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SEBI

Grade A - (Phase 1 & 2)

Securities and Exchange Board of India (SEBI)

Volume - 1

Quantitative Aptitude & Reasoning

SIERRA INNOVATIONS

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1 CHAPTER

Number Series

What is Series?

- A series is a systematic arrangement of numbers or letters within a defined group. In competitive exams, sequences of numbers, letters, or a mix of both are presented. One position in the sequence is either left blank or contains an incorrect number or letter. Candidates are tasked with completing the series by selecting the correct option to fill the blank or identify the incorrect element.

Here's a clearer and concise way to solve number series problems:

- **Look for Patterns**: Check if the numbers are increasing or decreasing by a constant (addition/subtraction) or multiplied/divided by a constant (multiplication/division).
- **Find Differences**:
  - ✓ If the difference between consecutive numbers is the same, it's an arithmetic series (e.g., 2, 5, 8, 11).
  - ✓ If the second difference (difference of differences) is constant, it's a quadratic series.
- **Check for Multiplication/Division**: Look if each number is multiplied or divided by a constant to get the next number (e.g., 3, 6, 12, 24 where each number is doubled).
- **Recognize Special Sequences**:
  - ✓ **Squares**: 1, 4, 9, 16, ...
  - ✓ **Cubes**: 1, 8, 27, ...
  - ✓ **Fibonacci**: Each number is the sum of the two preceding ones (e.g., 0, 1, 1, 2, 3, 5).
- **Check for Ratios**: If the numbers increase by a constant ratio, it's a geometric progression (e.g., 2, 4, 8, 16).
- **Test the Options**: If options are provided, check which one follows the identified pattern.

Type-1 Series in Increasing Order

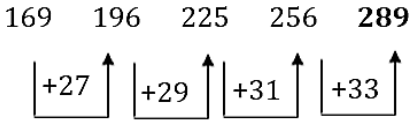
Ex: In the following question, select the missing number from the given series.

169, 196, 225, 256,_?

1. 289 **2. 324**

3. 441 **4. 361**

Ans:



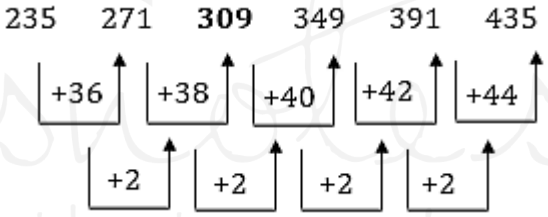
Ex: Which of the following numbers will replace the question mark (?) in the given series?

235, 271, ?, 349, 391, 435

1. 311 **2. 307**

3. 313 **4. 309**

Ans:



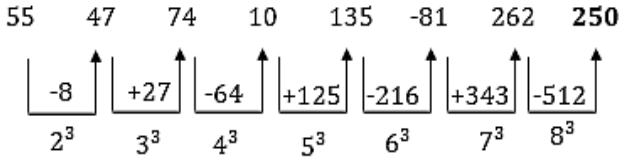
Ex: What number should replace question Mark (?) in the series given below.

55, 47, 74, 10, 135, -81, 262, ?

1. 774 **2. -250**

3. 343 **4. -343**

Ans:



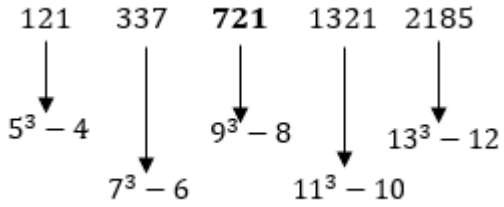
Ex: Find the missing number in the following series:

121, 337,?, 1321, 2185

1. 713 **2. 720**

3. 721 **4. 737**

Ans:



Type-2 Multiplication Based Increasing Series

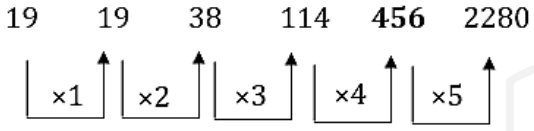
Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

19, 19, 38, 114, ?, 2280

1. 344 **2. 1140**

3. 456 **4. 224**

Ans:



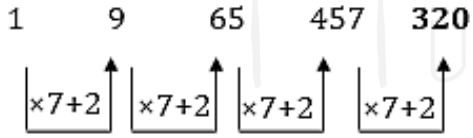
Ex: Which of the following numbers will replace the question mark (?) in the given series?

1, 9, 65, 457, ?

1. 4258 **2. 3125**

3. 3201 **4. 5289**

Ans:



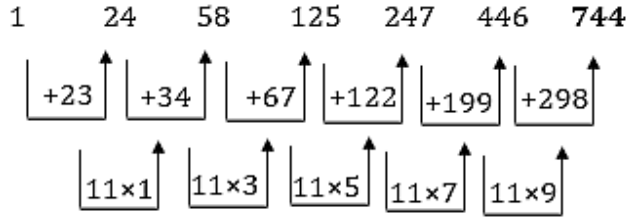
Ex: Which of the following numbers will replace the question mark (?) in the given series?

1, 24, 58, 125, 247, 446, ?

1. 774 **2. 747**

3. 744 **4. 777**

Ans:



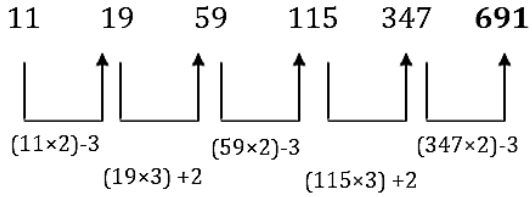
Ex: Select the number from among the given options that can replace the question mark (?) in the following series.

11, 19, 59, 115, 347, ?

1. 697 **2. 619**

3. 679 **4. 691**

Ans:



Type-3 Addition and Subtraction Series

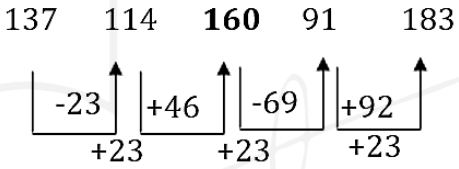
Ex: Which of the following number will replace the question mark (?) in the given series?

137, 114, ?, 91, 183

1. 145 **2. 160**

3. 125 **4. 112**

Ans:

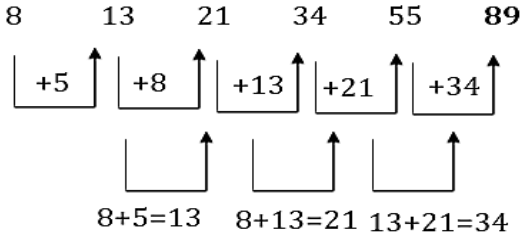


Ex: Which of the following numbers will replace the question mark (?) in the given series?

