positive and negative numbers.
(i) Adding two positive integers

For e.g., add 3 and 2



3 + 2 = 5

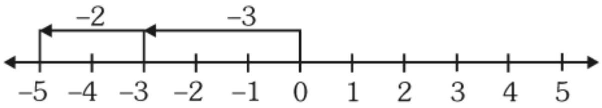
First move 3 places to the right from the origin. Then move 2 units further to the right.



Start at 3 and move 2 units to the right.

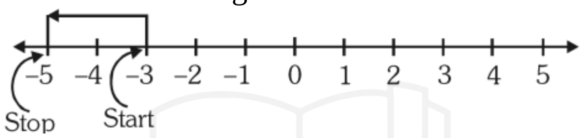
(ii) Adding two Negative integers

For e.g. add – 3 and – 2



-3 + (-2) = -5

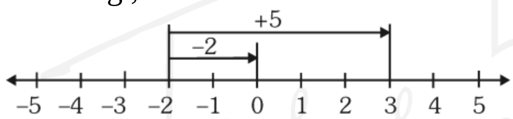
First move 3 places to the left from the origin. Then move 2 units further to the left.



Start at -3 and move 2 units to the left.

(iii) Adding a positive integer and a negative integer

For e.g., add - 2 and 5



(–2) + 5 = 3

First move 2 units to the left from 0. Then move 5 units to the right from this point.

5.2 <u>Properties of addition of Integers</u>

(I) Closure property of addition

The sum of two integers is always an integer. E.g.

(i) 5 + 4 = 9, which is an integer.

(ii) 4 + (-8) = - 4, which is an integer.

(iii) (-3) + (- 8) = - 11, which is an integer.

(iv) 15 + (- 9) = 6, which is an integer.

Hence, the sum of two integers is always an integer.

(ii) Commutative law of addition

If a and b are any two integers, then a + b = b + a.

E.g.

(i) (-4) + 9 = 5 and 9 + (- 4) = 5.

∴ (- 4) + 9 = 9 + (- 4).

(ii) (- 5) + (- 8) = -13 and (- 8) + (- 5) = - 13.

∴ (- 5) + (- 8) = (- 8) + (- 5).

(III) Associative law of addition

If a, b, c are any three integers, then (a + b) + c = a + (b + c).

E.g. {(-6) + {- 8)} + 5 = (-14) + 5 = - 9

and, (- 6) + {(-8) + 5} = (- 6) + (- 3) = - 9

∴ {(-6) + (-8}} + 5 = (-6) + {(-8) + 5)}

Similarly, other examples may be taken up.

(IV) Existence of additive identity

For any integer a, we have: a + 0 = 0 + a = a.
0 is called the additive identity for integers.
E.g.
(i) 9 + 0 = 0 + 9 = 9
(ii) (-6) + 0 = 0 + (-6) = (-6).

(v) Existence of additive inverse

For any integer a, we have : a+ (-a) = (a) + a= 0.
The opposite of an integer a is (-a).
The sum of an integer and its opposite is 0.
Additive inverse of a is (-a).
Similarly, additive inverse of (-a) is a.
E.g. We have : 5 + (- 5) = (- 5) + 5 = 0.
So, the additive inverse of 5 is (- 5).
And, the additive inverse of (-5) is 5.

Subtraction of integers

If a and bare two integers then a-6 is equal to a + (-b), i.e., to subtract b from a, change the sign of b and add to a.

Rule: (i) Change the sign of the subtrahend.
(ii) Add by the rules for adding integers.
In general, 'a-b' means the displacement from the point of b to the point of a. E.g.

(i) $\begin{array}{r} 8 \\ (-)\ 4 \\ \hline 4 \end{array}$ (ii) $\begin{array}{r} -8 \\ (-)\ -4 \\ \hline -4 \end{array}$

Properties of subtraction of integers Closure property for subtraction
If a and bare any two integers, then (a - b) is always an integer. E.g.

(i) 2 - 5 = 2 + (-5) = -3, which is an integer.
(ii) (-2) - 6 = (-2) + (-6) = -8, which is an integer.
(iii) 3 - (-5) = 3 + 5 = 8, which is an integer.
(iv) -4 - (-6) = -4 + 6 = 2, which is an integer.

Subtraction of integers is not commutative

(i) Consider the integers 3 and 5. We have :
(3 – 5) = 3 + (- 5) = -2 and (5 – 3) = 5 + (-3) = 2.
∴ (3 – 5) ≠ (5 – 3)
(ii) Consider the integers (−4) + (−2) = −6 and 2 – (−4) = 2 + 4 = 6.
∴ (−4) – 2 ≠ 2 – (-4)
(iii) Consider the integers (-6) and (-4). We have :
(-6) - (-4) = (-6) + 4 = -2 and (-4) - (-6) = (-6) + 6 = 2.
∴ (-6) - (-4) ≠ (-4) - (-6)
If a, b, c are three integers, then a- b ≠ b - a and a- (b - c) ≠ (a- b) - c.

Rules of Signs in addition and Subtraction

If a is any number, then
+(+a) = +a
+(-a) = -a
-(+a) = -a
-(-a) = +a
The sign of the number inside the brackets remains unchanged if there is a positive sign before it. The sign of the number inside the bracket changes if there is a negative sign before it.

