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## Physics

**Volume - 1 || Part - I**



# NEET - UG

## PHYSICS - CLASS - 12<sup>th</sup>

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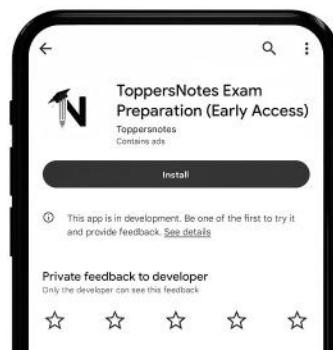
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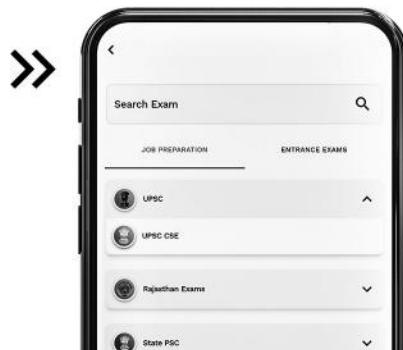
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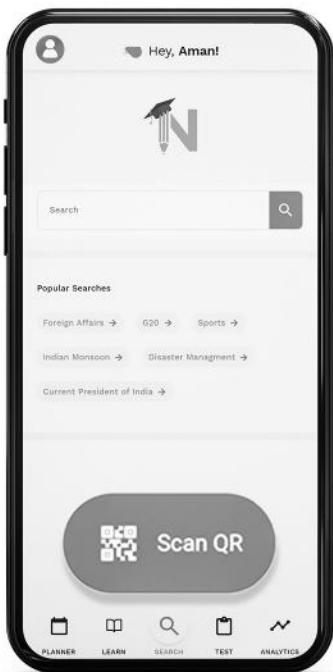
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## CHAPTER

# 1

# Alternating Current

### CHAPTER OUTLINE

- Introduction
- Power in AC Circuits
- Impedance
- Power Factor
- Transformers
- Sinusoidal Waveform
- Simple Circuits
- Mixed AC Circuit
- Wattless Current
- Choking Coil

### INTRODUCTION

- The bulk of the world's electrical power is generated, distributed, and consumed as sinusoidal alternating current (AC) and voltage. It is utilised in both domestic and industrial settings.
- AC has a number of advantages versus DC. The primary advantage of alternating current (AC) is that it can be changed into any form, whereas direct current (DC) cannot.
- A transformer allows voltage to be stepped up or down for transmission purposes. Transmission of high voltage (measured in KV) suggests that less current is needed to generate the same amount of power. Less current allows for the use of thinner cables for gearbox.
- In this chapter, we will look at a sinusoidal signal and its underlying mathematical equation. We will explore and analyse circuits in which currents and voltages vary with time.
- The phasor analysis techniques will be utilised to examine electronic circuits under sinusoidal steady-state operating circumstances. The chapter will conclude with single-phase power.

### SINUSOIDAL WAVEFORMS

- Unlike DC, AC flows in one direction first, then the other direction. A sine (or sinusoidal) waveform is the most frequent.
- When describing an AC signal, the current and voltage must be expressed in terms of maximum or peak values, peak-to-peak values, effective values, average values, or instantaneous values. Each of these figures represents a different quantity of current or voltage and has a distinct significance.  
$$V(t) = V_0 \sin \omega t$$
- Where  $V_0$  is the peak voltage,  $\omega = 2\pi f$  is the angular frequency expressed in radian per second (rad/s),  $f$  is the frequency expressed in Hertz,  $t$  is time expressed in second (s).

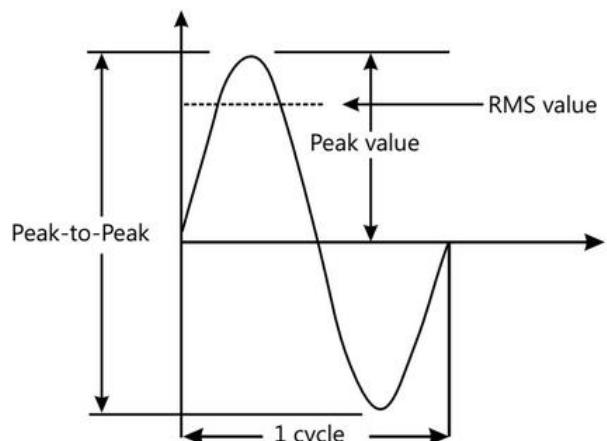
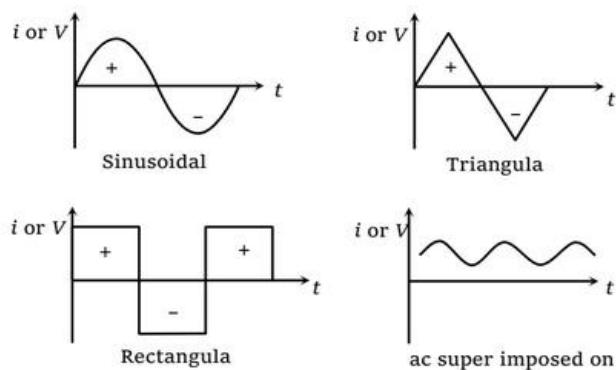