8, 13, 21, 34, 55, ?

1. 74 **2. 68** **3. 72** **4. 89**

Ans:



Type-4 Square and Cube Based Series

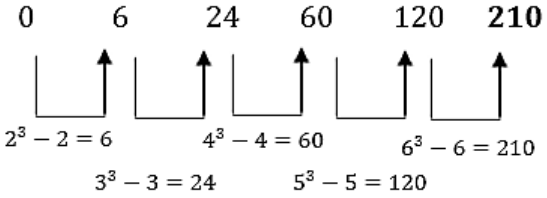
Ex: Find the next term of the series:

0, 6, 24, 60, 120, ?

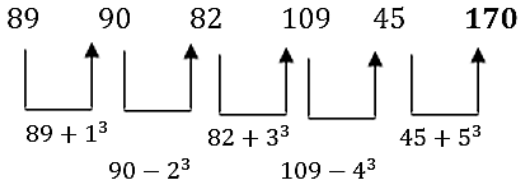
1. 180 **2. 210**

3. 216 **4. 240**

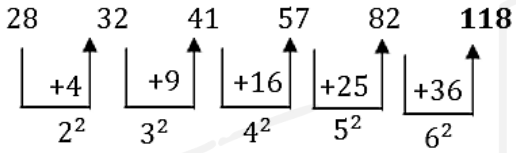
Ans:



Ex: Which number will replace the question mark (?) in the following series?
89, 90, 82, 109, 45, ?
Ans:

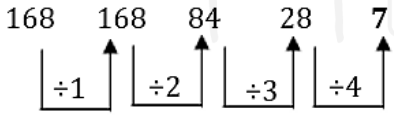


Ex: Which number will replace the question mark (?) in the following series?
28, 32, 41, 57, 82, ?
Ans:

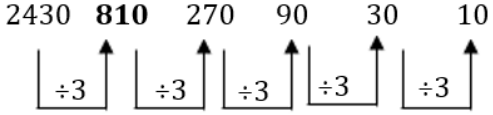


Type-5 Division Based Decreasing Series

Ex: Which number will replace the question mark (?) in the following number series?
168, 168, 84, 28, ?
Ans:



Ex: Which number will replace the question mark (?) in the following series?
2430, ?, 270, 90, 30, 10
Ans:

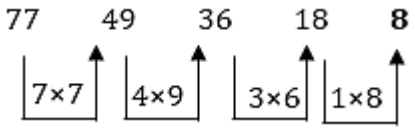


Type-6 Mixed Number Series

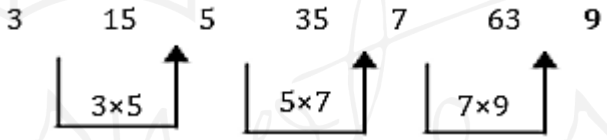
Ex: Find the next terms
11,13,17,19, 23,_?
1. 27 **2. 29**
3. 31 **4. 33**

Ans:
11 → Prime number
13 → Prime number
17 → Prime number
19 → Prime number
23 → Prime number
Next prime number = 29

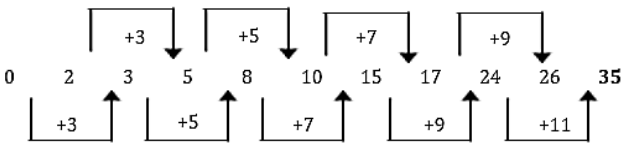
Ex: What will come in place of the question mark (?) in the following series?
77, 49, 36, 18, ?
1. 10 **2. 12**
3. 8 **4. 16**
Ans:



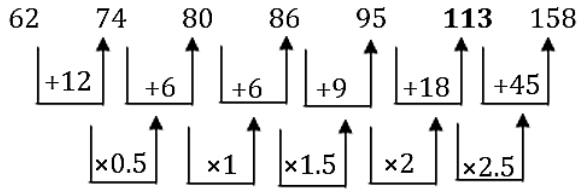
Ex: What will come in place of the question mark (?) in the following series?
3, 15, 5, 35, 7, 63, ?
1. 10 **2. 126**
3. 9 **4. 84**
Ans:



Ex: In the following series, what comes in place of the question mark (?)?
0, 2, 3, 5, 8, 10, 15, 17, 24, 26, ?
1. 28 **2. 30**
3. 32 **4. 35**
Ans:



Ex: Select the number from among the given options that can replace the question mark (?) in the following series.
62, 74, 80, 86, 95, ?, 158
1. 113 **2. 100**
3. 108 **4. 122**
Ans: The correct answer is 113



Ex: Find the wrong number in the FIRST series and then find the value of the following question.

(i) 455, 212, 131, 104, 94

(ii) Analyze the series and find the missing number (The exact place of the missing number is not known it can be on 1st, 2nd... 6th position) 57, 82, 167, 231, 312

1. **Wrong term's position from left.**
2. **Find the position of the missing term from the right side.**

Ans:

First Series:

455-212=243 = 3^5

212-131=81 = 3^4

131-104=27 = 3^3

104-95 = 3^2

95-92 = 3^1 (Wrong term 5^{th} from left)

Second Series:

57 + 5^2 = 82

82 + 6^2 = 118 (Missing term 4^{th} from right)

118 + 7^2 = 167

167 + 8^2 = 231

231 + 9^2 = 312

Type-7 Finding the Wrong Term in a Series

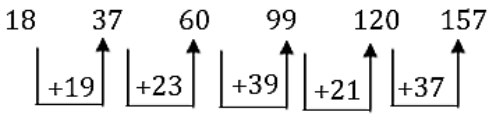
Ex: Find the wrong number in the following series

18, 37, 60, 99, 120, 157

1. 18 **2. 37**

3. 60 **4. 99**

Ans:



Logic used: (+) Increasing prime

18+19=37 37 +23=60

60+29=89 89+31=120

120+37=157

But given number is 99 instead of 89.

So, wrong number is 99.

Ex: Find the wrong number in the following series

2, 5, 14, 41, 124, 371

1. 14 **2. 41**

3. 371 **4. 2** **5. 124**

Ans:

2 × 3 -1 = 5 5 ×3 - 1 = 14

14 × 3 - 1 = 4141 × 3 - 1 = 122 (Not 124)

122 × 3 - 1 = 365

Wrong number = 124

Ex: In the following number series, a wrong number is given. Find out the wrong number.