Illustrations

Illustration 1. Subtract the sum of 837 and -487 from the sum of -392 and 792.
Solution Sum of 837 and -487 = 837 + (-487)
= 837 - 487 = 350
Sum of -392 and 792 = -392 + (792) = 400
Now 400 - 350 = 50

Illustration 2. The sum of two integers is -449. If one of them is -336, determine the other.
Solution Sum of two integers = -449
-336 + other integer = -449
other integer = sum -(-336)
= -449 - (-336)
= -449 + 336
= -113

Illustration 3. Find the value of -12 - [(-15) + (-3) - 8].
Solution -12 - [(-15) + (-3) - 8]
= -12 - [(-15) + (-3) + (-8)]
= -12 - [(-15) + (-11)]
= -12 - [-26] = -12 + 26 = 14

Additive Inverse

The sum of an integer and its opposite is 0.
Thus, if a is an integer, then a + (- a) = 0
a and -a are called opposites or negatives or additive inverses of each other.
Successor and predecessor of an integer
Let a be an integer then (a + 1) is called the successor of a and (a - 1) is called the predecessor of a. E.g.
The successor of-18 is -18 + 1= -17 and the predecessor of-18 -1 = -19.

5.3 Multiplication of integers

Rule 1: To find the product of two integers with unlike signs, we find the product of their values regardless of their signs and give a minus sign to the product.
Rule 2: To find the product of two integers with the same sign, we find the product of their values regardless of their signs and give a plus sign to the product.

Properties of multiplication of integers
Closure property for multiplication
The product of two integers is always an integer. E.g.
(i) 7 × 5 = 35 which is an integer.
(ii) (- 8) × 4 = - 32 which is an integer.

Commutative law for multiplication
For any two integers a and b, we have: (a × b) = (b × a)
E.g. 5(- 8) = - 40 and (- 8) × 5 = - 40
∴ 5(- 8) = (- 8) × 5

Associative law for multiplication
For any integers a, b, c, we have: (a × b) × c = a × (b × c)
E.g. Consider the integers 3, -5 and -8. We have
{3 × (-5)} × (-8) = (-15) × (-8) = 120
and 3 × {(-5) × (-8}} = (3 × 40) = 120
∴{3 × (-5)} × (-8) = 3 × {(-5) × (-8)} = 120

Distributive law for multiplication over addition
For any integers a, b, c, we have : a × (b + c) = (a × b) + (a × c)
(i) Consider the integers 5, (-6) and (-8). We have
5 × {(-6) + (-8)} = 5 × (-14) = -70
and {5 × (-6)} + {5 × (-8)} = (-30) + (-40) = -70.
5 × {(-6) + (-8)} = {5 × (-6)} + {5 × (-8)}.

Existence of multiplication identity
For every integer a, we have : (a × 1) = (1 × a) = a.
1 is called the multiplicative identity for integers.
(i) (12 × 1) = 12. (ii) (-16) × 1 = -16
Existence of multiplication inverse
Multiplicative inverse of a non-zero integer 'a' is the number $\frac{1}{a}$, as a.$\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)$. a = 1
(i) Multiplicative inverse of 6 is $\frac{1}{6}$.
(ii) Multiplicative inverse of -6 is - $\frac{1}{6}$.

Property of zero
For every integer a, we have : (a × 0) = (0 × a) = 0.
(i) 8 × 0 = 0 × 8 = 0 (ii) (-6) × 0 = 0 × (-6) = 0
Important results
(i) $(-a_1) \times (-a_2) \times (-a_3) \times \ldots \times (-a_n) = -(a_1 \times a_2 \times a_3 \times \ldots \times a_n)$, when n is odd.
(ii) $(-a_1) \times (-a_2) \times (-a_3) \times \ldots \times (-a_n) = (a_1 \times a_2 \times a_3 \times \ldots \times a_n)$, when n is even.
(iii) (-a) × (-a) × (-a) × … n times = $-a^n$, when n is odd.
(iv) (-a) × (-a) × (-a) × … n times = a^n, when n is even.
(v) (-1) × (-1) × (-1) × … n times = -1, when n is odd.
(vi) (-1) × (-1) × (-1) × … n times = 1, when n is even.

5.4 Division of integers

Rule 1: For dividing one integer by the other, the two having unlike signs, we divide their values regardless of their signs and give a minus sign to the quotient.
Rule 2: For dividing one integer by the other having like signs, we divide their values regardless of their signs and give a plus sign to the quotient.

Properties of division of integers
(i) If a and b are integers then (a ÷ b) is not necessarily an integer.
(a) 16 and 5 are both integers, but (16 ÷ 5) is not an integer.
(b) (-9) and 4 are both integers, but {(- 9) ÷ 4} is not an integer.
(ii) If a is an integer and a ≠ 0, then a ÷ a = 1
(a) 16 ÷ 16 = 1 (b) (- 8) ÷ (- 8) = 1
(iii) If a is an integer, then (a ÷ 1) = a
(a) 7 ÷ 1 = 7 (b) (- 6) ÷ 1 = (- 6)
(iv) If a is an integer and a ≠ 0, then (0 ÷ a) = 0 but (a ÷ 0) is not meaningful.
(a) 0 ÷ 6 = 0 (b) 0 ÷ (- 4) = 0 (c) 6 ÷ 0 is meaningless.
(v) If a, b, c are integer, then (a ÷ b) ÷ c ≠ a ÷ (b ÷ c), unless c = 1.
Thus, division on integers is not associative.
Let a = − 8 b = 4 and c = − 2 Then,
(a ÷ b) ÷ c = {(- 8) ÷ 4} ÷ (- 2) = (- 2) ÷ (- 2) = 1
a ÷ (b ÷ c) = (- 8) ÷ {4 ÷ (- 2)} = (- 8) ÷ (- 2) = 4
∴ (a ÷ b) ÷ c ≠ a ÷ (b ÷ c).
However, if a = - 8 b = 4 and c = 1 then