Figure: Sinusoidal Waveform

## SOME OTHER WAVE FORM FOR I-V



## Instantaneous Value

- An AC signal's instantaneous value is the value of voltage or current at a specific point in time. If the precise instant is the time in the cycle when the polarity of the voltage changes, the value may be zero.
- It could also be the same as the peak value if the chosen instant is the point in the cycle when the voltage or current stops increasing and begins falling. Between zero and the peak value, there exists an endless number of instantaneous values.

## Average Value

- Average value of a function, from  $t_1$  to  $t_2$ , is defined as.

$$\langle f \rangle = \frac{\int_{t_1}^{t_2} f dt}{t_2 - t_1}$$

We can find the value of  $\int_{t_1}^{t_2} f dt$  graphically if the graph is linear. It is the area of  $f$ - $t$  graph from  $t_2 - t_1$

$$I_{avg} = \frac{\int_0^t i dt}{\int_0^t dt}$$

where  $I$  is the instantaneous value of the current.

## For Sinusoidal Variation of Current and Voltages

### Case I: Average value over complete cycle.

$$\frac{\int_0^t i_0 \sin(\omega\tau + \theta) dt}{\int_0^t dt}$$

Similarly  $V_{av} = 0$

### Case II: Average value over half cycle

$$I_{avg} = \frac{\int_0^{t/2} i_0 \sin(\omega\tau + \theta) dt}{\int_0^{t/2} dt} = \frac{2i_0}{\pi}$$

Similarly

$$V_{avg} = \frac{2i_0}{\pi}$$

## Effective Value (RMS Value)

- This is the value of an alternating current signal that has the same effect on a resistance that a comparable value of direct voltage or current does on the same resistance. To a high degree of accuracy, the effective value of a sine wave of current can be computed by taking equally spaced instantaneous values of current along the curve and calculating the square root of the average of the squared values. As a result, effective value is also known as RMS value.
- Root mean square value of a function, from,  $t_1$  to  $t_2$  is defined as

$$I_{rms} = \frac{\int_{t_1}^{t_2} i^2 dt}{t_2 - t_1}$$

The magnitude of  $I_{rms}$  is given by

$$I_{rms}^2 = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{\int_0^T i_0^2 \sin^2(\omega\tau) dt}{\int_0^T dt} = \frac{i_0^2}{2}$$

$$I_{eff} = I_{rms} = \frac{i_0}{\sqrt{2}} = 0.707i_0$$

Where  $I_0$  is the peak value of the current.

Similarly

$$V_{\text{eff}} \text{ or } V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707 E_0 A$$

### Difference between Sine and Cosine Representation of AC Signal

- The sine and cosine functions are essentially the same, but with a  $90^\circ$  phase difference. For instance,  $\sin \omega t = \cos(\omega t - 90^\circ)$ . Any sinusoidal function's argument can have multiples of  $360^\circ$  added or deleted without altering the function's value. Consider the following to see what I mean.

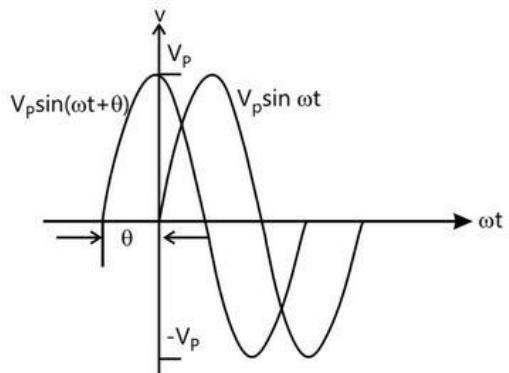
$$V_1 = V_{p1} \cos(10t + 20^\circ) = V_{p1} \sin(10t + 90^\circ + 20^\circ) \quad \dots(i)$$