781, 783, 793, 791, 805, 800

1. 783 **2. 793**

3. 791 **4. 805** **5. 800**

Ans:

156 × 5 + 1 = 781 157 × 5 - 2 = 783

158 ×5 + 3 = 793 159 × 5 - 4 = 791

160 × 5 + 5 = 805 161× 5 - 6 = 799

Wrong number in the given series is 800

Ex: Find the wrong number in the following series

2, 12, 66, 404, 2828, 22716

1. 2 **2. 404**

3. 2828 **4. 22716** **5. 12**

Ans:

Each term is obtained by multiplying the previous term with successive numbers (4, 5, 6, 7, 8) and then adding successive even numbers (4, 6, 8, 10, 12).

2 × 4 + 4 = 8 + 4 = 12

12 × 5 + 6 = 60 + 6 = 66

66 × 6 + 8 = 396 + 8 = 404

404 × 7 + 10 = 2828 + 10 = 2838

2838 × 8 + 12 = 22704 + 12 = 22716

The wrong number is 2828.

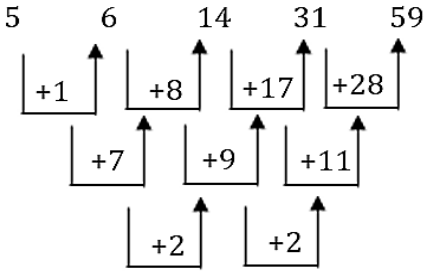
Ex: Find the wrong term from the given number series.

5, 7, 14, 31, 59

1. 31 **2.5**

3. 59 **4.7**

Ans: Correct answer in 7



In the place of 6, it is written 7, so the answer is 7.

Ex: Find the Wrong number in the given number series: 6, 7, 10, 13, 21, 37

1. 10 **2. 37**

3. 6 **4. 13** **5. 21**

Ans:

The given number series follows the pattern:

$6 + 2^0 = 7$ $7 + 2^1 = 9$

$9 + 2^2 = 13$ $13 + 2^3 = 21$

$21 + 2^4 = 37$

Wrong Number in the given number series is 10.

Type 8: Miscellaneous Type

Ex: Given below there are three number series (I, II, and III). One of the numbers in each series (I, II, and III) is wrong or (odd one out).

Series I: 590, 615, 651, 711, 795, 927

Series II: 64, 164, 285, 510, 546, 770

Series III: 18, 25.5, 40.5, 63, 93, 131

If the wrong number of the series (i), (ii), and (iii) are represented as (P, Q, and R) respectively, then which of the following equations is/are correct?

(A) Q - (P + R) = 40

(B) P < Q > R

(C) (R + P) < Q

(D) Both 1 and 3

(E) Both 2 and 3

Sol: For the three series, the wrong terms are $P = 590, Q = 770, R = 131$.

Now compare: $P < Q > R$ i.e. $590 < 770 > 131$, so statement 2 is true. Also, $R + P = 131 + 590 = 721$, and $721 < 770$, so statement 3 is true. But statement 1 is false because $Q - (P + R) = 770 - 721 = 49 \neq 40$. Hence, both 2 and 3 are correct ⇒ (E)

Ex: **Given below there are three number series (I, II, and III). One of the numbers in each series (I, II, and III) is wrong or (odd one out).**

Series I: 590, 615, 651, 711, 795, 927

Series II: 64, 164, 285, 510, 546, 770

Series III: 18, 25.5, 40.5, 63, 93, 131

If the correct number of the series (i), (ii), and (iii) are represented as (X, Y, and Z) respectively, then which of the following equations is/are correct?

I. (X - Z) + 5 = Y - 152 + 2

II. (252 - Y) = (X + Z) - 785

III. Z + 300 - (X + Y) = 940

(A) Only I

(B) Only II

(C) Only I and III

(D) Only III

(E) None of the above

Sol:

Series I follows pattern $+12 \times 2, +12 \times 3, +12 \times 5, +12 \times 7, +12 \times 11$, so first term should be 591, hence $\bar{X} = 591$.

Series II follows (sum of digits) ^{2}, so last term should be 771, hence $Y = 771$.

Series III follows +7.5, +15, +22.5, +30, +37.5, so last term should be 130.5, hence $Z = 130.5$.

Now substitute values in equation I: LHS ≠ RHS, so false.

In equation II: both sides are unequal again, so false.

In equation III: LHS ≠ RHS, so false. Since none of the equations satisfy the values, no relation holds.

Hence, correct answer is (E) None of the above

2 CHAPTER

Simplification

Simplification means reducing an expression to its simplest and most compact form by applying standard mathematical rules.

Order to Solve

——	Vinculum/Line/Bar bracket
B	Bracket
O	Of
D	Division
M	Multiplication
A	Addition
S	Subtraction

Types of brackets and solving order

( )	Small bracket
{ }	Curly bracket
[ ]	Square bracket

Important Exam – Oriented Types

Type 1: VBODMAS based questions

Ex: **Find the value of x. 42 ÷ 6 × 3 + 5 - (8 + 7) + 123 = x**

Sol: $42 \div \&6 \times 3 + 5 - (8 + 7) + 123$

$= \&7 \times 3 + 5 - 15 + 123$

$= \&21 + 5 - 15 + 123$

$= 134 \Rightarrow \& \Rightarrow x = 134$

Ex: **"What should come in place of question mark '?' in the following question?**

999 ÷ 3 - 165 = 75 + 111 - ?

Sol: $999 \div 3 - 165 = 333 - 165 = 168$

$75 + 111 - ? = 186 - ?$

$168 = 186 - ? \Rightarrow ? = 186 - 168 = 18$

Type 2: Square and Square Root Based Questions

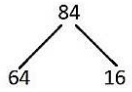
Trick 1: Find the Square (Less than 100)

Steps:

- Split the number and find the square of each digit separately.
- Now write these numbers together. Write both ones and tens place of each number (say, if it's 32, write as 09).
- Now find the double of the product of each digit in the base number
- Add this to the answer obtained in step 2 after leaving one's place at the right.
- The sum will give you the final answer.

Ex: $84 \rightarrow 8 \times 4 \times 2 = 64$

$6416 + 64 = 7056$



Trick 2: Find the Square (Greater than 100)

1. Check the nearest base number having zeroes.

For example, for **112**, the nearest base is **100**.