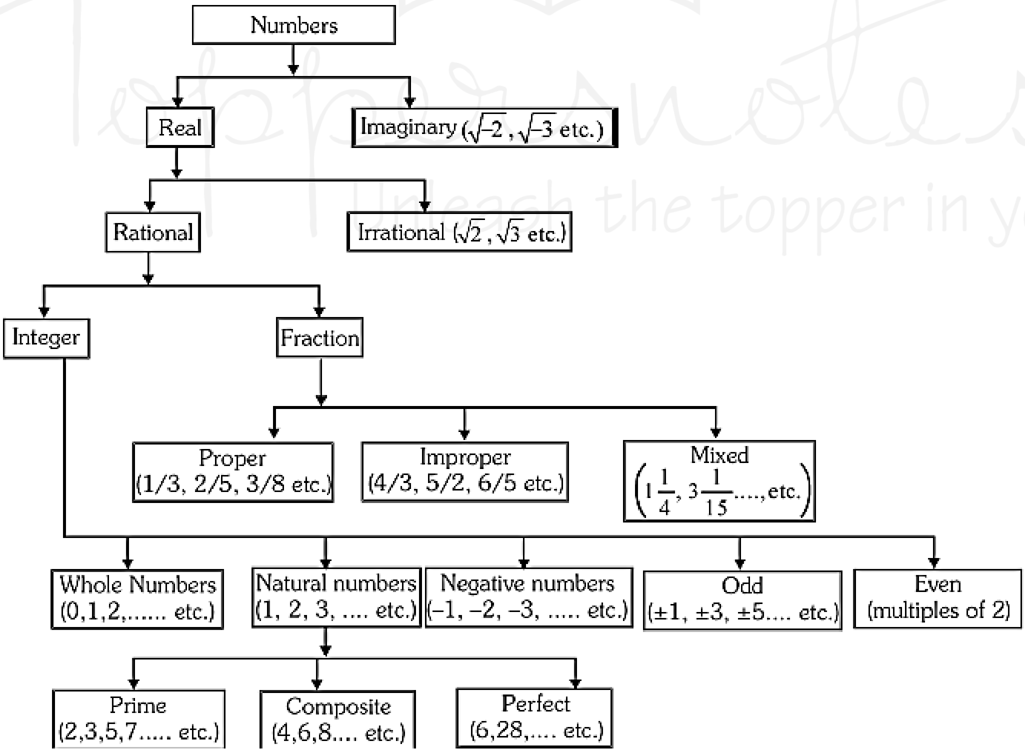
(a ÷ b) ÷ c = {(- 8) ÷ 4} ÷ 1 = (- 2) ÷ 1 = (- 2)
a ÷ (b ÷ c) = (- 8) ÷ {4 ÷ 1} = (- 8) ÷ 4 = (- 2)
So, in this case, (a ÷ b) ÷ c =a ÷ (b ÷ c.)
(vi) If a, b, c are nonzero integers and a > b then
(a ÷ c) ÷ (b ÷ c) if c is positive.
(a ÷ c) < (b ÷ c) if c is negative.
(a) 27 > 18 and 9 is positive.
$\frac{27}{9} > \frac{18}{9}$
(b) 27 > 18 and (-9) is negative.
$\frac{27}{-9} > \frac{18}{-9}$
If a, b and c are three integers then
a ÷ (b^{1}) b ÷ a and a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c

6. Absolute Value of an Integer

(i) The absolute value of an integer is its numerical value regardless of its sign is negative or positive on a number line, it is the distance between the number and zero.
E.g. (i) The absolute value of - 15 is 15
(ii) The absolute value of 15 is 15
(ii) The symbol of absolute value is to enclose the number between vertical bars such as $|-a|$ = a
E.g. $|-20|$ = 20
It is read as 'the absolute value of - 20 equals 20'.

7. Classification of Numbers

All the numbers that we see or use on a regular basis can be classified as given in the chart below.



7.1 Identification of prime numbers

Step 1: Find approximate square root of the given number.
Step 2: Divide the given number by prime numbers less than the approximate square root of that given number. If the given number is not divisible by any of these prime numbers, then the number is prime otherwise not. e.g., approximate square root of 571 is 24 Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19 & 23. 571 is not divisible by any of these prime numbers. So, 571 is a prime number.

Co-prime numbers
Two numbers are said to be relatively prime to each other or co-prime, if their HCF is 1.
e.g., (i) 9 and 28 (ii) 3 and 5 etc.
Twin primes: Prime numbers differing by 2 are called twin primes. Ex. 3 and 5, 5 and 7, 11 and 13, 17 and 19 etc.
Prime triplet: The set {3, 5, 7} of three consecutive primes is called the prime triplet.

Other special numbers
Perfect numbers
If the sum of all factors of a number N is equal to 2N, then N is called a perfect number.
e.g., 6, 28, 496, 8128 etc.
6= 1 + 2 + 3, where 1, 2 and 3 are divisors of 6.
28 = 1 + 2 + 4 + 7 + 14.

8. HCF of Numbers

It is the highest common factor of two or more given numbers.
It is also called GCD (greatest common divisor).
For example, HCF of 10 and 15 = 5, HCF of 55 and 200 = 5, HCF of 64 and 36 = 4 etc.

Factorisation Method to find HCF
To find the HCF of given numbers, first resolve the numbers into their prime factors. After expressing the numbers in terms of the prime factors, the HCF is the product of common factors.