$$= V_{p1} \sin(10t + 110^\circ)$$

$$\text{Leads } V_2 = V_{p2} \sin(10t - 40^\circ) \quad \dots(ii)$$

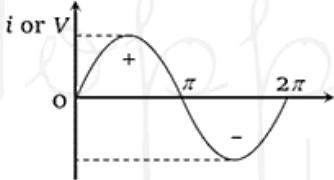
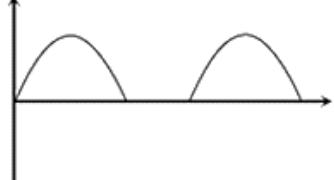
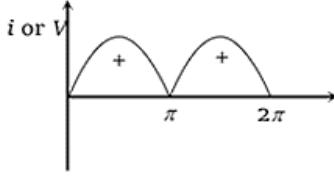
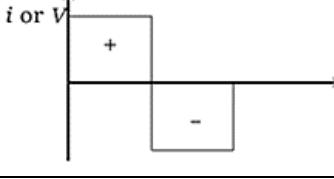
by  $150^\circ$  It is also correct to say that  $V_1$  lags  $V_2$  by  $210^\circ$  since  $V_1$  may be written as

$$V_1 = V_{p1} \sin(10t - 250^\circ) \quad \dots(iii)$$



**Figure:** Representation of voltage as sine and cosine function

### SOME IMPORTANT VALUES

Nature of wave form	Wave form	r.m.s. value	Average value	Form factor $R_f = \frac{\text{r.m.s. value}}{\text{Average value}}$	Peak factor $R_p = \frac{\text{Peak value}}{\text{r.m.s. value}}$
Sinusoidal		$\frac{i_0}{\sqrt{2}}$	$\frac{2}{\pi} i_0$	$\frac{\pi}{2\sqrt{2}} = 1.11$	$\sqrt{2} = 1.41$
Half wave rectified		$\frac{i_0}{2}$	$\frac{i_0}{\pi}$	$\frac{\pi}{2} = 1.57$	2
Full wave rectified		$\frac{i_0}{\sqrt{2}}$	$\frac{2i_0}{\pi}$	$\frac{\pi}{2\sqrt{2}}$	$\sqrt{2}$
Square or Rectangular		$i_0$	$i_0$	1	1

## POWER IN AC CIRCUITS

Average power in alternating current circuit over time  $t$  is defined as,

$$P_{\text{avg}} = \frac{\int_0^t V i dt}{t}$$

where  $V$  and  $i$  are the instantaneous values of voltage and current respectively.

Let

$$V = V_0 \sin \omega t;$$

$$i = i_0 \sin(\omega t - \phi)$$

Average power over a cycle

$$P_{\text{avg}} = \frac{\int_0^T V_0 i_0 \sin \omega t \cdot \sin(\omega t - \phi) dt}{T} = \frac{V_0 i_0 \int_0^T \left( \sin^2 \omega t \cos \phi - \frac{1}{2} \sin^2 \omega t \sin \phi \right) dt}{T} = \frac{1}{2} V_0 i_0 \cos \phi = V_{\text{rms}} i_{\text{rms}} \cos \phi$$

The term  $\cos \phi$  is known as power factor.

If current precedes voltage, it is considered to be leading; if current lags voltage, it is said to be trailing. Thus, a power factor of 0.5 trailing indicates that current is 600 times slower than voltage.

We see that the phase difference between potential differences across resistance,  $V_R$  and  $I_R$  is 0.

$$V \rightarrow \rightarrow i \rightarrow \rightarrow$$

$$I_m = \frac{V_m}{R} \Rightarrow I_{\text{rms}} = \frac{V_{\text{rms}}}{R} < P > = V_{\text{rms}} I_{\text{rms}} \cos \phi = \frac{V_{\text{rms}}^2}{R}$$

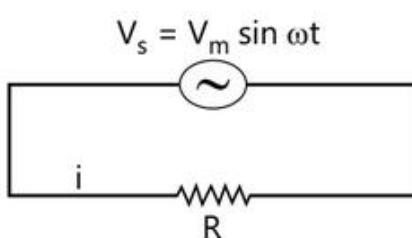


Figure: AC voltage applied to resistive load

## Purely Capacitive

Writing KVL along the circuit shown in Fig. 23.4

And current in the circuit is

(as  $\cos^{-1} 0.5 = 60^\circ$ ). The apparent power is given by the product of  $V_{\text{rms}}$  and  $I_{\text{rms}}$ , while the true power is produced by multiplying the apparent power by the power factor  $\cos \phi$ . As a result, apparent power =  $V_{\text{rms}} * I_{\text{rms}}$ , and true power = apparent power \* power factor.

For  $\phi = 0^\circ$ , the current and voltage are in phase. The power is maximum  $V_{\text{rms}} * I_{\text{rms}}$ .

For  $\phi = 90^\circ$  the power is zero.

The current is then specified as wattless. This is the case when the resistance in the circuits is zero. The circuit is either entirely inductive or entirely capacitive. The situation is comparable to that of a frictionless pendulum, in which gravity does no work on the pendulum cycle.

