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Rajasthan – CET

Senior Secondary Level

Common Eligibility Test (CET)

Volume - 5

Logical Reasoning & Mental Ability

SIERRA INNOVATIONS

INDEX

1 CHAPTER

LCM & HCF

LCM- least common multiple

- The smallest number that is exactly divisible by each of the given numbers is called their LCM.

  Example:

  Factor of 12 – 12,24,36, 48, …

  Factor of 16 - 16,32,48,64, …

  Common multiple – 48, 96...

  Least common multiple – 48

HCF- highest common factor

- The largest number that exactly divides each of the given numbers is called their HCF.

  Example

  Factors of 24 - 1, 2, 3, 4, 6, 8, 12, 24

  Factors of 36 - 1, 2, 3, 4, 6, 9, 12, 18, 36

  Common factors of 24 and 36 - 1, 2, 3, 4, 6, 12

  Highest common factor – 12

Important Exam – Oriented Types

Type 1: Least common Multiple – based

Find the smallest number that is exactly divisible by x, y and z.
$= LCM\ of\ (x, y, z)$

1. Find the smallest number that is divided by x, y and z and leaves remainder r in each case.
   $= LCM\ of\ (x, y, z) + r$
2. Find the smallest number that is divisible by x, y, and z, and leaves remainders of a, b, and c, respectively.
   $= LCM\ of\ (x, y, z) - k$
   $Where\ \ k = (x - a) = (y - b) = (z - c)$

Ex: Find the smallest number which when divided 5,6,7 and 8 gives remainder 3 in each case.

Sol: Smallest number $=$ LCM of (x, y, z) + r
$= LCM(5,6,7,8) + 3 = 840 + 3 = 843$

Ex: Find the smallest number that, when divided by 25, 15, and 30, leaves remainders of 21, 11, and 26, respectively.

Sol:
$k = (25 - 21) = (15 - 11) = (30 - 26) = 4$
$= LCM\ of\ (25,15,30) - 4 = 146$

Ex: When the numbers 12, 16, 18, 20, and 25 divide the least number x, the remainder in each case is 4, but x is divisible by 7. What is the digit at the thousands' place in x

Sol:
$LCM = LCM\ of\ (12,16,18,20,25) = 3600$
Given, 3600 should be divisible by 7.
Let $(3600k + 4)$ is divisible by 7.
So, we can write

$$\frac{3600k + 4}{7}$$

Put $k = 1,2,3,4,5..$ if we put $k = 5$ then expression is divisible by 7.
Number $= 3600 \times 5 + 4 = 18004$
Digit at thousands' place in 18004 is 8.

Type 2: Highest Common factor – Based

1. Find the largest number which is divisible by x, y and z exactly.
   $= HCF\ of\ (x, y, z)$
2. Largest number which can divide x, y and z and leaves remainder r.
   $= HCF\ of\ (x, y, z) + r$
   If r is absent.
   $= HCF\ of\ |x - y|, |y - z|, |z - x|$
3. Largest number which can be divide x, y, z and leaves remainder a, b, c respectively.
   $= HCF\ of\ (x - a)(y - b)(z - c)$

Ex: If $X = 2^3 \times 3^{10} \times 5^1$ and $Y = 2^5 \times 3^1 \times 7^1$, then find the HCF of X and Y?

Sol: The largest number that exactly divides each of the given numbers is called their HCF.
$HCF = 2^3 \times 3^1$

Ex: Find the greatest possible length which can be used to measure exactly the length 7m,3m 85cm, 12m 95cm.

Sol:

$= \text{HCF of } (700,385,1295)$

$= \text{HCF of } (x-a)(y-b)(z-c)$

$\text{HCF of } (700-385)(1295-385)(1295-700)$

$= \text{HCF of } (315)(1225)(595)$

$\text{HCF of } (315)(1225)(595) = 35 \text{ cm}$

Type 3: LCM and HCF of fractions

$$\text{LCM of } \left(\frac{a}{b},\frac{c}{d},\frac{e}{f}\right) = \frac{\text{LCM of (a, c, e)}}{\text{HCF of (b, d, f)}}$$

$$\text{HCF of } \left(\frac{a}{b},\frac{c}{d},\frac{e}{f}\right) = \frac{\text{HCF of (a, c, e)}}{\text{LCM of (b, d, f)}}$$

Key point – The fraction must be in its simplest form.

Ex: What is the LCM of 3.6,1.8 and 0.144?

Sol:

$\frac{36}{10} = \frac{18}{5}, \quad \frac{18}{10} = \frac{9}{5}, \quad \frac{144}{1000} = \frac{18}{125}$

$$\text{LCM of } \left(\frac{18}{5},\frac{9}{5},\frac{18}{125}\right) = \frac{\text{LCM of (18,9,18)}}{\text{HCF of (5,5,125)}}$$

$= \frac{18}{5} = 3.6$

LCM & HCF of powers

$a^p + 1, \quad a^q + 1$

$\text{LCM of } (a^p+1)(a^q+1) = a^{\text{LCM of (p,q)}} + 1$

$\text{HCF of } (a^p+1)(a^q+1) = a^{\text{HCF of (p,q)}} + 1$

Ex: What is the HCF $(2^{36}-1)$ and $(2^{45}-1)$.

Sol:

$HCF\ of\ (a^p+1)(a^q+1) = a^{HCF\ of\ (p,q)} + 1$

$\text{HCF of } (2^{36}-1)(2^{45}-1)$

$= 2^{\text{HCF of (36,45)}} + 1 = 2^9 + 1 = 511$

Type 4: Relationship between LCM and HCF Based

Let the 2 numbers are

$N_1 = ha, N_2 = hb \rightarrow$ 'a' and 'b' are coprime

$\rightarrow h(a,b) = 1$

$LCM \times HCF = N_1 \times N_2$

$LCM = hab$

$sum = (a+b)h$

$difference = (a-b)h$

$Product = h^2ab$

$\frac{LCM}{HCF} = \frac{ab}{1}$

Ex: Two numbers are in the ratio of 5:7. the product of their LCM and HCF is 12635. Then the sum of the number will be.

Sol: Let HCF = x, then numbers are 5x, 7x

$5x \times 7x = 12635$

$x = 19$

$sum = (a+b)h = (5+7)19 = 228$

Ex: The sum of and difference between the LCM and HCF of two numbers are 512 and 496, respectively. If one number is 72, then the other number is.