Illustrations

Illustration 4. Find the HCF of 88, 24 and 124.
Solution $88 = 2 \times 44 = 2 \times 2 \times 22 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11^1$
$24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$
$124 = 2 \times 62 = 2 \times 2 \times 31 = 2^2 \times 31^1$
$\therefore$ HCF = $2^2 = 4$
Illustration 5. Find the HCF of 72, 60 and 96.
Solution $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3^1 \times 5^1$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3^1$
$\therefore$ HCF = product of the highest number of common factors = $2 \times 2 \times 3 = 12$
If we have to find the greatest number that will exactly divide p, q and r, then the required number = HCF of p, q and r.
Illustration 6. Find the greatest number that will exactly divide 65, 52 and 78.
Solution Required number = HCF of 65, 52 and 78 = 13
If we have to find the greatest number that will divide p, q and r leaving remainders a, b and c respectively, then the required number = HCF of (p - a), (q -b) and (r - c).
Illustration 7. Find the greatest number that will divide 65, 52 and 78 leaving remainders 5, 2 and 8 respectively.
Solution Required number = HCF of (65 - 5), (52 - 2) and (78 - 8)
= HCF of 60, 50 and 70 = 10
If we have to find the greatest number that will divide p, q and r leaving the same remainder in each case, then required number = HCF of the absolute values of (p - q), (q - r) and (r - p).
Illustration 8. Find the greatest number that will divide 65, 81 and 145 leaving the same remainder in each case.
Solution Required number is HCF of (81- 65), (145 - 81) and (145 - 65)
= HCF of 16, 64 and 80 = 16

9. Lcm of Numbers

Least common multiple of two or more numbers is the smallest number which is exactly divisible by all of them.

e.g., LCM of 5, 7, 10 = 70,

LCM of 2, 4, 5 = 20,

LCM of 11, 10, 3 = 330

Factorization Method to find LCM

To find the LCM of the given numbers, first resolve all the numbers into their prime factors and then the LCM is the product of highest powers of all the prime factors.

Illustrations

Illustration 9. Find the LCM of 40, 120, 380.

Solution $40 = 4 \times 10 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$,

$120 = 4 \times 30 = 2 \times 2 \times 2 \times 5 \times 3 = 2^3 \times 5^1 \times 3^1$,

$380 = 2 \times 190 = 2 \times 2 \times 5 \times 19 = 2^2 \times 5^1 \times 19^1$

$\therefore$ Required LCM = $2^3 \times 5^1 \times 3^1 \times 19^1 = 2280$.

Division method to find LCM

Write the given numbers separately. Then divide by 2 and write the result below the numbers divisible by 2. If it is not divisible by 2 then try with 3, 5, 7....etc. Leave the others (those not divisible) untouched. Do the same for all steps till you get 1 as the quotient in each column.

Illustrations

Illustration 10. Find the LCM of 6, 10, 15, 24, 39.

Solution

2	6,	10,	15,	24,	39
2	3,	5,	15,	12,	39
2	3,	5,	15,	6,	39
3	3,	5,	15,	3,	39
5	1,	5,	5,	1,	13
13	1,	1,	1,	1,	13
	1,	1,	1,	1,	1

LCM = $2 \times 2 \times 2 \times 3 \times 5 \times 13 = 1560$

If we have to find the least number which is exactly divisible by p, q and r, then the required number

= LCM of p, q and r.

Illustration 11. Find the least number that is exactly divisible by 6, 5 and 7.

Solution Required number = LCM of 6, 5 and 7 = 210. If we have to find the least number which when divided by p, q and r leaves the remainders a, b and c respectively, then if it is observed that (p - a) = (q - b) = (r - c) K (say), then the required number = (LCM of p, q and r) - (K).

Illustration 12. Find the least number which when divided by 6, 7 and 9 leaves the remainders 1, 2 and 4 respectively.

Solution Here, (6 - 1) = (7 - 2) = (9 - 4) = 5.

$\therefore$ Required number = (LCM of 6, 7 and 9) - 5

= 126 - 5 = 121

If we have to find the least number which when divided by p, q and r leaves the same remainder 'a' in each case, then required number = (LCM of p, q and r) + a.

Illustration 13. Find the least number which when divided by 15, 20 and 30 leaves the remainder 5 in each case.

Solution Required number = (LCM of 15, 20, 30) + 5

= 60+5 = 65

LCM × HCF = Product of two numbers. (Valid only for "two")

Illustration 14. Find the LCM of 25 and 35, if their HCF is 5.

Solution LCM = $\frac{\text{Product of the numbers}}{\text{HCF}}$

$= \frac{25 \times 35}{5} = 175$

Exercise - 1

Multiple choice questions

1. **Which set of integers is written in ascending order?**
(1) 112, -200, 315, -48
(2)-64, -101,-260
(3)-361, -316, -163, -136
(4) 45, 80, -100, -125

2. **The successor of -18 is**
(1) -19 (2) 17
(3)-17 (4) 19

3. **The predecessor of - 16 is**
(1)-15 (2)-17
(3) 15 (4) 17

4. **The additive inverse of -5 is**
(1) 5 (2) 0
(3)-4 (4)-6

5. **Which of the following is false.**
(1) Zero is an integer
(3) All whole numbers are integers
(2) All negative nos. are integer
(4) All integers are whole numbers

6. **Which of the following represent a positive integer?**
(1) Loss of Rs. 350
(2) Rs. 350 due
(3) Withdrawal of Rs 200
(4) Credit of Rs. 10

7. **Which number will we reach if we move 5 numbers to the right of - 1.**
(1)-5 (2) 5
(3) 4 (4)-4

8. **If the deepest point in the sea is 11,600 m below sea level and the highest mountain top is 8846 metres above sea level, then the difference in these elevations is.........**
(1) 2754 m (2) 20,446 m
(3) 21,406 m (4) 2952 m

9. **If p and q are two integers such that p is the predecessor of q, then pq is equal to**
(1) 1 (2) 0
(3) 2 (4)-1

10. **Which of the following is false ?**
(1) -1 < -2 (2) 79 < 89
(3) -1 < 1 (4) 1 > 0

11. **Which of the following expression are equal to -20?**
(1) - 4 × 5
(II)-3210-(-2)
(III) - 6 × 2 - [- 2 × (- 4)]
(IV) 5(- 2) + (- 3) × 4
(1) I only (2) I and II
(3) I, II and III (4) I, II, III and IV