2. Find the difference from the base:
$$112 - 100 = 12$$
3. Add and subtract this difference from the number:
$$112 + 12 = 124, 112 - 12 = 100$$
4. Multiply these two numbers:
$$124 \times 100 = 12400$$
5. Now find the square of the difference:
$$12^2 = 144$$
6. Since the base is 100 (two zeroes), take the last two digits of 144, i.e. 44, and carry 1 to the left part.
7. Add the carry to 124:
$$124 + 1 = 125$$
8. Final answer:
$$112^2 = 12544$$

Square Root

First, we need to remember the unit digits of all squares from 1 to 10

Number	Unit Digit of Square
1	1
2	4
3	9
4	6

5	5
6	6
7	9
8	4
9	1
10	0

Key Observations (Shortcut Tips)

- Square ending in **1** → **root ends in 1 or 9**
- Square ending in **4** → **root ends in 2 or 8**
- Square ending in **9** → **root ends in 3 or 7**
- Square ending in **6** → **root ends in 4 or 6**
- Square ending in **5** → **root ends in 5**
- Square ending in **0** → **root ends in 0**

3-Digit Numbers (Always 2-digit root)

Steps:

1. Pair digits from right → e.g. 144 → 1 | 44
2. Check unit digit → 4 → root ends with 2 or 8
3. Look at first part (1): $1^2 = 1$ → so tens digit = 1
4. Choose correct unit digit → **12** or **18**
   - ✓ Closest is 12

$$\sqrt{144} = 12$$

4-Digit Numbers (Always 2-digit root)

Steps:

1. Pair digits → e.g. 2025 → 20 | 25
2. Unit digit check → 5 → root ends with 5
3. First part (20): $4^2 = 16, 5^2 = 25$ → so tens digit = 4
4. Combine:

$$\sqrt{2025} = 45$$

Shortcut Rule (Important)

- Find a such that:

$$a^2 \leq \text{ first part } < (a+1)^2$$

- Tens digit = a
- Unit digit → from last digit rule

5-Digit Numbers (3-digit root)

Example: $\sqrt{40401}$

Steps:

1. Pair digits → 404 | 01
2. Unit digit = 1 → root ends in 1 or 9
3. First part (404): $20^2 = 400, 21^2 = 441$ → so, ten's part ≈ **20**
4. Check closer value → Answer = 201

$$\sqrt{40401} = 201$$

Ex: **"What will come in the place of the question mark '?' in the following question?**
$\sqrt{6561} + \sqrt{289} \times 2 + 45\%$ of 80 = ? + 1

Sol: $\sqrt{6561} + \sqrt{289} \times 2 + 45\%$ of $80 = ? + 1$

$$= 81 + 17 \times 2 + 36$$

$$= 81 + 34 + 36 = 151$$

$$151 = ? + 1 \Rightarrow ? = 150$$

Type 3: Cube and Cube Root Based Questions

Cube Root: The cube root of a number x is a value y such that y multiplied by itself three times equals x, or mathematically, $y^3 = x$

Cube Numbers & Unit Digit Tricks (1–10)

Number	Cube	Unit Digit
1^3	1	1
2^3	8	8
3^3	27	7
4^3	64	4
5^3	125	5
6^3	216	6
7^3	343	3
8^3	512	2
9^3	729	9
10^3	1000	0

Key Shortcut Rules (Very Important)

- Cube ending in **1** → **root ends in 1**
- Cube ending in **8** → **root ends in 2**
- Cube ending in **7** → **root ends in 3**
- Cube ending in **4** → **root ends in 4**
- Cube ending in **5** → **root ends in 5**
- Cube ending in **6** → **root ends in 6**
- Cube ending in **3** → **root ends in 7**
- Cube ending in **2** → **root ends in 8**
- Cube ending in **9** → **root ends in 9**
- Cube ending in **0** → **root ends in 0**

Cube Root Trick (5-Digit Number)

Example: $\sqrt[3]{19683} = 27$

1. Pair digits from right → **19** | 683
2. Last digit = 3 → unit digit = 7 (since $7^3 = 343$)
3. Remaining part = 19 → nearest cube $2^3 = 8, 3^3 = 27$ → take 2
4. Combine → 27

$$\sqrt[3]{19683} = 27$$

Cube Root Trick (6-Digit Numbers)

Example: $\sqrt[3]{175616} = 56$

1. Pair digits → **175** | **616**
2. Last digit = 6 → unit digit = 6 (since $6^3 = 216$)
3. Remaining part = 175
   $5^3 = 125, 6^3 = 216 \Rightarrow$ take 5
4. Combine → 56

$$\sqrt[3]{175616} = 56$$

Ex: "What will come in the place of the question mark '?' in the following question?

$$\sqrt[3]{\mathbf{1728}} \times \sqrt[3]{\mathbf{4096}} \div \sqrt[3]{\mathbf{512}}$$

Sol:

$$\sqrt[3]{1728} = 12, \sqrt[3]{4096} = 16, \sqrt[3]{512} = 8$$

$$= 12 \times 16 \div 8$$
$$= 2 \times 2 = 24$$

Type 4: Approximate Value based questions

Range-Based Approximation: Range-based approximation means estimating an answer by creating an upper and lower boundary.

Example: Actual expression: 249 × 3.9 Lower bound: 250 × 4 = 1000 Upper bound: 248 × 4.1 ≈ 1016 So the approximate answer should lie between 1000 and 1016.

Tricks to Solve Approximation Based Questions:

1. **Rounding Off Numbers:**
   - ✓ Round off numbers to the nearest ten, hundred, or thousand to simplify addition, subtraction, multiplication, and division.
   - ✓ For example, rounding 498 to 500 or 1523 to 1500.
2. **Using Significant Figures:**
   - ✓ Retain a few significant digits and drop less significant ones to simplify calculations.
   - ✓ For example, approximating 123.456 to 123.5 or 0.004567 to 0.00457.
3. **Multiplication and Division by Powers of 10:**
   - ✓ Simplify calculations by multiplying or dividing by powers of 10.
   - ✓ For example, 423 × 99 can be approximated as 423 × 100 = 42300 and then subtract 423 (42300 - 423 = 41877).
4. **Using Common Fractions:**
   - ✓ Convert complex fractions into simpler ones.
   - ✓ For example, instead of using 0.3333, use 1/3, or instead of 0.142857, use 1/7.