We'll talk about power and power factor more later, once we define impedance and its attributes.

## SIMPLE AC CIRCUITS

### Purely Resistive Load

Writing KVL along the circuit (see Fig. 23.3),

$$V_s - IR = 0$$

Or

$$I = \frac{V_s}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$V_s - \frac{q}{C} = 0$$

$$I = \frac{dq}{dt} = \frac{d(cv)}{dt} = \frac{d(cv_m \sin \omega t)}{dt} = cv_m \omega \sin \omega t = \frac{V_m}{1/\omega C} \cos \omega t$$

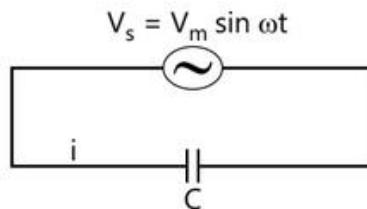


Figure: AC voltage applied to capacitive load

$$= \frac{V_m}{X_C} \cos \omega t = I_m \cos \omega t$$

where

$$X_C = \frac{1}{\omega C}$$

and is called capacitive reactance. Its unit is  $\Omega$

The graphs of current vs time and voltage versus time show that current reaches its maximum value  $T/4$  before voltage reaches its maximum value.

Corresponding to  $T/4$  phase difference.

$$\omega t = \frac{2\pi T}{T/4} = \frac{\pi}{2}$$

I leads V by diagrammatically (phase diagram) represented as

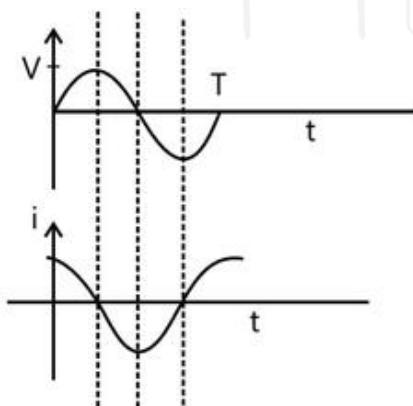
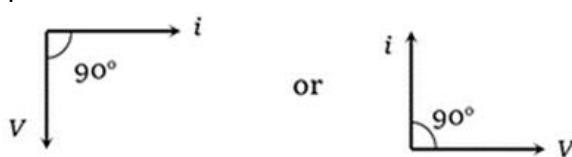


Figure:

Since  $\phi = 90^\circ$ ,  $\langle P \rangle = V_{rms} I_{rms} \cos \phi = 0$

The current leads the voltage by  $\pi/2$  in a capacitive circuit



### Pure Inductive Circuit

Writing KVL along circuit, ;

$$V_s - L \frac{di}{dt} = 0; L \frac{di}{dt} = V_m \sin \omega t; \int L di = \int V_m \sin \omega t dt$$

$$i = -\frac{V_m}{\omega L} \cos \omega t + C; \langle i \rangle = 0; C = 0;$$

$$\therefore i = -\frac{V_m}{\omega L} \cos \omega t \quad I_m = \frac{V_m}{X_L}$$

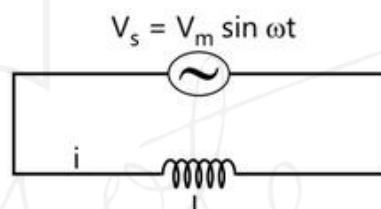


Figure: AC voltage applied to inductive load

The graphs of current versus time and voltage versus time show that voltage reaches its maximum value  $T/4$  before current reaches its maximum value. Corresponding to  $T/4$ , the phase difference

$$\omega t = \frac{2\pi T}{T/4} = \frac{2\pi}{T} = \frac{\pi}{2}$$

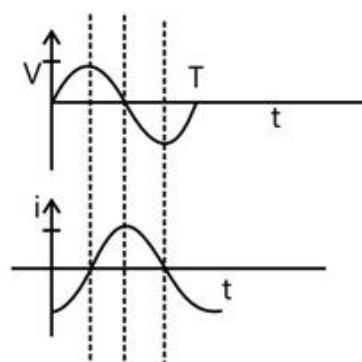
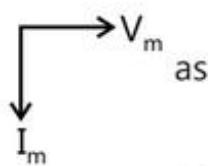
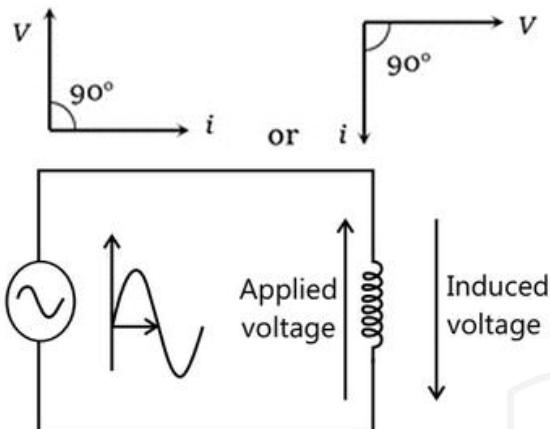