12. **1763 × (-2) + (-1763) x 98 =**
(1) 176300 (2) 169248
(3)-176300 (4)-169248

13. **A tanker contains 500 litres of milk. Due to small hole in the tanker, the quantity of milk is decreasing at the rate of 9 litres every hour. What will be the quantity of milk after 10 hours?**
(1) 491 litres (2) 410 litres
(3) 90 litres (4) 450 litres

14. **In a test, (+4) marks are given for every correct answer and (-2) marks for every wrong answer. Rohit answered all the questions and scored 68 marks. 25 of Rohit's answers were correct. How many of the questions he attempted were incorrect?**
(1) 22 (2) 5
(3) 16 (4) 8

15. **$(-1)^{217} \times (-4)^{11} \times (-3)^{13}$ results in a**
(1) negative integer (2) positive integer
(3) zero (4) none of these

16. **The predecessor of $|-1|$ is**
(1)-2 (2) 2
(3) 0 (4) 1

18. **Which of the following is true?**
(1) -5 > -7 (2) -8 < -11
(3) -6 <- 8 (4) +10 - 10

19. **0 is**
(1) a positive integer
(2) a negative integer
(3) neither positive nor negative
(4) None of these

20. **The smallest positive integer is**
(1) 0 (2) 1
(3)-1 (4) Not determinable

21. The greatest negative integer is
(1) 0 (2) 1
(3)-1 (4) Not determinable

22. The integer 4 more than - 5 is
(1) 1 (2) 9
(3) -1 (4) none of these

23. What must be added to -135 to get -142?
(1) 7 (2)-7
(3) 277 (4)-277

24. The value of the expression 10000 ÷ {(80+ 100 ÷ 5) x 100} is
(1) 0 (2) 1
(3) 10 (4) 100

25. The difference in temperatures +50°C and -50°C is
(1) 1°C (2) 0°C
(3) 50°C (4) -100°C

26. Additive inverse of (pqrs) where p, q, r and s are nonzero integers is
(1) (- p)(- q)(- r)(- s) (2) (- p) qr(- s)
(3) pqrs (4) (- p)(- q)(- r)s

27. Simplify: 38 - 2(5 – $\overline{8 - 3}$) [2 {7+ (-3) × (-4)}]
(1) 35 (2) 37
(3) 38 (4) 39

28. The value of [(- 4) × (- 9) × (- 25)] ÷ [(- 2)(-3)(- 5)] is
(1) 10 (2) 20
(3) 30 (4) 40

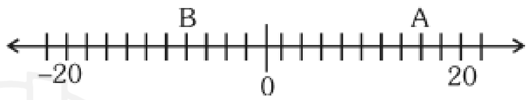
29. Which of the following sets is not closed under subtraction?
(1) N (2) Z
(3) Q (4) R

30. Next three consecutive integers in the pattern 11, 8, 5, 2, _, _,_?
(1) 0, -3, -6 (2) -1,-5,-8
(3) -2,-5,-8 (4) -1, -4, -7

31. Which of the following is not the additive inverse of a?
(1) -(-a) (2) a × (- 1)
(3) -a (4) a ÷ (- 1)

32. Simplify: (23 (23 – $\overline{23 - 23}$))}
(1) 0 (2) 1
(3) 2 (4) 3

33. Which of the following options is not true with respect to given number line?



(1) B is greater than -20
(2) A is greater than 0
(3) A is greater than B
(4) B is greater than 0

34. Which of the following shows the maximum rise in temperature?
(1) 23° to 32° (2) - 10° to 1°
(3) - 18° to - 11° (4) - 5° to 5°

35. The number of integers between -2 and 2 are
(1) 4 (2) 3
(3) 2 (4) 0

Exercise 2

1. $|10 - 4| \div |-3|$ is equal to
(1) -2 (2) -18
(3) 2 (4) 3

2. The sum of two integers is 45. If one of them is -23, the other is
(1) 68 (2) 22
(3) -68 (4) -22

3. Which of the following integers in the smallest?
(1) $|-14|$
(2) $|-(-24)|$
(3) $|-28|$
(4) $-|-30|$

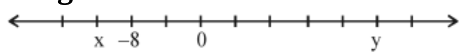
4. Which one of the following is correct about the statements given below?
Statement 1: The product of all the integers between -123 to 123 is equal to zero.
Statement 2: The sum of all the integers between -123 to 123 is equal to zero.
(1) Statement 1 is true and 2 is false.
(2) Statement 2 is true and 1 is false.
(3) Both the statement 1 and 2 are true.
(4) Both the statement 1 and 2 are false.

5. The sum of two integers having the negative signs is
(1) always negative
(2) always positive
(3) sometimes positive and sometimes negative
(4) none of these

6. **Which one of the following expressions does not give an integer?**
(1) (- 56) × (- 8) (2) (- 56) ÷ (- 8)
(3) (- 56) - (- 8) (4) (- 8) ÷ (- 56)

7. **In a quiz, each student is required to answer 40 questions. 5 marks are given for every correct answer and 3 marks are deducted for every wrong answer. If Mahesh answered 35 questions correctly and rest incorrect and Suresh answered 32 questions correctly and rest incorrect, what is the difference in the total marks obtained by them?**
(1) 15 (2) 24
(3) 160 (4) 175

8. **The diagram shows a number line.**



The value of x + y is
(1) -32 (2) -8
(3) 8 (4) 32

9. **A lady parked her car on the 6th floor and took a lift up 17 floors to the Finance Department. She then went down 9 floors to the Tax Department. On which floor would you find the lady?**
(1) 14th (2) 15th
(3) 23rd (4) 26th