Sol: Let second number is N_1

$LCM + HCF = 512$

$LCM - HCF = 496$

After solving the equations

$LCM = 504, \quad HCF = 8$

$72 \times N_2 = 504 \times 8$

$N_2 = 56$

Type 5: LCM and HCF questions based on ratios

Ex: Three numbers are in the ratio $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$. The difference between the largest and the smallest number is 21. The sum of all the numbers is.

Sol:

$\text{largest number} - \text{smallest number} = 21$

$\frac{1}{2}:\frac{2}{3}:\frac{3}{4} = 6:8:9$

Now, let the ratio be x then, numbers are $6x, 8x$ and $9x$

$9x - 6x = 21$

$x = 7$

Sum of all three numbers

$= (6+8+9) \times 7 = 161$

Type 6: Questions based on bells and traffic signals

Ex: There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 p.m., they together turn green. At what time will they change to green next, simultaneously?

Sol: The three traffic signals turn from green to red at and interval 25 second, 39 second, and 60 seconds.

$LCM(25,39,60) = 3900\ sec = 65\ min$

$Total = 2\ times = 65 + 65 = 130\ min$

Time = 4:10 P.M.

Ex: Four ropes of lengths 102m,119m,153m and 204 m are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?

Sol:

HCF= equal length in which ropes can be cut

$HCF(102,119,153,204) = 17$

$$\text{Total parts} = \frac{102}{17} + \frac{119}{17} + \frac{153}{17} + \frac{204}{17}$$

$$= 6 + 7 + 9 + 12 = 34$$

Type 7: LCM and HCF of polynomials

Ex: What is the HCF of $(x^8 - y^8)$ and $(x^7 - y^7 + x^5y^2 - x^2y^5)$.

Sol:

$$(x^8 - y^8) = (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

$$(x^7 - y^7 + x^5y^2 - x^2y^5)$$
$$= x^7 + x^5y^2 - y^7 - x^2y^5$$

$$x^7 + x^5y^2 - y^7 - x^2y^5 = (x^2 + y^2)(x^5 - y^5)$$

$$(x^2 + y^2)(x^5 - y^5)$$
$$= (x^2 + y^2)(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$$

HCF of $(x^8 - y^8)(x^7 - y^7 + x^5y^2 - x^2y^5)$

$$= (x^2 + y^2)(x - y)$$

$$HCF = (x^2 + y^2)(x - y)$$
$$= x^3 - x^2y + xy^2 - y^3$$

2 CHAPTER Average

Definition

- The average of two or more numbers is calculated by adding all the numbers together and then dividing the total by how many numbers there are.

  Average $= \frac{\text{Sum of all terms / observations}}{\text{numbers of terms / observations}}$

Ex: The marks of the five friends are 66kg, 86 kg, 71 kg, 91 kg and 105 kg. Find the average of their marks.

Sol:

$$\text{Average} = \frac{66 + 86 + 71 + 91 + 105}{5}$$

$$\text{Average} = \frac{419}{5} = 83.8$$

Important Exam-Oriented Types

Type 1: Change in average when 'a' is added, subtracted, multiplied, or divided

If Average of 'n' number is = x

Change	Average
If 'a' is added	x + a
If 'a' is subtracted	x - a
If 'a' is multiplied	a × x
If 'a' is divided	x/a

Ex- The average of 10 numbers is 92. What will be the new average if 5 is added to each number?

Sol- old average = 92

Final average = 92+5=97

Type 2: Consecutive Natural numbers

Sum of first 'n' number $= \frac{n(n+1)}{2}$

Average of first 'n' number $= \frac{(n+1)}{2}$

Sum of square of first 'n' numbers $= \frac{n(n+1)(2n+1)}{6}$

Average of square of first 'n' numbers $= \frac{(n+1)(2n+1)}{6}$

Sum of cube of first 'n' numbers $= \left[\frac{n(n+1)}{2}\right]^2$

Average of cube of first 'n' numbers $= \frac{n(n+1)^2}{4}$

Ex: Find the average of square of first 47 natural numbers.

Sol-

$$\text{Average} = \frac{(47+1)(2\times 47+1)}{6}$$

$$= \frac{48 \times 95}{6} = 760$$

Type 3 Average of consecutive even and odd numbers

Average of first 'n' even numbers $= (n+1)$

Average of first 'n' odd numbers $= n$

Ex- What is the difference between the average of the first 148 even positive numbers and the average of the first 129 odd positive numbers?

Sol-

Average of first even numbers $= 148 + 1 = 149$

Average of first odd numbers = 129

Difference $= 149 - 129 = 20$

Key Points:

- For consecutive numbers, even numbers and odd numbers
- **Average** = Middle number = Average of first and last numbers

Ex- If the average of 97 consecutive number is 82. Find the sum of first and last numbers.

Sol-

Average $= \frac{first+last}{2} = 82$

$sum = 82 \times 2 = 164$

Type 4 When change in data happens

For a series of natural numbers having common difference 'x' and average 'A', if 'n' **terms are added at the end or removed from the beginning** of the series, the resulting change in the average is

$= A + \frac{n \times x}{2}$

For a series of natural numbers having common difference 'x' and average 'A', if 'n' **terms are added at the beginning or removed from the end** of the series, the resulting change in the average is

$= A - \frac{n \times x}{2}$

For a series of even or odd numbers having an average 'A', **adding 'n' terms at the end or removing 'n' terms from the beginning results** in a change in the average of

$= A + n$

For a series of even or odd numbers having an average 'A', if 'n' **terms are added at the beginning or removed 'n' terms from the end**, the change in the average is

$= A - n$

Ex: The average of 35 consecutive natural numbers is N. After dropping the first 10 numbers and including the next 10 numbers, the average changes to M. If $M^2 - N^2 = 600$ find the average of 3M and 5N.

Sol: Difference = 1 for natural number

After dropping the numbers then average

$= N + (10/2) = N + 5$

After including the numbers then average

$= N + (10/2) = N + 5$

Total change $M = N + 10$

$M - N = 10$

$(M + N)(M - N) = 600$

$M = 35, N = 25$

$\frac{3M + 5N}{2} = \frac{230}{2} = 115$

Ex: Average of 20 consecutive even number is 'M

(i) Next 8 numbers included then what will be the new average

Sol- New average = M + 8

(ii) Last four number excluded then what will be the new average

Sol- New average = M – 4

Type 5 Based on Inclusion and exclusion

When new person is added

= old average ± average change × total person (final)

When 'n' person is added

= old average ×n ± average change × total person (final)

Ex: The average of 4 numbers is 36. A fifth number is added, and the average becomes 40. What is the fifth number?