Ex: "Directions: Determine the approximate value of '?' in the following question. (You are not expected to calculate the exact value)

14% of 75 + ?% of 90 = 31.9

Sol: 14% of 75 ≈ 10.5

$10.5 + ?\% \text{ of } 90 = 31.9 \Rightarrow ?\% \text{ of } 90$

$$\approx 31.9 - 10.5 = 21.4$$

$$? = \frac{21.4 \times 100}{90} \approx 23.8 \approx 24$$

Step to Solve (Exam Approach)

1. First find known percentage → 14% of 75 = 10.5
2. Subtract from total → $31.9 - 10.5 = 21.4$
3. Convert to percentage → $\frac{21.4}{90} \times 100$
4. Approximate →≈ 24

Ex: "What approximate value should come in place of question mark (?) in the following question?

$$? = \sqrt{723.99 + \left(\frac{1998.970}{4.795}\right)^{1/2}} \div 4.0019$$

Sol:

Approximate values:

$$723.99 \approx 724, 1998.970 \approx 2000, 4.795$$
$$\approx 5, 4.0019 \approx 4$$

$$\frac{2000}{5} = 400 \Rightarrow \sqrt{400} = 20$$

$$20 \div 4 = 5$$

$$? = \sqrt{724 + 5} = \sqrt{729} = 27$$

Step to Solve (Exam Trick)

1. Round numbers → 2000,5,4,724
2. Solve inside → $2000 \div 5 = 400$
3. Square root → $\sqrt{400} = 20$
4. Divide → $20 \div 4 = 5$
5. Final → $\sqrt{724 + 5} = \sqrt{729} = 27$

Type 5: Fraction Based Questions

Ex: What will be the value of x in the given equation?

$$\frac{1}{3^3} \times \frac{2}{3^3} \times \left(2\frac{1}{4}\right)^2 = \frac{x}{18}$$

Sol:

Step 1: Combine powers: $3^3 \times 3^3 = 3^6 = 729$

$$\Rightarrow \frac{2}{729}$$

Step 2: Convert & square: $2\frac{1}{4} = \frac{9}{4} \Rightarrow \left(\frac{9}{4}\right)^2 = \frac{81}{16}$

Step 3: Cancel smartly

$$\frac{2}{729} \times \frac{81}{16}$$

$$729 = 81 \times 9 \Rightarrow \frac{81}{729} = \frac{1}{9}$$

$$\Rightarrow \frac{2}{9 \times 16} = \frac{1}{72}$$

Step 4: Compare: $\frac{1}{72} = \frac{x}{18} \Rightarrow x = \frac{18}{72} = \frac{1}{4}$

Ex: find the value of $\frac{7}{9} - \frac{17}{12} + \frac{21}{6} + \frac{35}{4}$.

Sol:

Step 1: Take LCM of denominators

LCM of 9, 12, 6, 4 = 36

Step 2: Convert all into denominator 36

$$\frac{28 - 51 + 126 + 315}{36}$$

Step 4: Solve numerator (fast grouping)

$$(28 - 51) = -23$$
$$(-23 + 126) = 103$$
$$(103 + 315) = 418$$

Step 5: Final Answer $= \frac{418}{36}$

Simplify: $= \frac{209}{18}$

Type 6: Percentage Based Questions

Fraction	Percentage	Percentage	Fraction	Percentage	Percentage
1	100%	100%	$\frac{1}{16}$	6.25%	$6\frac{1}{4}\%$
$\frac{1}{2}$	50%	50%	$\frac{3}{8}$	37.5%	$37\frac{1}{2}\%$
$\frac{1}{3}$	33.33%	$33\frac{1}{3}\%$	$\frac{5}{8}$	62.5%	$62\frac{1}{2}\%$
$\frac{1}{4}$	25%	25%	$\frac{7}{8}$	87.5%	$87\frac{1}{2}\%$
$\frac{1}{5}$	20%	20%	$\frac{2}{3}$	66.66%	$66\frac{2}{3}\%$
$\frac{1}{6}$	16.66%	$16\frac{2}{3}\%$	$\frac{5}{6}$	83.33%	$83\frac{1}{3}\%$
$\frac{1}{7}$	14.28%	$14\frac{2}{7}\%$	$\frac{1}{25}$	4%	4%
$\frac{1}{8}$	12.5%	$12\frac{1}{2}\%$	$\frac{1}{40}$	2.5%	$2\frac{1}{2}\%$
$\frac{1}{9}$	11.11%	$11\frac{1}{9}\%$	$\frac{4}{5}$	80%	80%
$\frac{1}{11}$	9.09%	$9\frac{1}{11}\%$	$\frac{3}{4}$	75%	75%
$\frac{1}{12}$	8.33%	$8\frac{1}{3}\%$	$\frac{1}{19}$	5.26%	$5\frac{5}{19}\%$
$\frac{1}{15}$	6.66%	$6\frac{2}{3}\%$	$\frac{1}{20}$	5%	5%

Basic Concept

1. A percentage is converted into a fraction or a decimal by removing the percent sign and dividing the value by 100.

$$14\frac{5}{8}\% = \frac{117}{8}\% = \frac{117}{8 \times 100} = \frac{117}{800}$$

2. Determine *y*% of the number *x*

$$y\%\ of\ x = x \times \frac{y}{100}$$

To express *x* as a percentage of *y*, the required percentage is $= \frac{x}{y} \times 100\%$

Ex: What will come in the place of the question mark '?' in the following question?
36% of 250 - 40% of 130 = 150% of (?) - 22

Sol: Convert % to fraction: $36\% = \frac{9}{25}, 40\% = \frac{2}{5}, 150\% = \frac{3}{2}$.

LHS: $\frac{9}{25} \times 250 - \frac{2}{5} \times 130 = 90 - 52 = 38.$

So, $38 = \frac{3}{2}x - 22 \Rightarrow 60 = \frac{3}{2}x. \Rightarrow x = 40$

Ex: What will come in the place of the question mark '?' in the following question?
50% of 170 + 60% of 180 + 110 = ? + 10

Sol: 50% of $170 = 85$ and 60% of $180 = 108$.

LHS $= 85 + 108 + 110 = 303$.

So, $303 = ? + 10. \Rightarrow ? = 293$

Some Surdes and Indices Formulae

- $a^{m+n} = a^m \times a^n$
- $a^m \times a^n \times a^n \times \ldots = a^{m+n+p\ldots}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(d^m)^n = d^{nm} = (d^n)^m$
- $a^{m^z} = a^{m \times m \times m \times m \times}$ ... upto n times $\neq (a^m)^n$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
- $(-a)^n = [a^n$ when n is even, $-a^n$ when n is odd]
- $a^n = a^{(-1)n} = (a^{-1})n = \left(\frac{1}{a}\right)^n = \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \ldots n$ times
- $a^{\frac{p}{q}} = a^{\frac{1}{q} \times p} = \left(a^{1/q}\right)^p$ is positive integer where $q \neq 0 = a^{1/q} \times a^{1/q} \times \ldots$ upto p times
- $a^m = a^n \Rightarrow m = n$ where $a \neq 0,1$.
- $a^m = b^m \Rightarrow a = b$

Some Algebraic Formulae

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- $(a+b)^2 - (a-b)^2 = 4ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

3 CHAPTER

Algebra: Quadratic Equation

What is Quadratic Equation?

- A quadratic equation is a polynomial equation of degree two, typically represented as $ax^2 + bx + c = 0$
  where a, b, and c are real numbers, and $a \neq 0$.
- The solutions to quadratic equations are known as roots or zeroes (two distinct real roots, one real root, or complex roots, depending on the values of a, b, and c.)