10. **Universal Tuition Centre has 1,070 students. Of these, 598 are boys. How many students in the centre are girls?**
(1) 1,668 (2) 488
(3) 472 (4) 462

11. **Shyam earns ₹3500. If he spent ₹749 on buying a headphone, what is the balance left in his salary?**
(1) ₹2,751 (2) ₹2,251
(3) ₹2,151 (4) ₹2,059

12. **In an examination, Jessy's score was 92. Nissi obtained 15 marks less than Jessy. Rishi scored 4 marks more than Nissi. What is the difference between Jessy's and Rishi's scores?**
(1) 11 (2) 19
(3) 73 (4) 77

13. **If a is the predecessor of b, then the values of (a - b) and (ba) are respectively**
(1) -1 and 1 (2) 1 and -1
(3) 0 and 1 (4) 1 and 0

14. **A man lost ₹500 in one transaction and gained 300 in another. His net profit or loss is**
(1) profit ₹800 (2) loss ₹800
(3) loss ₹200 (4) gain ₹200

15. **Simplify: $19 + \frac{1}{5}\{-20 \times (55 - \overline{13 - 3})\} + (-5)]$**
(1) 50 (2) 55
(3) 60 (4) 65

16. **The difference of the sum of even numbers and sum of odd numbers between 10 and 20 is**
(1) 13 (2) 14
(3) 15 (4) 16

17. **The value of $12 - [7 - \{16 - (18 - \overline{6 + 3 - 12})\}]$ is**
(1) 3 (2) 2
(3) 1 (4) 0

18. **Simplify : $222- \left[\frac{1}{3}42 + (56 - \overline{8 + 9})\} + 108\right]$**
(1) 87 (2) 88
(3) 89 (4) 90

19. **Integers for which $|1 - x| = 3$ are**
(1) 3,0 (2) 4, -2
(3) -2, 3 (4) 4, -25

20. **Find the value of |7| + |5| - |-3|, where | | is absolute value of an integer.**
(1) 0 (2) 1
(3) 9 (4) 3

21. **An insect crawls up 5 cm every second on a 60 cm vertical rod and then falls down 2 cm over the next second. How many seconds will it take to climb the rod?**
(1) 20 seconds (2) 39 seconds
(3) 60 seconds (4) 30 seconds

22. **A tanker contains 500 litres of water. Due to a small hole in the tanker, the quantity of water is decreasing at the rate of 9 litres every hour. What will be the quantity of water in the tank after 10 hours?**
(1) 410 litres (2) 491 litres
(3) 400 litres (4) 90 litres

23. **If a, b, c, d are first 4 positive odd integers, then their sum is**
(1) 5^2 (2) 2^2
(3) 4^2 (4) 6^2

24. **If m = (- 1)2000 and n = (- 1)2002, then find the value of $\frac{m}{n}$.**
(1) -1 (2) 1
(3) 2000 (4) 2002

25. **If a = (- 1)2009 and b = (- 1)2010, then find the value of ab.**
(1) 1 (2)-1
(3) 2009 (4) 2010

26. **If x = (- 23) + 22 + (- 23) + 22 (40 terms) and y = 11 + (- 10) + 11 + (- 10) +........(20 terms), then y - x is**
(1) 40 (2) 10
(3) 20 (4) 30

27. **x= $(-1)^1 + (-1)^2 + ... + (-1)^{2009}$; and y= $(-1)^1 - (-1)^2 + (-1)^3 - (-1)^4 + ... + (-1)^{2009}$, then find x - y**
(1) 2009 (2) 2008
(3) 0 (4) 1004

28. **The LCM of two numbers is 48. If the numbers are in the ratio 2 : 3, then the sum of the numbers is**
(1) 28 (2) 32
(3) 40 (4) 64

29. **The greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case is**
(1) 4 (2) 7
(3) 9 (4) 13

30. **There are two ropes each measures 496 m and 624 m respectively. The largest scale that should be used to measure both the ropes exactly is of**
(1) 4 m (2) 8 m
(3) 16 m (4) 24 m

31. **Two bridges have to be build of length 1024 m and 960 m respectively. The largest length of concrete beams that is required to built both bridges as the combination of beams exactly fit for the required lengths, is**
(1) 12 m (2) 16 m
(3) 32 m (4) 64 m

32. **Simplify: [29 -(- 2) {6 - (7 - 3)}] ÷ [3 ×{5+(- 3) × (-2)}]**
(1) 0 (2) 1
(3) 2 (4) 3

33. **Study the below statements carefully and answer the question given below.**
(i) The successor of 0 × (-52) is 1 × (-52)
(ii) Integers are closed under division.
(iii) (-20) × (5 -3) = (-20) × (-2)
(iv) (-2) + (-9) is less than (-9) - (-2)

34. $4+\frac{1}{5}\left[\left\{-10\times\left(25-\overline{13-3}\right)\right\}\div(-5)\right]$
(1) 0 (2) 1
(3) 2 (4) 10

35. **Which of the following statements is/are true?**
(1) Only (i)
(2) Both (i) and (iv)
(3) Only (iv)
(4) Both (i) and (iii)

35. **Which of the following statements is not true?**
(1) When two positive integers are added, we always get a positive integer.
(2) When two negative integers are added, we always get a negative integer.
(3) When a positive and a negative integer is added we always get a negative integer.
(4) Additive inverse of an integer 2 is (-2) and additive inverse of (-2) is 2.