Sol:

Fifth number $= 36 + 4 \times 5 = 36 + 20 = 56$

Ex: The average weight of 10 students in a class is 'x' kg. After 5 students with an average weight of 62.8 kg join, the class average increases by 0.55 kg. Find the value of 'x'.

Sol-

Sum of the 5 students $= 62.8 \times 5 = 314$

Form formula $314 = 5 \times x + 0.55 \times 15$

$x = 61.15kg$

Ex- The average age of 24 boys and their teacher is 15 years. When the teacher 's age is excluded, the average age decreased by 1 year. The age of the teacher is?

Sol-

Age of teacher $= 15 - (-1) \times 24 = 39$

Ex- The average number of runs scored by a cricketer in 42 innings is 30. The difference between his maximum and minimum scores in an inning is 100. If these two innings are not taken into consideration, the average score of the remaining 40 innings becomes 28. What is the maximum number of runs scored by him in an inning.

Sol-Sum of both the innings

$L + H = 2 \times 30 - (-2) \times 40 = 140$

$L + H = 140$

$L - H = 100$

After solving the equations $H = 120$

Type 6 Based on replacement

When one person is replaced

New person = Replace ± average change × total quantity

When 'n' person is replaced

n× (New person) = Replace × n ± average change × total quantity

Ex- The average weight of three men is increased by 4 kg when one of them, whose weight is 100 kg is replaced by another man. What is the weight of the new man?

Sol- Replace

$= 100 + 4(3) = 100 + 12 = 112\ kg$

Ex- The average weight of 49 students in a class is 39 kg. Seven students whose average weight is 40 kg leave the class, and another seven students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class?

Sol- Using formula

$7 \times 54 = 7 \times 40 + a \times 49$

$a = 2$

so final average $= 39 + 2 = 41$

Ex- There are some children in a camp, and their average weight is 40 kg. If 5 children with an average weight of 36 kg join the camp, or if 5 children with an average weight of 43.2 kg leave the camp, the average weight of the children in both cases remains the same. How many children were there in the camp initially?

Sol- Let, total number of students initially = x

The average weight of the children remains the same in both cases, which means that the change in both cases is equal.

$$\frac{5 \times (40 - 36)}{x + 5} = \frac{(43.2 - 40) \times 5}{x - 5}$$

$$\frac{5 \times 4}{x + 5} = \frac{3.2 \times 5}{x - 5}$$

$x = 45$

Type 7 Based on wrongly entered data

If the average of 'n' numbers is 'A', but it is later discovered that one number 'a' was mistakenly read as 'B'

$$\text{Correct average} = A + \frac{(a - b)}{n}$$

If the average of 'n' numbers is 'A', but it is later discovered that the numbers 'a' and 'b' were wrongly taken as 'p' and 'q' respectively, then

$$Correct\ average = A + \frac{(a + b - p - q)}{n}$$

Ex- The average of 25 students is 48 kg. But it was later found that the weight of one student was wrongly written as 36 kg instead of 63 kg, and another as 65 kg instead of 56 kg. What is the correct average?

Sol- Correct average

$$= 48 + \frac{(63 + 56 - 36 - 65)}{25}$$

$= 48 + 0.72 = 48.72\ kg$

Type 8 Based on alligation (Average)

Ex- A student scored 80 marks in subject A, 70 in subject B, and 90 in subject C. The weights (credits) of the subjects are 2, 3, and 5 respectively. What is the weighted average?

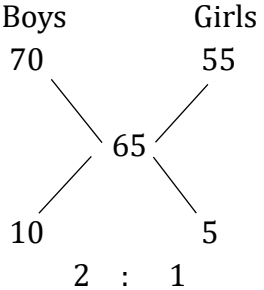
Sol-

$$\text{Average} = \frac{(80 \times 2) + (70 \times 3) + (90 \times 5)}{2 + 3 + 5}$$

$$= \frac{820}{10} = 82$$

Ex- The average marks of boys in a class are 70, and that of girls is 55. If the average marks of the entire class are 65, find the ratio of boys to girls.

Sol-



2 : 1

Type 9 Number-based

A. Middle number (no overlapping)

Ex- The average of 9 numbers is 18. Out of these, the average of the first four numbers is 17 and the average of the last four numbers is 20. Find the middle number:

Sol- $9 \times 18 = 4 \times 17 + x + 4 \times 20$

$x = 14$

B. Middle number (overlapping)

The extra term must be subtracted.

Ex- The average of 11 numbers is 30. Out of these, the average of the first six numbers is 32 and the average of the last six numbers is 29. Find the middle number?

Sol- 6^{th} number is repeated 2 times

$11 \times 30 = 6 \times 32 + 6 \times 29 - x$

$x = 36$

Ex- The average of 15 numbers is 45. The average of the first six numbers is 42 and the average of the last six numbers is 43. The 7th number is double the 8th number but is 5 more than the 9th number. Find the 7th number.

Sol-

All	15		45
First	6	-2	42 -3
Last	6		43

Net deviation $= (-3 \times 6) + (-2 \times 6) = -30$

8^{th} number $= x$ 7^{th} number $= 2x$

9^{th} number $= 2x - 5$

Now, $5x - 5 = 45 \times 3 + 30$

$x = 34$

then 7^{th} number $= 2 \times 34 = 68$

Type 10 Based on cricket

$$\text{Bowling average} = \frac{\text{total runs given}}{\text{total wickets taken}}$$

$$\text{Batting average} = \frac{\text{total runs scored}}{\text{total numbers of innings played}}$$

Ex- The average run of a batsman in his 23 matches is 62. He scores 122 in his 24^{th} match. Find the new average.