Types of Quadratic Equation

Types	Description/Definition
Factorizable Quadratic Equations	Can be written as product of linear factors Form: $(x-a)(x-b)=0$ Example: $(x-2)(x+3)=0$
Roots of Quadratic Equations	Use quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
Nature of Roots	Using Discriminant $D = b^2 - 4ac$ ➢ $D > 0 \rightarrow$ Real & distinct roots ➢ $D = 0 \rightarrow$ Real & equal roots ➢ $D < 0 \rightarrow$ Complex (imaginary) roots

How to solve Quadratic Equations

Factorization Method

- Use when equation is easily factorable
- Write in form: $ax^2 + bx + c = 0$
- Factor into: $(x-p)(x-q) = 0$
- Solve each factor $= 0$

Example: $x^2 - 5x + 6 = 0$

$$\Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2,3$$

Quadratic Formula Method

Use when factorization is difficult:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Steps:

- Identify a, b, c
- Find discriminant $D = b^2 - 4ac$
- Substitute values and solve

Example:

$$2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{25+24}}{4} \Rightarrow x = \frac{-5 \pm 7}{4} \Rightarrow x = \frac{1}{2}, -3$$

Word Problems Method

- Convert statement into equation
- Assume variable (let x)
- Form quadratic equation
- Solve using factorization or formula

Example: "The product of two consecutive integers is 42 "

$$x(x+1) = 42$$
$$\Rightarrow x^2 + x - 42 = 0 \Rightarrow (x-6)(x+7) = 0$$
$$x = 6, x+1 = 7$$

Tricks to Solve Quadratic Equations Questions:

Trick 1: Quadratic equations solution based on the sign of coefficients in the equation:

Signs visible in the X/ Y's Question Equation	Signs of actual values of **X** or **Y**
	Larger Value's Sign, Smaller Value's sign
+, +	-, -
-, +	+, +
+,-	- , +
-, -	+, −

Ex:

1. $x2 - 14x + 45 = 0$
2. $y2 + 7y + 6 = 0$

In this example, the signs visible in the x equation are $(-,+)$ and signs visible in the y equation are $(+,+)$. So, signs of actual value of x and y in the final answer will be $(+,+)$ and $(-,-)$ So, "Relationships can't be established".

Trick 2: Constant term negative in both equations = Relationship can't be established. Ex:

1. $2 \times 2 - 11x - 15 = 0$
2. $21\ y\ 2 + 23y - 6 = 0$

In this example, the constant term is negative in both equations. So, the correct answer is "Relationships can't be established".

Example

Ex: Direction: In the following questions two equations numbered I and II are given. You have to solve both the equations and find relation between x and y.

I. $x^2 + 14x + 48 = 0$

II. $y^2 + 12y + 32 = 0$

(A) $x \geq y$

(B) $x > y$

(C) Relationships cannot be established between x and y

(D) $x \leq y$

(E) $y > x$

Sol 1: Basic Method

Equation I: $x^2 + 14x + 48 = 0 \Rightarrow (x + 6)(x + 8) = 0 \Rightarrow x = -6, -8$

Equation II: $y^2 + 12y + 32 = 0 \Rightarrow (y + 4)(y + 8) = 0 \Rightarrow y = -4, -8$

Possible values:

$$x = -6, -8$$
$$y = -4, -8$$

Different cases give different relations

Sol 2: According to Trick 1:

Equation I: $x^2 + 14x + 48 = 0 \rightarrow$ signs (+,+) $\Rightarrow$ roots negative $\Rightarrow x = -6, -8$.

Equation II: $y^2 + 12y + 32 = 0 \rightarrow$ signs (+,+) $\Rightarrow$ roots negative $\Rightarrow y = -4, -8$.

Compare values: -6 vs $-4 \Rightarrow x < y$, but -6 vs $-8 \Rightarrow x > y$.

Different relations are possible. So, no fixed relation between x and y. Answer: (C)

Ex: **Direction: Given below are two equations named I and II. Based on the given information, you have to determine the relation between the two equations.**

I: $x^3 + x^2 + 4x - 63 = x^3 - 3x^2$

II: $y^3 + 4y^2 - 77y = 0$

(A) x = y or relation between x and y cannot be established.

(B) x > y

(C) x < y

(D) $x \leq y$

(E) $x \geq y$

Sol 1: Basic Method

Equation I: $x^3 + x^2 + 4x - 63 = x^3 - 3x^2$

Cancel x^3 from both sides:

$$x^2 + 4x - 63 = -3x^2$$
$$4x^2 + 4x - 63 = 0$$
$$(2x + 9)(2x - 7) = 0$$
$$x = -\frac{9}{2}, \frac{7}{2}$$

Equation II: $y^3 + 4y^2 - 77y = 0$

Take common factor:

$$y(y^2 + 4y - 77) = 0$$
$$y(y + 11)(y - 7) = 0$$
$$y = 0, -11, 7$$

Comparison

Possible values:

$$x = -\frac{9}{2}, \frac{7}{2}$$
$$y = 0, -11, 7$$

Now compare:

- if $x = \frac{7}{2}, y = 0 \rightarrow x > y$
- if $x = \frac{7}{2}, y = 7 \rightarrow x < y$
- if $x = -\frac{9}{2}, y = -11 \rightarrow x > y$

Relation changes.

Final Answer: (A) $x = y$ or relation cannot be established

Sol 2: Trick Method

Equation I $\Rightarrow 4x^2 + 4x - 63 = 0 \rightarrow$ signs $(+, +, -) \Rightarrow$ roots are one + ve and one - ve.

Equation II $\Rightarrow y(y^2 + 4y - 77) = 0 \rightarrow$ roots: one +ve, one -ve and 0.

So x has (+, -) and y has (+,-, 0).

Different combinations give different relations. $\therefore$ Relation cannot be established (A)

Ex: Find the relation between x and y. If m = 3n, $\sqrt{(m^2 + n^2)} = 4\sqrt{10}$

I: x^2- (m + n)x + 39 = 0

II: y^2- (m + 2n)y + 75 = 0

(A) x ≥ y

(B) x < y

(C) x ≤ y

(D) x > y

(E) x = y or relation between x and y cannot be established

Sol:

Given $m = 3n$ and $\sqrt{m^2 + n^2} = 4\sqrt{10}$, so $\sqrt{9n^2 + n^2} = \sqrt{10n^2} = 4\sqrt{10}$

$\Rightarrow n = 4, m = 12.$

Equation I: $x^2 - (m + n)x + 39 = 0$

$\Rightarrow x^2 - 16x + 39 = 0 \Rightarrow x = 13,3.$

Equation II: $y^2 - (m + 2n)y + 75 = 0$

$\Rightarrow y^2 - 20y + 75 = 0 \Rightarrow y = 15,5.$

Now compare: $x = 13, y = 5 \Rightarrow x > y$,

but $x = 13, y = 15 \Rightarrow x < y$.