Figure: Variation of current and voltage with respect to time



$I_L$  lags behind  $V_L$  by  $\pi/2$  since  
 $\phi = 90^\circ$ ,  $\langle P \rangle = V_{rms} I_{rms} \cos \phi = 0$

The current lags voltage by  $\pi/2$  in a purely inductive circuit.



**Figure:** AC voltage applied to purely inductive circuit

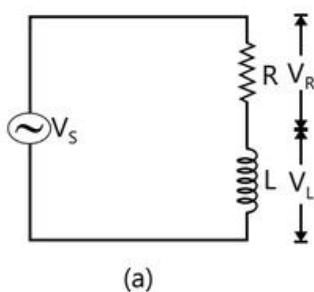
## IMPEDANCE

as we know inductive reactance  $X_L = \omega L$  and capacitance reactance  $X_C = 1/\omega C$   
 the function of an effective resistance in a purely inductive and capacitive circuit. The effective resistance in the series RLC circuit is described as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The graphic in Fig. 23.9 represents the relationship between  $Z$ ,  $X_L$ , and  $X_C$ .

The link between  $Z$ ,  $X_L$ , and  $X_C$  is depicted in the diagram below.

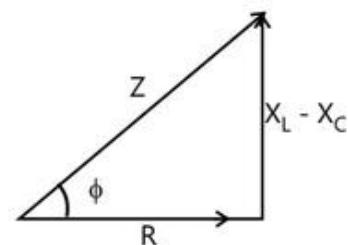


(a)

The impedance is measured in SI units. The current can be expressed in terms of  $Z$  as  $I(t)$ .

$$= \frac{V_0}{Z} \sin(\omega t - \phi)$$

Notice that the impedance  $Z$  also depends on the angular frequency  $\omega$ , as do  $X_L$  and  $X_C$ .



**Figure:** Impedance Triangle

We can easily derive the limit for a basic circuit (with only one element) using the previous formulae for phase  $\phi$  and  $Z$ .

The following circuits would be simple to understand because they are simply a superposition of individual phasor diagrams.

## MIXED AC CIRCUITS

### LR Circuit

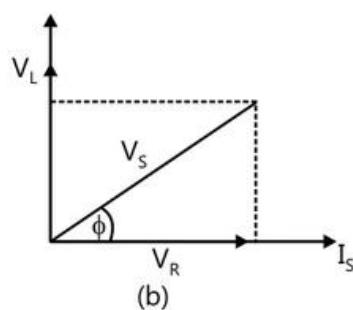
If  $V_R$ ,  $V_L$ , and  $V_C$  are the RMS voltage across are  $R$ ,  $L$  and  $C$  the source respectively. Then,

$$V_s = \sqrt{V_R^2 + V_L^2} = I_s \sqrt{R^2 + X_L^2}$$

Where  $I_s$  = r.m.s value of current source

The overall resistance to the current is known as impedance, and it is indicated by  $Z$ .

$$Z = \frac{V_s}{I_s} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$



**Figure:** (a) AC voltage applied to LR circuit (b) Phasor diagram of voltage drops across R and L

The phase angle  $\phi$  by which the applied voltage leads the current is

$$\phi = \tan^{-1} \left( \frac{X_L}{R} \right) = \tan^{-1} \left( \frac{\omega L}{R} \right)$$

## RC Circuits

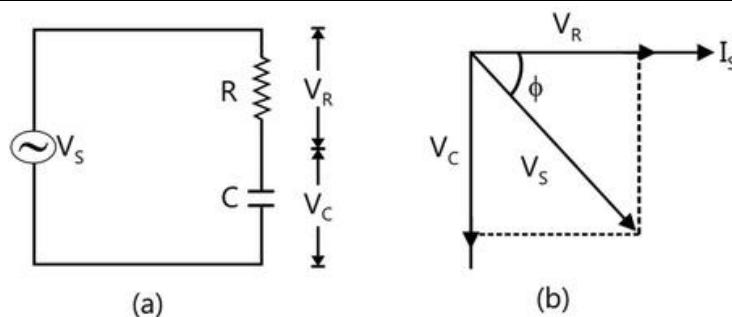


Figure: (a) AC voltage applied to RC circuit (b) Phasor diagram of voltage drops across R and C

If  $V_s$ ,  $V_R$  and  $V_C$  are RMS voltages across a source, resistance and capacitor respectively

$$V_s = \sqrt{V_R^2 + V_C^2} = I_s = \sqrt{R^2 + X_C^2}$$

impedance of circuit

$$Z = \frac{V_s}{I_s} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$V_s \text{ leads } I_s \text{ by } \phi = \tan^{-1} \left( \frac{X_C}{R} \right) = \tan^{-1} \left( \frac{1}{\omega C R} \right)$$

The current leads the applied voltage by angle.