Sol - Using formula

$122 = 62 + (x)\ 24$

$x = 2.5$

New average $= 62 + 2.5 = 64.5$

3 CHAPTER Percentage

Definition

- The literal meaning of percentage is "per hundred," and it is represented by the symbol %.
  For example, 25 percent means 25 out of 100 parts, i.e.,
  $25\% = \frac{25}{100}$

Fraction Percentage chart

Fraction	Percentage	Percentage	Fraction	Percentage	Percentage
1	100%	100%	$\frac{1}{16}$	6.25%	$6\frac{1}{4}\%$
$\frac{1}{2}$	50%	50%	$\frac{3}{8}$	37.5%	$37\frac{1}{2}\%$
$\frac{1}{3}$	33.33%	$33\frac{1}{3}\%$	$\frac{5}{8}$	62.5%	$62\frac{1}{2}\%$
$\frac{1}{4}$	25%	25%	$\frac{7}{8}$	87.5%	$87\frac{1}{2}\%$
$\frac{1}{5}$	20%	20%	$\frac{2}{3}$	66.66%	$66\frac{2}{3}\%$
$\frac{1}{6}$	16.66%	$16\frac{2}{3}\%$	$\frac{5}{6}$	83.33%	$83\frac{1}{3}\%$
$\frac{1}{7}$	14.28%	$14\frac{2}{7}\%$	$\frac{1}{25}$	4%	4%
$\frac{1}{8}$	12.5%	$12\frac{1}{2}\%$	$\frac{1}{40}$	2.5%	$2\frac{1}{2}\%$
$\frac{1}{9}$	11.11%	$11\frac{1}{9}\%$	$\frac{4}{5}$	80%	80%
$\frac{1}{11}$	9.09%	$9\frac{1}{11}\%$	$\frac{3}{4}$	75%	75%
$\frac{1}{12}$	8.33%	$8\frac{1}{3}\%$	$\frac{1}{19}$	5.26%	$5\frac{5}{19}\%$
$\frac{1}{15}$	6.66%	$6\frac{2}{3}\%$	$\frac{1}{20}$	5%	5%

Derived fraction from base fractions

- Fractions can be derived from different fractions

1. $\frac{1}{7}$ **= 14.28% then** $\frac{2}{7}$ multiply $14.28 \times 2 = 28.56\%$ **2.** $\frac{1}{9}$ **= 11.11% then** $\frac{2}{9}$ multiply $11.11 \times 2 = 22.22\%$

Type 1: Basic concept of percentage

1. A percentage is converted into a fraction or a decimal by removing the percent sign and dividing the value by 100.
$14\frac{5}{8}\% = \frac{117}{8}\% = \frac{117}{8 \times 100} = \frac{117}{800}$

2. Determine $y\%$ of the number x
$y\%\ of\ x = x \times \frac{y}{100}$

3. To express x as a percentage of y, the required percentage is $= \frac{x}{y} \times 100\%$

Ex: 0.09% of 25% of 1200 =?

Sol:

$$= \frac{0.09}{100} \times \frac{25}{100} \times 1200 = 0.27$$

Ex: If $x\%$ of $x + y\%$ of $y = 2\%$ of xy then what % of $2x$ is $3y$?

Sol:

$$\frac{x \times x}{100} + \frac{y \times y}{100} = \frac{2 \times xy}{100}$$

$$x^2 + y^2 - 2xy = 0$$

From an algebraic formula

$$(x - y)^2 = 0$$

$$x = y$$

$$\frac{3y}{2x} \times 100 = 150\%$$

Ex: By what percentage is the percentage of vowels in the word CONSONANT less than the percentage of consonants in it?

Sol: vowels = 3 consonant = 6

$$\%\ \text{change} = \frac{3}{6} \times 100 = 50\%$$

Ex: A's income is 25% more than the income of B. Income of B is how much percent less than the income of A?

Sol- $25\% = \frac{1}{4}$

A B

5 $\xrightarrow{+1}$ 4

$$\% = \frac{1}{5} \times 100 = 20\%$$

Ex: If the numerator of a fraction is increased by 25 percent and the denominator is decreased by 50 percent; the resulting fraction becomes 2/5. Find the original fraction.

Sol:

$$\frac{N \times 125}{D \times 50} = \frac{2}{5}$$

$$\frac{N}{D} = \frac{4}{25}$$

Type 2: Based on successive percentage change

- If $x\%$ and $y\%$ change occur successively then overall change %

  overall $\% = x + y + \frac{xy}{100}$

- If $x\%$, $y\%$ and $z\%$ changer occur successively then overall change %

  overall $\% = x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{10000}$

- Use a negative sign for a decrease and a positive sign for an increase.

Ex: A number is increased by 25% and then decreased by 20%. By what percentages is resulting number less than or more than the original number?

Sol: Using formula

$$\text{overall } \% = 25 - 20 - \frac{25 \times 20}{100} = 5 - 5$$

$$= 0\ \%\ \text{change}$$

Ex: The price of a scooter increases successively by 10%, 5%, and 15%. Find the total percentage increase in the price of the scooter.

Sol:

$$P\ \% = 10 + 5 + 15 + \frac{10 \times 5 + 5 \times 15 + 15 \times 10}{100} + \frac{10 \times 5 \times 15}{10000}$$

$$= 30 + 2.75 + 0.075 = 32.825\%$$

Type 3: Percentage increase/decrease based - Series method

Final Value $= \textbf{Given Value} \times \frac{100 \pm x}{100} \times \frac{100 \pm y}{100} \times \frac{100 \pm z}{100} \ldots\ldots$

Use a negative sign for a decrease and a positive sign for an increase.

Ex: A number, 200, is increased by 25%, then decreased by 20%, again increased by 40%, and finally decreased by 10%. What is the final number?

Sol:

$$200 \times \frac{125}{100} \times \frac{80}{100} \times \frac{140}{100} \times \frac{90}{100} = 252$$

Type 4: Question based on area and volume

If the radius, diameter, or circumference of a circle; each side or altitude of an equilateral triangle; each side, diagonal, or perimeter of a square; each edge of a cube or cuboid; or the radius or diameter of a sphere or hemisphere is increased or decreased by **x**%, then the percentage increase or decrease in its area (or surface area) is:

Percentage change $= 2x \pm \frac{x^2}{100}$

- Use + when there is an increase.
- Use - when there is a decrease.

Ex: The length and breadth of a square are increased by 30% and 20% respectively. The area of resulting rectangle exceeds the area of the square by.