So relation cannot be established, hence (***E***)

Ex: $3x^2$- 26x + C = 0 Roots of the given equation are A and B, where, A > B and B = 8/3. Which of the following/s is /are true?

1) HCF of C and A is equal to 50% more than B
2) Value of C is a perfect cube
3) C is multiple of A

(A) Only I and Il

(B) Only III

(C) Only ll

(D) Only I and III

(E) All of these

Sol: $3x^2 - 26x + C = 0, B = \frac{8}{3}$

Step 1: Use product of roots

$$A \times B = \frac{C}{3}$$

$$A \times \frac{8}{3} = \frac{C}{3} \Rightarrow A \times 8 = C$$

Step 2: Use sum of roots

$$A + B = \frac{26}{3}$$

$$A + \frac{8}{3} = \frac{26}{3} \Rightarrow A = 6$$

Step 3: $C = 8A = 8 \times 6 = 48$

Step 4: Check statements

1. $\text{HCF}(48,6) = 6 \neq \frac{3}{2} \times \frac{8}{3} = 4$
2. 48 is not perfect cube
3. 48 is multiple of 6

Final Answer: (B) Only III

Ex: In the given question, two equations numbered l and II are given. Solve both the equations and mark the appropriate answer.

I. x^2 - 38x + 360 = 0

II. y^2 + 10y + 24 = 0

(A) x > y

(B) x < y

(C) x ≥ y

(D) x ≤ y

(E) x = y or relationship between x and y cannot be established.

Sol:

Equation I: $x^2 - 38x + 360 = 0$

$\Rightarrow (x - 20)(x - 18) = 0 \Rightarrow x = 20,18.$

Equation II: $y^2 + 10y + 24 = 0$

$\Rightarrow (y + 6)(y + 4) = 0 \Rightarrow y = -6, -4.$

So x values are positive and y values are negative.

Hence, in all cases $x > y$.

Answer: (A) $x > y$

Ex: In the given question, two equations numbered l and II are given. Solve both the equations and mark the appropriate answer.

I. x^2 + 19x + 78 = 0

II. $45y^2$ + 83y + 28 = 0

(A) x > y

(B) x < y

(C) x ≥ y

(D) x ≤ y

(E) x = y or relationship between x and y cannot be established.

Sol:

Equation I: $x^2 + 19x + 78 = 0$

$\Rightarrow (x + 13)(x + 6) = 0 \Rightarrow x = -13, -6.$

Equation II: $45y^2 + 83y + 28 = 0$

$\Rightarrow (9y + 4)(5y + 7) = 0 \Rightarrow y = -\frac{4}{9}, -\frac{7}{5}.$

So x values are more negative than y values.

Hence, in all cases $x < y$.

Answer: (B) $x < y$

Ex: **In the given question, two equations numbered l and II are given. Solve both the equations and mark the appropriate answer.**

I. $2x^2 - 33x + 136 = 0$

II. $2y^2 - 37y + 171 = 0$

(A) x > y

(B) x < y

(C) x ≥ y

(D) x ≤ y

(E) x = y or relationship between x and y cannot be established

Correct: B

Sol:

Equation I: $2x^2 - 33x + 136 = 0$

$\Rightarrow (2x - 17)(x - 8) = 0 \Rightarrow x = \frac{17}{2}, 8.$

Equation II: $2y^2 - 37y + 171 = 0$

$\Rightarrow (2y - 19)(y - 9) = 0 \Rightarrow y = \frac{19}{2}, 9.$

Compare: $x = \frac{17}{2} < \frac{19}{2}$ and $8 < 9$.

So in all cases $x < y$.

Answer: (B) $x < y$

4 CHAPTER Percentage

Definition

- The literal meaning of percentage is "per hundred," and it is represented by the symbol %.
  For example, 25 percent means 25 out of 100 parts, i.e.,
  $25\% = \frac{25}{100}$

Type 1: Based on successive percentage change

- If x% and y% change occur successively then overall change %
  overall % $= x + y + \frac{xy}{100}$
- If x%, y% and z% changer occur successively then overall change %
  overall % $= x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{10000}$
- Use a negative sign for a decrease and a positive sign for an increase.

Ex: A number is increased by 25% and then decreased by 20%. By what percentages is resulting number less than or more than the original number?

Sol: Using formula

$$\text{overall \%} = 25 - 20 - \frac{25 \times 20}{100} = 5 - 5 = 0\ \%\ \text{change}$$

Ex: **The price of a scooter increases successively by 10%, 5%, and 15%. Find the total percentage increase in the price of the scooter.**

Sol:

$$P\ \% = 10 + 5 + 15 + \frac{10 \times 5 + 5 \times 15 + 15 \times 10}{100} + \frac{10 \times 5 \times 15}{10000}$$

$$= 30 + 2.75 + 0.075 = 32.825\%$$

Type 2: Percentage increase / decrease based -Series method

Final Value = $\boldsymbol{Given\ Value} \times \frac{100 \pm x}{100} \times \frac{100 \pm y}{100} \times \frac{100 \pm z}{100} \ldots\ldots$

Use a negative sign for a decrease and a positive sign for an increase.

Ex: A number, 200, is increased by 25%, then decreased by 20%, again increased by 40%, and finally decreased by 10%. What is the final number?

Sol:

$$200 \times \frac{125}{100} \times \frac{80}{100} \times \frac{140}{100} \times \frac{90}{100} = 252$$

Type 3: Income - Expenditure – Based

Z = X +Y (Use weighted average method)
Income = Expenditure + Saving

Ex: A family decided to spend 13% of their monthly income on traveling, 27% of their monthly income on household expenses, and 35% of their monthly income on family's medical expenses, and 5% on insurance. The family has the remaining amount of Rs. 13200 as cash. What is the monthly income of the family?

Sol: Total expenses $= 13\% + 27\% + 35\% + 5\% = 80\%$.

Remaining $= 100\% - 80\% = 20\%$.

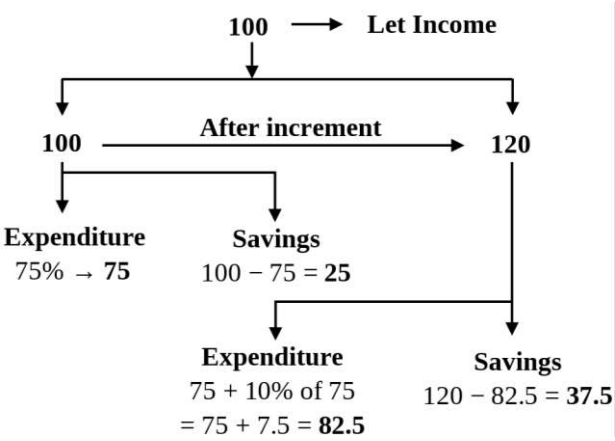
So, 20% of income = 13200.