## LC Circuits

From the phasor diagram

$$V = I |(X_L - X_C)| = IZ; \phi = 90^\circ$$

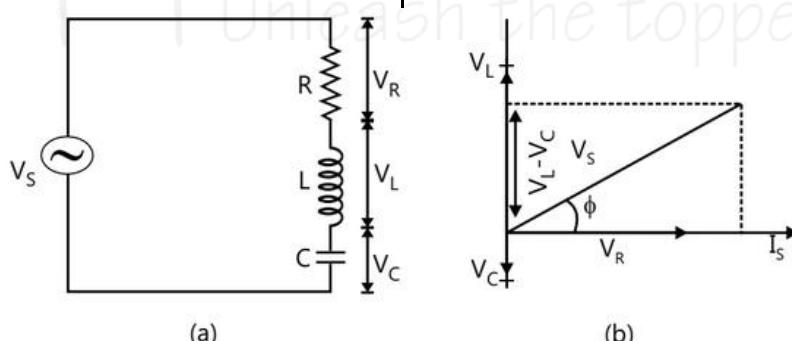


Figure: (a) AC voltage applied to LCR circuit, (b) Phasor diagram of voltage drops across L, C and R

and

Where  $\phi$  is called the power factor of the LCR circuit.

## Resonance in RLC Circuits

At a particular angular frequency  $\omega_0$  of the source, when  $X_L = X_C$  or

$$\omega_0 L = \frac{1}{\omega_0 C}$$

## RLC Circuits

For LCR series circuits

$$V_s = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Impedance of circuits

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2}$$

$V_s$  leads  $I_s$  by

$$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) = \tan^{-1} \left( \frac{\omega L - \frac{1}{\omega C}}{R} \right)$$

Power in LCR circuit =

$$= V_{\text{rms}} I_{\text{rms}} \cos \phi = V_{\text{rms}} I_{\text{rms}} \frac{R}{Z} = V_R I_{\text{rms}}$$

Unleash the topper in you

The circuit's impedance becomes minimum and equal to  $R$ , and so the current is maximum. The circuit is then considered to be in resonance. The resonance angular frequency  $\omega_0$  and frequency  $V_0$  are given by

$$\omega_0 = \frac{1}{\sqrt{2}} \cdot V_0 = \frac{1}{2\pi\sqrt{LC}}$$

Figure 23.16 depicts the fluctuation of RMS current with frequency of applied voltage. If the applied voltage has many frequency components, the current will be high for the components with frequency  $V_0$ . An LCR series circuit's Q factor is given by

$$Q = \frac{\omega_0 L}{R}$$

A direct current flows uniformly over the conductor's cross-section. An alternating current, on the other hand, runs primarily along the conductor's surface. The skin effect is the name given to this phenomena. The reason for this is that when alternating current travels through a conductor, the flux change in the inner region of the conductor is greater.

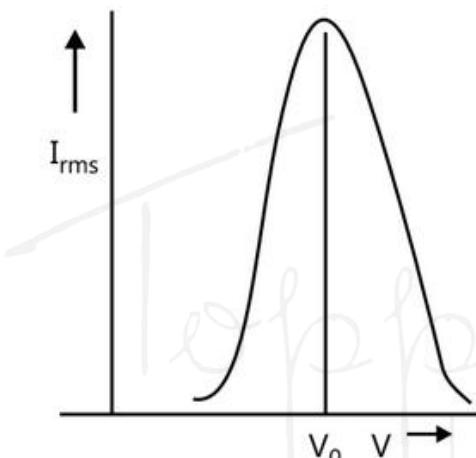
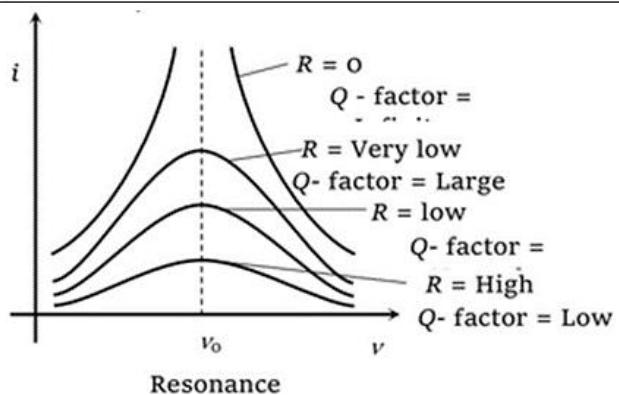


Figure:



### Parallel RCL Circuits

Consider the parallel RLC circuit illustrated in Fig. 23.19.