Sol:

Method 1: Ratio method

	Old	New
Length	10	13
Breath	5	6
Area	25	39

$$\% \text{ increase} = \frac{14}{25} \times 100 = 56\%$$

Method 2: Using formula

$$\text{overall } \% = 30 + 20 + \frac{30 \times 20}{100} = 56\%$$

Type 5 Income - Expenditure – Based

Z = X +Y (Use weighted average method)
Income = Expenditure + Saving

Ex: Reena saves 20% of her income. If her expenditure increases by 20% and her income increases by 29%, by what percentage do her savings increase?

Sol:

Method 1 $20\% = \frac{1}{5}$

5 (I) = 4 (E) + 1 (S)

5×29 = 4×20 + 1×S

S = 65%

Method 2

Let, Income = 500 Saving=100

Expenditure = 400

New income = 129% of 500 = 645

New expenditures = 120 % of 400 = 480

New saving = 645-480=165

Net increment in saving $= \frac{165-100}{100} \times 100 =$ 65%

Type 6: Price–Consumption Adjustment Concept

Z = X × Y

Expenditure = Price × Consumption

$$\text{Price} \propto \frac{1}{\text{consumption}}$$

Ex: If the price of a commodity be raised by $26\frac{2}{3}\%$, find how much percent must a householder reduce his consumption of that commodity, so as not to increase his expenditure.

Sol: $26\frac{2}{3}\% = \frac{80}{3\times100} = \frac{4}{15}$

	Initial		Final
Price	15	:	19
Consumption	19	:	15

$$\% \text{ change in consumption} = \frac{4}{19} \times 100$$

$$\% = 21\frac{1}{19}\%$$

Ex: A reduction of $14\frac{2}{7}\%$ in the price of tea enables Sakshi to obtain 4 kg more for ₹ 120. What is the original price of the tea?

Sol: $14\frac{2}{7}\% = \frac{1}{7}$

	Initial	Final
Price	7	6
Quantity	6	7

1 Unit ⟶ 4

Original quantity = 6×4=24

Original price $= \frac{120}{24} =$ ₹5

Type 7: Population increase/decrease based

Ex: The population of a town increases by 20% in the first year and decreases by 10% in the second year. If the present population of the town is 2,16,000, what was the population of the town two years ago?

Sol:

let population of the town two year ago = X

$X \times \frac{120}{100} \times \frac{90}{100} = 216000$

$X = 2{,}00{,}000$

Type 8: Fresh and day fruit concept

In fresh and dry fruits, the solid part (pulp) remains constant.

Ex: A fresh fruit contain 68% of water and dry fruit contain 20% of water. Then how much dry fruit can be obtained form 65 kg of fresh fruits?

Sol:

Original pulp amount = New pulp amount

65× (100-68) % = d × (100-20) %

d = 26kg

Type 9: Concept of income tax

Ex: The income of a man is increased by ₹1800, and the rate of income tax is reduced from 18% to 15%. In both cases 30% income is tax free. Find initial income if payable tax is same in both the cases.

Sol: $Tax_1 = Tax_2$

I × 70% × 18% = (I+1800) × 70% × 15%

I = 9000

Type 10: Based on elections

Ex: In an election between two candidates, the winning candidate secured 55% of the total votes and won by 3,000 votes. Find the total number of votes.

Sol: Let, Total vote = 100

winner = 55 loser = 100 – 55 = 45

Vote difference = 55 – 45 =10%

10% ⟶ 3000

100% ⟶ 30,000

Ex: In an election, 10% of the total votes were declared invalid. The winning candidate got 54% of the total votes and won by 4,320 votes. Find the total number of votes cast.

Sol: Invalid votes = 10% Valid votes = 90%

Winning candidate's votes = 54% of total votes

Losing candidate's votes = 90% − 54% = 36%

Margin of victory = 54% − 36% = 18% of total votes = 4,320

Total votes = $\frac{4320}{18} \times 100 = 24{,}000$

Ex: In an election between two candidates, 15 percent of the voters did not cast their votes and 10 percent of the votes polled were found invalid. The successful candidate secured 55 percent of the valid votes and won by a majority of 6120 votes. Find the total number of voters enrolled on the voters list.

Sol:

When some votes are not cast and some are declared invalid the valid votes are taken as = 100.

Winner = 55 % Loser = 45%

Margin of victory = 10%

10% ⟶ 6120

100% ⟶ 61200 (this valid vote)

Let total vote = x using series method

$x \times \frac{85}{100} \times \frac{90}{100} = 61200$

$x = 80000$

Ex: In an election between two candidates, 10 % of the voters did not cast their votes and 75 votes were found to be invalid. The winning candidate secured 50 % of the total votes expected and won the election by a margin of 170 votes. Find the total number of voters enrolled in the voters' list.

Sol-

Let total vote = $100x$

Vote cast = $90x$

Total valid vote = $90x - 75$

Winner = $50x$ loser = $40x - 75$

Margine of victory = $50x - (40x - 75) = 170$

$10x = 95$

$100x = 950$

Type 11: Commission based questions

Ex: An agent gets 5% commission on all sales up to ₹10,000 and 4% commission on all sales exceeding this amount. He gives ₹28,700 from his sales to his company after deducting his commission. Find his total sales.

Sol:

Method 1 Let the total sales be ₹ x

Commission on first ₹10,000

$= 5\% \, of \, 10{,}000 = ₹500$

Commission on remaining sales

$= 4\% \, of \, (x - 10{,}000)$

Total commission

$= 500 + 0.04(x - 10{,}000) = 0.04x + 100$

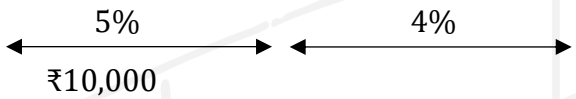
Amount given to the company after deducting commission

$= x - (0.04x + 100)$

$0.96x - 100 = 28{,}700$

$x = ₹\, 30{,}000$

Method 2



If the total commission is 4%, the company gets 1% more on ₹10,000

So, company gets

$= 28700 + 1\% \, of \, 10000 = ₹28{,}800$

28,800 represents 96% of the total shopping amount

Total shopping $= \frac{28800}{96} \times 100 = ₹30{,}000$

Important Formula:

1. Suppose the present value of a machine is **P**, and its value decreases at the rate of **R% per year**, then:

(i) Value of the machine after **n** years

$$\text{Value} = P \times \left(1 - \frac{R}{100}\right)^n$$

(ii) Value of the machine **n** years ago

$$\text{Value} = \frac{P}{\left(1 - \frac{R}{100}\right)^n}$$

Pass marks are x%

If a student gets y marks and fails by z marks, then:

$$\text{Maximum marks} = \frac{100(y+z)}{x}$$

2. **Two students' case (to find total marks / maximum marks = a%)**

In an exam:

- First student gets **b**% marks and becomes **x** marks pass/fail.
- Second student gets c% marks and becomes y marks pass/fail.