36. **On the following number line, value 'zero' is shown by the point.**

(1) X (2) Y
(3) Z (4) W

37. **-35×107 is not same as**
(1) -35 x (100 + 7)
(3) -35 x 7 + 100
(2) (-35) x 7+ (-35) × 100
(4) (-30-5) × 107

38. **Which list of integers is in order from least to greatest?**
(1)-52, -49,-5, 50, 57
(2)-52, 57, 50, -49, -5
(3) -5, -49, 50, 57, -52
(4) 57, 50, -5, -49, -52

39. For a non zero integer 'a', which of the following is not defined?

(1) a ÷ 0 (2) 0 ÷ a

(3) a ÷ 1 (4) 1 ÷ a

40. A diver was diving 70 meters below sea level. He went down 30 metres more and came up 45 metres. How far below sea level did he dive?

(1) 55 metres (2) -55 metres

(3) 145 metres (4) -145 metres

Answer Key

Exercise -1

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	3	2	1	4	4	3	2	4	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	3	2	3	1	3	2	1	3	2
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	3	3	2	1	4	4	3	3	1	4
Que.	**31**	**32**	**33**	**34**	**35**					
Ans.	1	1	4	2	3					

Exercise -2

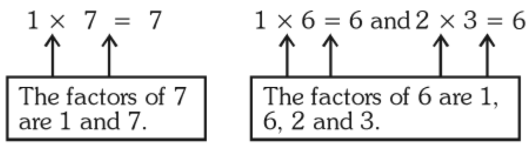
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	1	4	3	1	4	2	3	1	3
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	1	1	3	2	3	4	1	2	3
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	2	1	3	2	2	4	2	3	1	3
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	4	2	3	4	3	3	3	1	1	2

2 CHAPTER

Playing With Numbers

1. Factors and Multiples

When two or more numbers are multiplied, each number of the product is called a factor.
and the product is a multiple of each of its factors.



Facts about factors and multiples

(1) 1 × 2 = 2 1 × 3 = 3 1 × 5 = 5
So, 1 is a factor of every number.

(2) 2 × 1 = 2, 3 × 1 = 3, 4 × 1 = 4
and it is true for all numbers. So, it can be said that every number is a factor of itself.

(3) 1 × 8 = 8, 2 × 4 = 8, 4 × 2 = 8, 8 × 1 = 8
Clearly, 1 < 8, 2 <8, 4 <8, and 8 =8.
We can say that every factor of the number is less than or equal to the given number.

(4) Factors of 4 are 1, 2 and 4
Factors of 8 are 1, 2, 4 and 8
Factors of 16 are 1, 2, 4, 8 and 16
Factors of 64 are 1, 2, 4, 8, 16, 32 and 64
The number of factors are 3, 4, 5 and 7 respectively, i.e., the numbers of factors are countable.
Thus, the number of factors of a given number are always finite, i.e., they can be counted.

(5) 7 × 1 = 7, 7 × 2 = 14,7 × 3 = 21, 7 × 4 = 28.7 × 5 = 35
7 = 7, 14 > 7, 21 > 7, 28 > 7, 35 > 7. Thus 7, 14, 21, 28, 35 are multiples of 7 and all these are either equal to or greater than 7. Thus, every multiple is equal to or greater than the given number.

(6) Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, ... The number of multiples do not end.
So, it can be said that the number of multiples of a given number is infinite.

(7) 2 is a multiple of 2 and 32 is a multiple of 32.
Thus, every number is a multiple of itself.

✓ Every number is a multiple of 1. Every number is a factor of itself. Every number is a multiple of itself.

✓ Factors of a given number is finite. Multiples of a given number is infinte.

2. Perfect Number

Factors of 6 are 1, 2, 3 and 6
Now, the sum of the factors of 6
1+2+3+61 = 2 = 2 times 6
Factors of 28 are 1, 2, 4, 7, 14 and 28
Now, the sum of the factors of 28
= 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 times 28.
The numbers like 6 and 28 are called perfect numbers.
A number is called a perfect number if the sum of all its factors is equal to twice the number.

3. Prime And Composite Numbers

3.1 Prime numbers

Except 1 each natural number which is divisible by 1 and itself only is called as prime number, e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, …. etc.

There are total 25 prime numbers upto 100

There are total 46 prime numbers upto 200

2 is the only even prime number and the least prime number.

1 is neither prime nor composite number.

There are infinite prime numbers.

A list of all prime numbers upto 100 is given below.

Table of prime numbers (1-100) :

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Test to find whether a given number is a prime

Step-1: Select a least positive integer n such that n^2 > given number.

Step-2: Test the divisibility of given number by every prime number less than n.

Step-3: The given number is prime only if it is not divisible by any of these primes.

- ✓ 1 is neither prime nor composite.
- ✓ 2 is the smallest prime number which is even. Every prime number except 2 is odd.

Illustrations

Illustration 1. Investigate whether 571 is a prime number.

Solution Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore n = 24$

Prime numbers less than 24 are 2, 3, 5, 11, 13, 17, 19, 23. Since 571 is not divisible by any prime number less than 24. So, 571 is a prime number.

Illustration 2. Is 353 a prime number?

Solution Since 19 × 19 = 361 > 353, we test divisibility by 2, 3, 5, 7, 11, 13, 17. We find that it is not divisible by any one of them. Hence, 353 is a prime number.

3.2 Twin Primes

- ✓ A twin prime is a prime number that differs from another prime number by two. Some examples of twin prime pairs are (3, 5), (5, 7), (11, 13), (17, 19), (29, 31) and (41, 43). Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin.

3.3 Co-prime numbers

- ✓ Two numbers are said to be co-prime or relatively prime if they have only 1 as a common factor. E.g. 2 and 3 are co-prime.
- ✓ The two numbers in the pair of co-prime numbers need not be both prime. They both can be composite, or prime, or one composite and the other prime.

4. Tests For Divisibility of Numbers

Divisibility by 2

- ✓ A number is divisible by two, if it has any of the digits 0, 2, 4, 6, or 8 in its one's place. E.g. 24, 632, 478, 500 etc. are divisible by two whereas 301, 783, etc. are not divisible by two.