The voltage source is.

$$V(t) = V_0 \sin \omega t$$

In contrast to the series RLC circuit, the instantaneous voltage across all three circuit elements, R, L, and C, is the same, and each voltage is in phase with the current flowing through the resistor. The current flowing through each constituent, however, will be different.

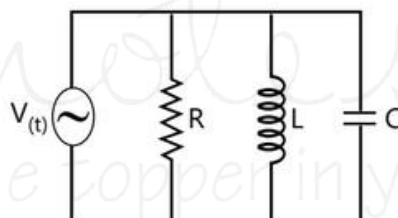


Figure: Parallel LRC circuit

We use the previously obtained results to analyse this circuit. The resistor's current is

$$I_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} = \sin \omega t = I_{R0} \sin \omega t \quad \dots(i)$$

Where

$$I_{R0} = V_0 / R$$

The voltage across the inductor is

$$V_L(t) = V(t) = V_0 \sin \omega t = L \frac{dI_L}{dt} \quad \dots(ii)$$

which gives

$$I_L(t) = \int_0^t \frac{V_0}{L} \sin \omega t' dt' = \frac{V_0}{\omega L} \cos \omega t = \frac{V_0}{X_L} \sin \left( \omega t - \frac{\pi}{2} \right) = I_{L0} \sin \left( \omega t - \frac{\pi}{2} \right) \quad \dots(iii)$$

where  $I_{L0} = V_0 / X_L$  and  $X_L = \omega L$  is the inductive reactance.

Similarly, the voltage across the capacitor is,

$$V_C(t) = V_0 \sin \omega t = Q(t) / C$$

which implies

$$I_c(t) = \frac{dQ}{dt} = \omega C V_0 \cos \omega t = \frac{V_0}{X_C} \sin \left( \omega t + \frac{\pi}{2} \right) = I_{c0} \sin \left( \omega t + \frac{\pi}{2} \right) \quad \dots(iv)$$

where

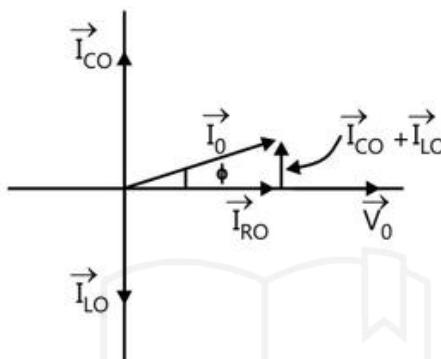
$$I_{c0} = V_0 / X_C \text{ and } X_C = 1 / \omega L$$

$X_C$  is the capacitive reactance.

The total current is simply the sum of the three currents using Kirchhoff's junction rule.

$$I(t) = I_R(t) + I_L(t) + I_C(t) = I_{R0} \sin \omega t + I_{L0} \sin \left( \omega t - \frac{\pi}{2} \right) + I_{c0} \sin \left( \omega t + \frac{\pi}{2} \right) \quad \dots(v)$$

The current can be represented with the phasor diagram shown in Fig. 23.20



**Figure:** Phase difference between current and voltage

From the phasor diagram, we see that.

$$\vec{I}_0 = \vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{c0} \quad \dots(vi)$$

And the maximum amplitude of the total current,  $I_0$ , can be obtained as

$$\begin{aligned} \vec{I}_0 = \vec{I}_0 &= \left| \vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{c0} \right| = \sqrt{I_{R0}^2 + (I_{c0} - I_{L0})^2} \\ &= V_0 \sqrt{\frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2} = V_0 \sqrt{\frac{1}{R^2} + \left( \frac{1}{X_C} - \frac{1}{X_L} \right)^2} \end{aligned} \quad \dots(vii)$$

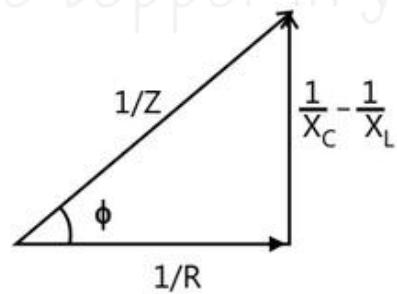
Note however, since  $I_R(t)$ ,  $I_C(t)$  and  $I_L(t)$  are not in phase with one another,  $I_0$  is not equal to the sum of the maximum amplitudes of the three currents: of the circuit is given by:

$$I_0 \neq I_{R0} + I_{L0} + I_{c0} \quad \dots(viii)$$

With  $I_0 = V_0/Z$ , the (inverse) impedance

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left( \omega C - \frac{1}{\omega L} \right)^2} = \sqrt{\frac{1}{R^2} + \left( \frac{1}{X_C} - \frac{1}{X_L} \right)^2} \quad \dots(ix)$$