Then the percentage of total marks (or maximum marks) is found by:

- If both have the same result (both pass or both fail) → subtract (-)
- If results are opposite (one pass, one fail) → add (+)

$$\text{Total marks} = \frac{xy}{|a-b|} \times 100$$

Rule: Same sign → subtract, opposite sign → add.

3. **Two subjects (Math and English) percentage facts**

In an exam:

- x% candidates pass/fail in Math
- y% candidates pass/fail in English
- **z**% candidates pass/fail in both subjects

Case 1: Candidates who pass/fail in neither subject

$= 100 - (x + y - z)\%$

Case 2: Candidates who pass/fail in exactly one subject

$= (x + y - 2z)\%$

4 CHAPTER

Profit and Loss

Definition

- **Cost price (CP)** - The price at which we buy any thing.
- **Selling price (SP)** – The price at which we sell things.
  - ✓ If SP > CP Profit (P)= SP – CP
  - ✓ If CP > SP Loss(L)= CP – SP
  - ✓ If CP = SP No profit no loss

$$SP = CP \times \frac{(100 \pm P/L\%)}{100}$$

$$(P/L)\% = \frac{P/L \text{ in rupees} \times 100}{CP}$$

- **Marked / Listed price (MP)** – The price printed on an article or increased price of article.

$$MP = CP \times \frac{(100 + Markup\ \%)}{100}$$

Important Exam – Oriented Types

Type 1: Based on Calculation of Profit/loss from Cost Price and Selling Price

Ex: A product is bought ₹ 12.5 and sold at 25% loss then find the selling price?

Sol:

Method 1

$$SP = 12.5 \times \frac{(100-25)}{100} = 12.5 \times \frac{3}{4} = ₹\ 9.375$$

Method 2

$25\% = \frac{1 \to loss}{4 \to CP}$ SP = CP – Loss

$4 \longrightarrow 12.5$

$3 \longrightarrow \frac{12.5}{4} \times 3 = ₹9.375$

Ex: The difference in selling prices of an article when at 15% profit and 17% loss is ₹ 96. If it is sold at 10% profit then what is the selling prices?

Sol:

SP_1	CP	SP_2
83%	100%	115%

(115-83) % = 32% ⟶ 96

$$110\% \longrightarrow \frac{96}{32} \times 110 = ₹\ 330$$

Type 2: When an Item Is Bought/sold Multiple Times

Final Price = Cost Price $\times \frac{100 \pm x}{100} \times \frac{100 \pm y}{100} \times \frac{100 \pm z}{100} \ldots\ldots$

"+" → Profit; "-" → Loss

Ex: Richa sold a cow to Virat at 10% profit. Virat sold the cow to Harman at 5% loss; Harman sold the cow to Dipti at a 20% profit. If Dipti bought the cow ₹ 12450 then find the cost price of cow at which Richa bought it.

Sol:

$$CP \times \frac{110}{100} \times \frac{95}{100} \times \frac{120}{100} = 12450$$

CP = ₹ 10000

Type 3– When CP of 'x' article is equal to the SP of 'y' article.

CP of 'x' = SP of 'y'

If the selling price = x_1 and the cost price = x_2, then:

$$\text{Profit } \% = \frac{(x_1 - x_2)}{x_2} \times 100$$

$$\text{Loss } \% = \frac{(x_2 - x_1)}{x_2} \times 100$$

Ex: The cost price of 20 articles is equal to selling price of 15 articles. Then find the profit or loss percentage.

Sol: CP× 20 = SP × 15

$\frac{CP}{SP} = \frac{3}{4} \circlearrowright +1$

$\text{Profit} = \frac{1}{3} \times 100 = 33.33\%$

Type 4: When Profit and loss are in the form of CP or SP.

If by selling **x** items, the total profit or loss is equal to the selling price of **y** items, then:

$\text{Profit\%} = \frac{y \times 100}{x - y}$, $\text{Loss\%} = \frac{y \times 100}{x + y}$

Ex: When 2 articles sell in ₹750, Gauttam gains the CP of 6 articles. The profit percent is?

Sol- Suppose CP of One article if 1₹ Profit = 6CP = 6₹, CP =2 ₹

$\text{Profit \%} = \frac{6}{2} \times 100 = 300\%$

Type 5: When Profit % is equal to the CP

Profit%= CP

$CP = 10\left(\sqrt{25 + SP} - 5\right)$

Ex: When the profit of wrist watch is equal to the cost price of wrist watch and selling price of wrist watch is ₹1200 then finds the CP.

Sol: Let CP = x, So Profit % = x

$x \times \left(1 + \frac{x}{100}\right) = 1200$

$x(100 + x) = 120000$

$x = 300, -400$

So, we consider the positive value of CP = 300

Trick- When Profit % is equal to CP, then SP divides into 2 parts. Then their difference must be 10. After that, the smaller value is multiplied by 10. It gives the answer.

1200 → 40, 30 Difference is 10

Answer is smaller value =30×10=300

Type 6: When Profit or loss count on SP.

Ex: If Anshul calculates his profit on selling price and according to him, his profit is 25%. Find his actual profit % is?

Sol: P = 25 % on SP

$25\% = \frac{1 \to P}{4 \to SP}$ CP = SP – P = 4 -1 = 3

Profit % on CP = $\frac{1}{3} \times 100 = 33.33\%$

Ex: If 20% profit on CP then finds the Profit % on SP.

Sol: let CP = 100, Profit = 20, SP =120

P % on SP = $\frac{20}{120} \times 100 = 16.66\%$

Type 7: When One SP is given and asks to find second

Ex: When an article sells in ₹166.44 and gains 14% profit. If he sells this article in ₹154.76 then find the profit or loss.

Sol- No need to take 100 as CP

166.44 is value of 114 so,

$= \frac{114}{166.44} \times 154.76 = 106$

106 means when he sells the article on 154.76 is gains 6% of profit.

Type 8: When profit gain is 'n' times of loss incurred.