Income $= \frac{13200 \times 100}{20} = 66000$.

Answer: Rs. 66,000

Ex: A man spends 75% of his income. If his income increases by 20% and his expenditure also increase by 10%. The percentage of increase in his savings is

Sol:



Increase in savings:
Increase = 37.5 − 25 = **12.5**
% increase in savings:

$$= \frac{12.5}{25} \times 100 = 50\%$$

Ex: Himanshu decided to donate 25% of his income to a fund for making arrangements for oxygen cylinders to COVID patients. On the day of donation, he changed his mind and donated Rs. 4250 which was 85% of what he had decided earlier. How much is Himanshu's income per month?

Sol:
Let **planned donation = 100%**
Given: 85% of planned = **4250**
So, **100% = 4250 × (100 / 85)**
Then 100% = 5000
Now, his planned donation = **25% of income**
25% of income = 5000
Income = **5000 × 4 = 20000**

Short Trick
85% = 4250 → then 100% = 5000 → which is 1/4th (25%) of his income →then income = 20000 (5000 X 4)

Type 4: Exam Based Questions

Pass marks are x%
If a student gets y marks and fails by z marks, then:

Maximum marks $= \frac{100(y+z)}{x}$

1. **Two students' case (to find total marks / maximum marks = a%)**

In an exam:
- First student gets **b**% marks and becomes **x** marks pass/fail.
- Second student gets c% marks and becomes y marks pass/fail.

Then the percentage of total marks (or maximum marks) is found by:
- If both have the same result (both pass or both fail) → subtract (-)
- If results are opposite (one pass, one fail) → add (+)

Total marks $= \frac{xy}{|a-b|} \times 100$

Rule: Same sign → subtract, opposite sign → add.

2. **Two subjects (Math and English) percentage facts**

In an exam:
- x% candidates pass/fail in Math
- y% candidates pass/fail in English
- **z**% candidates pass/fail in both subjects

Case 1: Candidates who pass/fail in neither subject
$= 100 - (x + y - z)\%$
Case 2: Candidates who pass/fail in exactly one subject
$= (x + y - 2z)\%$

Ex: A student got 30% marks in an exam of 150 marks and failed by few marks. If he had got 10% more of his present marks, he would have passed by 1.5 marks. Calculate the passing marks.

Sol: Total marks = 150
Student got 30% → **30% of 150 = 45 marks**
If he got 10% more of his present marks:
10% of 45 = **4.5**
→ New marks = 45 + 4.5 = **49.5**
He would pass by 1.5 marks
→ Passing marks = 49.5 − 1.5 = **48**

Short Trick
According to statement of Questions
(30% of 150) x 110% = m + 1.5
45 x $\frac{110}{100}$=m +1.5 → 49.5 = m+1.5 → m= 48 (Ans)

Ex: An engineering student has to secure 15% marks to pass. He gets 55 marks and fails by 20 marks. Find his maximum marks.

Sol: Student scored = **55**

Failed by 20 → Passing marks = **55 + 20 = 75**

Passing marks $= 15\%$ of total marks

$15\% = 75$

Total marks $= \frac{75\times100}{15} = 500$

Ex: In an examination of 100 marks, a student obtained 35 marks and failed by 5 marks. If another student wants to score 50% more than the passing marks, how many marks will he have to score?

Sol: Student got = **35**

Failed by 5 → Passing marks = **35 + 5 = 40**

Now second student wants **50% more than passing marks**

50% of $40 = 20$

Required marks $= 40 + 20 = 60$

Ex: Raman scored 456 marks in an exam and Seeta got 54% marks in the same exam which is 24 marks less than Raman. If the minimum passing marks in the exam is 34%, then how much more marks did Raman score than the minimum passing marks?

Sol:

Raman $= 456$

Seeta $= 456 - 24 = 432$

Seeta got 54% = 432

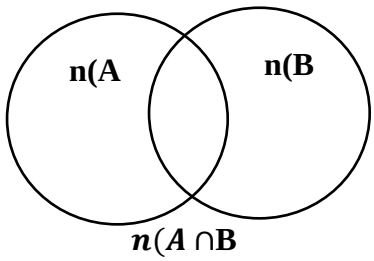
Total marks $= \frac{432\times100}{54} = 800$

Passing marks $= 34\%$ of 800

34% = 272

Extra marks by Raman: 456 – 272 = 184

Ven Diagram concept



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$A \cap B$ **'A and B '**

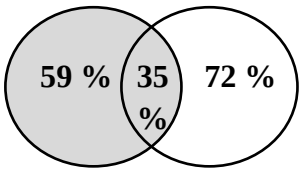
- The intersection of A and B.
- The elements in both sets A and B

$A \cup B$ **'A or B'**

- The union of A or B.
- Any element in set A or set B.

Ex: In a class of N students, 59% students passed in History, 72% students passed in Political Science and 35% students passed in both subjects. If 666 students passed in only Political Science, then, what is the value of N?

Sol:



Let total students = **N**

Only History $= 59\% - 35\% = 24\%$

Only Political Science $= 72\% - 35\% = 37\%$

Both $= 35\%$

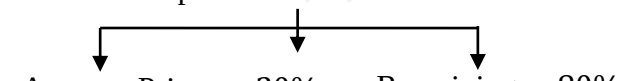
Only Political Science = 666 students

37% of N = 666

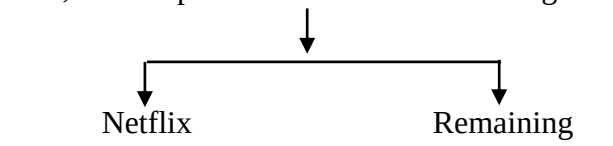
$$N = \frac{666 \times 100}{37} = 1800$$

Ex: **In a survey of prime membership users on four major platforms. It is found that out of the total prime members 20% are on Amazon Prime, 50% of the remaining are on Netflix premium and 30% of the rest are on Hotstar premium and rest 6300 are on Youtube premium. Then find the total number of prime members on these major platforms**.

Sol: Let total prime members $= 100\%$



Now, Netflix premium = 50% of remaining



Now, Hotstar premium = 30% of remaining

- Hotstar $= 30\%$ of $40 = 12\%$
- Remaining $= 40 - 12 = 28\%$

This remaining 28% = YouTube premium = 6300

$$28\% = 6300$$

$$100\% = \frac{6300 \times 100}{28} = 22500$$