Divisibility by 3

- ✓ A number is divisible by three, if the sum of its digits is a multiple of three. E.g. 27, 66 are divisible by three because the sum of their digits 2 +7 = 9, 6+6=12 are multiples of three, whereas 31, 88 not divisible by three because their sum 3+1 = 4, 8 + 8 = 16 is not a multiple of three.

Divisibility by 4

- ✓ A number with 3 or more digits is divisible by 4, if the number formed by its last two digits (i.e. ones and tens) is divisible by 4. E.g. 284 is divisible by 4 because 84 is divisible by 4 whereas 315 is not divisible by 4 because 15 is not divisible by 4.
  E.g. 1700, 1600, 1900, etc. are divisible by 4. (If last two digits are 00 divisible by 4)

Divisibility by 5

- ✓ A number which has either 0 or 5 in its one's place is divisible by 5. E.g. 15, 40 are divisible by 5 whereas 12, 78 are not divisible by 5.

Divisibility by 6

- ✓ A number is divisible by 6, if it is divisible by 2 and 3 both. E.g. 72 is divisible by 6 because it is divisible by 2 and 3 both whereas 81 is not divisible by 6 because it is divisible by 3 but not by 2.

Divisibility by 8

- ✓ A number with 4 or more digits is divisible by 8, if the number formed by its last three digits (i.e. ones, tens and hundreds) is divisible by 8. E.g. 74512 is divisible by 8 because 512 is divisible by 8. E.g. 5000, 6000, 2000, etc. are divisible by 8. (If last three digits are 000 divisible by 8)

Divisibility by 9

- ✓ A number is divisible by 9, if the sum of its digits is a multiple of 9. E.g. 756 is divisible by 9 because 7 + 5 + 6 = 18, is divisible by 9.

Divisibility by 10

- ✓ A number is divisible by 10, if it has 0 in the ones place. E.g. 20, 10, 900 etc. are divisible by 10.

Divisibility by 11

- ✓ A number is divisible by 11, if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number is either 0 or divisible by 11.
  E.g. 121, 1331 are divisible by 11 whereas 3456788 is not. The following table will make it more clear to you.

Number	Sum of the digits (at odd places from right)	Sum of the digits (at even places from right)	Difference	Divisible by 11
121	1+1 = 2	2	2 - 2 = 0	yes
1331	1+3= 4	3 + 1 = 4	4-4 4 = 0	yes
3456788	8 +7+5+3= 23	8 + 6 + 4 = 18	23 - 18 = 5	no

Divisibility by 25

- ✓ A number is divisible by 25, if the number formed by the digits at the tens and the unit place is divisible by 25.
- ✓ E.g. 8750, 23275, 8926825 are divisible by 25 because the number formed by tens and unit place in these numbers viz., 50, 75, 25 respectively are divisible by 25.

Some more divisibility rules

(1) If a number is divisible by another number, then it is divisible by each of the factors of that number.

E.g. 48 is divisible by 12 and the factors of 12 are 1, 2, 3, 4, 6, 12. Clearly, each of the factors of 12 divide 48 exactly.

(2) If a number is divisible by two co-prime numbers, then it is divisible by their product also. E.g. 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by $3 \times 5 = 15$.

(3) If two given numbers are divisible by a number, then their sum is also divisible by that number.

E.g. The number 27 and 33 both are divisible by 3, then their sum 27 + 33 = 60 is also divisible by 3.

(4) If two given numbers are divisible by a number, then their difference is also divisible by that number.

E.g. The number 16 and 20 are divisible by 4. Then, their difference 20 - 16 = 4 is also divisible by 4.

5. Prime Factorisation

- ✓ When a number is expressed as a product of its factors, we say that the number has been factorised. E.g. we can write 72 as product of 12 and 6 or as a product of 8 and 9 or as a product of 2, 4 and 9, in each of these cases, we will say that the number 72 has been factorised into two or three factors, as the case may be.
- ✓ Most uniquely we can express 72 as 72 = 2 × 2 × 2 × 3 × 3.
- ✓ In this factorization of 72, all factors are prime numbers. It is representing the prime factorisation of 72.
- ✓ Such a factorisation of a number in which the number is expressed as a product of primes is called prime factorisation of the number.
- ✓ Observe that 2 × 3 + 1=7 is a prime number. Here, 1 has been added to a multiple of 2 to get a prime number. Can you find some more numbers of this type?

6. Common Factors And Common Multiples

- ✓ The factor of a number can divides the number and the multiple of a number is divisible by the number. E.g. 2 being a factor of 12 can divide 12 and 16. As 12 and 16 are multiples of 4, is divisible by 4.
- ✓ Think of the smallest possible difference between two primes.

Illustrations

Illustration 3. Find the common factors of 54 and 72.

Solution The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54.

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72

The common factors are 1, 2, 3, 6, 9 and 18.

7. Highest Common Factor (Hcf)

The highest common factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as **greatest common divisor (GCD).**

You can find HCF of given numbers in three ways as follows:

(a) Listing factors

(b) Prime factorisation method

(c) Continued division method

Illustrations

Illustration 4. Find the HCF of 18, 27 and 45 by listing factors.

Solution: The factors of 18 are 1, 2, 3, 6, 9 and 18. The factors of 27 are 1, 3, 9 and 27.

The factors of 45 are 1, 3, 5, 9, 15 and 45.

The common factors are 1, 3, and 9.

The highest common factor (HCF) is 9.

Illustration 5. Find the HCF of 98 and 112 using the prime factorisation method.

Solution

2	98
7	49
	7

2	112
2	56
2	28
2	14
2	7

98 = 2 × 7 × 7

112 = 2 × 2 × 2 × 2 × 7

The highest common factor =2×7

So, HCF = 2 × 7 = 14