Ex: A shopkeeper sold an article for ₹ 450 at a loss. If he sells it for ₹ 500, then he would gain an amount four times the loss. At that price should be selling the articles to gain 25%?

Sol:

1 : 4

SP_1	CP	SP_2
450	X	500

$= \frac{500-450}{4+1} = \frac{50}{5} = 10$

CP= 450+ (1×10) = 460

$SP = \frac{460}{100} \times 125 = ₹575$

Type 9: Question based on number of articles

Tips: Every time, the articles must be the same

Ex: If I would purchase 11 articles of ₹ 10 and sold all the articles at the rate of 10 for ₹11, the profit percent would have been?

Sol:	Price	Articles
CP	$10_{\times 10}=100$	$11_{\times 10}=110$
SP	$11_{\times 11}=121$	$10_{\times 11}=110$

$$\text{Profit \%} = \frac{121-100}{100} \times 100 = 21\%$$

Ex: A shopkeeper buys 60 oranges at 10 for ₹ 72, and an equal number at 12 for ₹ 90. He spends ₹ 118 on the transaction and sells all the oranges that he buys. If there is a profit of 26% in the entire transaction, then what is the selling price of 32 oranges?

Sol:	Price	Article
CP_1	$72_{\times 6}=432$	$10_{\times 6}=60$
CP_2	$90_{\times 5}=450$	$12_{\times 5}=60$
Total CP= 882		Total article=120

Actual CP = CP + transaction charges

Actual CP = 882+118=1000

SP= 26% of 1000= 1260 SP of One orange= $\frac{1260}{120} = 10.5$

SP of 32 oranges= 10.5×32=₹336

Type 10: Concept of weighted average in CP

Ex: A fruit merchant bought some bananas. One fifth of them got rotten and was thrown away. He sold two-fifth of the bananas with him at 15% profit and the remaining bananas at 10% profit. Find his overall loss or profit percent?

Sol: Method 1

Let the merchant bought 25 bananas.

Bananas	5	8	12
Profit/Loss	-100%	+15%	+10%

$$P/L\% = \frac{-500+120+120}{25} = \frac{-260}{25} = -10.4\%$$

Method 2 rotten means total loss

$$= \frac{1}{5}(-100) + \frac{4}{5}\left\{\frac{2}{5}(+15)\right\} + \frac{4}{5}\left\{\frac{3}{5}(+10)\right\} = -10.4\%$$

Key point: If the question states that the remaining part of the total is (4/5) and two-fifths of the remaining part is

$$\frac{4}{5}\left(\frac{2}{5}\right) = \frac{8}{25}$$

Type 11: Net Profit / Loss If CP is same = average of profit or loss

Ex: A shopkeeper sold two items which have same CP. First one is sold on 10% profit and second item sold on 25 % loss then find the overall profit/loss % is?

Sol: Given is CP is same of both the items

$$\text{Overall P/L\%} = \frac{+10-25}{2} = \frac{-15}{2} = -7.5\%$$

Type 12: Based on Selling Two Articles at the Same Price

If SP is same and P% = L% is also same. (In this case every time loss occurs)

$$\text{Overall L\%} = \frac{-a^2}{100}\%$$

Ex: Two cows were sold at ₹ 1600. First was sold at 25% profit and second at 25% loss. Find the over profit/loss%.

Sol: overall L%= $\frac{-25^2}{100} = -6.25\%$

Type 13: If SP is same and P/L% is different

$$P/L\% = \frac{100(x+y)+2xy}{200+x+y}$$

Where x and y are profit /loss %

If both are profit then both are "+"

If both are loss then both are "- "

If one is profit and one is loss then profit in "+" and Loss in "- "

Ex: Two cows were sold at ₹ 1600. First was sold at 25% profit and second at 20% loss find the overall profit or loss%.

Sol- Method 1

	CP	SP
25% Profit=$\frac{1}{4}$	$4_{\times 4}=16$	$5_{\times 4}=20$
20% Loss=$\frac{-1}{5}$	$5_{\times 5}=25$	$4_{\times 5}=20$
	41	40

$$\text{Loss\%} = \frac{1}{41} \times 100 = 2.43\%$$

Method 2 $x = +25\%$ $y = -20\%$

$$L\% = \frac{100(25-20)+2(25)(-20)}{200+25-20}$$

$$l\% = -2.43\%$$

Type 14: When the quantity and price are given.

$$\frac{\text{Price}_1}{Q_1(100 \pm P_1/L_1)} = \frac{\text{Price}_2}{Q_2(100 \pm P_2/L_2)}$$

Ex: By selling 3 dozen oranges for ₹ 405, a trader loses 25%. How many oranges should he sell for ₹288 if he needs to earn a profit of 20% in the transaction?

Sol:

$$\frac{405}{12 \times 3 \times 75} = \frac{228}{x \times 120}$$

$$x = 16$$

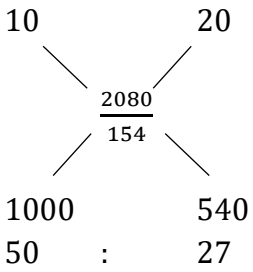
Alligation in profit and loss

Alligation on Profit/Loss= Ratio of CP and Ratio of quantity

Alligation on CP= Ratio of quantity

Ex: A person buys 5 tables and 9 chairs for ₹ 15400. He sells tables at 10% profit and chairs at 20% profit. If his total profit on selling all the tables and chairs is ₹2080. What is the cost price of 3 chairs?

Sol: Overall profit $= \frac{2080}{15400} \times 100 = \frac{2080}{154}$



50 : 27

77 units= 15400

1 unit= 200

Cost of 9 chairs= $27 \times 200 = ₹\,5400$

Cost of 1 chair= ₹ 600

Cost of 3 chair= $3 \times 600 =$ ₹ 1800

Type 15: When CP of one article is equal to SP of second article.

(To equate SP and CP – Take LCM of them)

Ex: Tarun sold Table and chair at ₹1600. The selling price of table equals to cost price of chair. He sold the table at a profit of 20% and the chair at a loss of 10%. What is his overall profit/loss%?

Sol:

	CP	SP
$20\% = \frac{1}{5}$	$5_{\times 5}=25$	$6_{\times 5}=30$
$-10\% = \frac{-1}{10}$	$10_{\times 3}=30$	$9_{\times 3}=27$
	55	57

$$P\% = \frac{2}{55} \times 100 = 3.64\%$$

5 CHAPTER Simple Interest

Definition

- If an institution (i.e. a bank or an individual or an organisation) receives a sum of money from another institution, the first institution is referred to as the borrower, while the second institution is referred to as the lender. Interest is a fee to use money for a specific period. Simple interest is a method used to calculate interest on the principal amount. It is calculated based on the original principal amount and remains unchanged for each period.
- **Formula**

$$SI = \frac{P \times T \times R}{100} = RT\ \%\ of\ P$$

Where;

- Principal (P) →Money borrowed;
- Accumulated principal or Amount(A) →Principal + Interest;
- Rate(r/R) → The fee charged per year, in term of %.

Important Exam – Oriented Types

Type- 1: Simple Questions (finding Principal, Rate and Time)

(To calculate interest, the day on which amount is deposited, is not counted but the day on which amount is withdrawn, is counted)

Ex: What would be the simple interest and amount obtained on the principal of ₹ 10,000 if the rate is 12.5 % and the time is 4 years?

Sol: Using formula of SI

$$\text{SI} = \frac{10000 \times 12.5 \times 4}{100} = ₹5{,}000$$

$$\text{Amount} = \text{P} + \text{SI} = 10000 + 5000 = ₹\,15{,}000$$

Type-2: When the Rate or Timing of Interest Is Different

Ex: Amar invested ₹10,000 and earns a simple interest at 7.25% p.a. for the first eight years, at 8.5% for the next six years, and at 6.5% for the final four years. the total interest is?

Sol:

$$\text{SI} = \frac{10000 \times (7.25 \times 8 + 8.5 \times 6 + 6.5 \times 4)}{100}$$

$$\text{SI} = ₹13{,}500$$

Type-3: When Money Becomes "n" Times in "t" Years

If a sum with simple interest rate, amounts to 'n_1' in t_1 years and 'n_2' in t_2 years, then.

$$\frac{n_1 - 1}{t_1} = \frac{n_2 - 1}{t_2}$$

Ex: When a sum becomes 4 times in 3 years, then in what time will the same amount become 8 times?

Sol: Using formula

$$\frac{4-1}{3} = \frac{8-1}{\text{t}}$$

$$\text{t} = 7$$

Ex: If a principal becomes 1.25 times itself in 3 years, then find the principal if the amount becomes ₹76000 in 7 years.

Sol: Using formula

$$\frac{1.25-1}{3} = \frac{\text{n}-1}{7}$$

$$\text{n} = \frac{19}{12}$$

$$19 \longrightarrow 76000$$

$$12 \longrightarrow \frac{76000}{19} \times 12 = ₹48{,}000$$

Type 4: If a certain sum becomes 'n' times of itself in 't' years on simple interest, then the rate is.

$$R\% = \frac{n-1}{t} \times 100$$

Ex: A certain sum becomes 5 times itself in 8 years on simple interest. Find the rate of interest.

Sol: Using formula

$$R\% = \frac{5-1}{8} \times 100 = \frac{4}{8} \times 100 = 50\%$$

Type-5: Concept of total SI and total amount

Ex: A certain amount is lent at 5 % p.a. for 3 years, 6% p.a. for 4 years and 9 % p.a. for 7 years. If the period is 12 years and the simple interest is ₹46704 then what is the sum?

Sol:

Total SI= sum of all (RT%) = (5×3) + (6×4) + (9×5) = 84

$$\text{Sum} = \frac{46704}{84} \times 100 = ₹55,600$$

Ex: A certain amount is lent at 4 % p.a. for 3 years, 5% p.a. for 4 years and 8 % p.a. for 7 years. If period of 12 years and the amount is ₹ 2232 then what is the principal?

Sol:

Total Amount = (100+RT%) = (100+4×3) + (100+5×4) + (100+8×5) = 372

$$\text{Principal} = \frac{2232}{372} \times 100 = ₹600$$

Type-6: The Simple Interest on One Principal Is Equal to the Simple Interest on the Other Principal

If the SI is equal then

$$P_1 : P_2 : P_3 = \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3}$$

If the amount is equal

$$P_1 : P_2 : P_3 = \frac{1}{100+r_1 t_1} : \frac{1}{100+r_2 t_2} : \frac{1}{100+r_3 t_3}$$

Ex: Sakshi invested ₹ 37000 in 3 different parts at 4% ,5% and 6% p.a. at the end of the 2 years, she received an equal amount of SI and then the principal at 5 %.

Sol:

$$P_1 : P_2 : P_3 = \frac{1}{4 \times 2} : \frac{1}{5 \times 2} : \frac{1}{6 \times 2}$$

$$P_1 : P_2 : P_3 = 30 : 24 : 20 = 15 : 12 : 10$$

$$P_2 = \frac{37000}{37} \times 12 = ₹12,000$$

Ex: ₹21750 is invested by a person in the bank account of his two sons, whose ages are 9 years and 13 years, in such a way that they will get equal amounts at the age of 21 years at the rate of 4.5% p.a. Find the share of the younger son.

Sol: Age of younger son = 21- 9= 12 years

Age of older son = 21-13= 8 years

$$P_1 : P_2 = \frac{1}{100 + 4.5 \times 12} : \frac{1}{100 + 4.5 \times 8}$$

$$P_1 : P_2 = 136 : 154 = 68 : 77$$

$$P_1 = \frac{21750}{(68+77)} \times 68 = ₹10,200$$

Type- 7: Based on Decrease/increase in Simple Interest

Ex: At the same rate of interest and with the same principal, Kavya gets ₹ 5400 in 3 years and ₹ 6400 in 7 years. Then find the principal and rate of interest.

Sol:

$$\text{SI of one year} = \frac{6400-5400}{7-3} = \frac{1000}{4}$$

$$= ₹\ 250$$

$$\text{Principal} = 5400 - (3 \times 250) = ₹\ 4650$$

$$\text{Rate} = \frac{250}{4650} \times 100 = 5.37\%$$

Type-8: When the rate is changed

When the rate of Simple Interest is increased or decreased.

If the rate of interest changes from $R_1\%$ to $R_2\%$, and due to this change the simple interest for T years increases or decreases by x, then the principal (P) is:

$$P = \frac{x \times 100}{(R_2 - R_1) \times T}$$