The relationship between  $Z$ ,  $R$ ,  $X_L$  and  $X_C$  is shown in Fig. 23.21 which shows a relationship between  $Z$ ,  $R$ ,  $X_L$  and  $X_C$  in a parallel RLC circuit.



**Figure:** Impedance triangle

from phasor diagram we see that phase can be obtain as

$$\tan \phi = \left( \frac{I_{c0} - I_{L0}}{I_{R0}} \right) = \frac{\frac{V_0}{X_C} - \frac{V_0}{X_L}}{\frac{V_0}{R}} = R \left( \frac{V_0}{X_C} - \frac{V_0}{X_L} \right) = R \left( \omega t - \frac{\pi}{2} \right) \quad \dots(x)$$

The resonance condition for the parallel RLC circuit is given by  $\phi = 0$ , which implies:

$$\frac{1}{X_C} = \frac{1}{X_L} \quad \dots(xi)$$

The resonant frequency is:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \dots(\text{xii})$$

which is the same as the series RLC circuit. We can see from Eq. (xii) that  $1/Z$  is smallest (or  $Z$  is maximal) at resonance. The current in the inductor perfectly cancels out the current in the capacitor, therefore the overall current in the circuit achieves a minimum and equals the current in the resistor:

$$I_0 = \frac{V_0}{R} \quad \dots(\text{xiii})$$

As in the series RLC circuit, power is wasted solely through the resistor. The average power is

$$\langle P(t) \rangle = \langle I_R(t)V(t) \rangle = \langle I_R^2(t)R \rangle = \frac{V_0^2}{R} \langle \sin^2 \omega t \rangle = \frac{V_0^2}{2R} = \frac{V_0^2}{2Z} = \left( \frac{Z}{R} \right) \quad \dots(\text{xiv})$$

Thus, the power factor in this case is

$$= \frac{\langle P(t) \rangle}{V_0^2 / 2Z} = \frac{Z}{R} = \frac{1}{\sqrt{1 + \left( R\omega C - \frac{R}{\omega L} \right)^2}} \cos \phi \quad \dots(\text{xv})$$

In this situation, the current flows equally via each resistor. To determine the current, calculate the equivalent resistor and divide the voltage by the equivalent resistance.

Series Resonant Circuit	Parallel Resonant Circuit
$X_L = X_C$	$\frac{1}{X_L} = \frac{1}{X_C}$
$V_r = \frac{1}{2\pi\sqrt{LC}}$	$V_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left( C\omega - \frac{1}{L\omega} \right)^2}$

## MORE ON POWER FACTOR

(a) The power factor is the factor present in the relationship for average power of an alternating current circuit.

So,

$$\cos \phi = \frac{P_{ac}}{E_{rms} I_{rms}} = \frac{P_{avg}}{P_v}$$

Thus, ratio of average power and virtual power in the circuit is equal to power factor.

(b) Power factor is also equivalent to the ac circuit's resistance to impedance ratio.

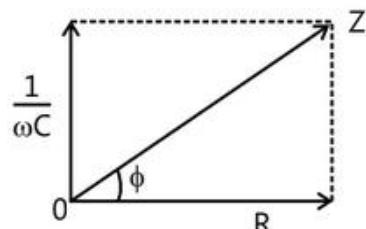
Thus,

$$\cos \phi = \frac{R}{Z}$$

(c) Power factor is determined by the type of components utilised in the circuit.

(d) If a pure resistor is connected in the alternating current circuit, then

$$\phi = 0, \cos \phi = 1; \quad P_{av} = \frac{E_0 I_0}{2} = \frac{E_0^2}{2R} = E_{rms} I_{rms}$$



**Figure:**

Thus, the power loss is maximum and electrical energy is converted in the form of heat.

(e) If a pure inductor or capacitor are connected in the ac circuit, then

$$\phi = 90^\circ, \cos \phi = 0 \therefore P_{av} = 0 \text{ (minimum)}$$

Thus is no loss of power.

(f) If a resistor and an inductor or a capacitor are connected in an ac circuit, then

$$\phi \neq 0 \text{ or } \phi \neq 90^\circ$$

Thus  $\phi$  is in between  $0^\circ$  &  $90^\circ$

(g) If the components L, C and R are connected in series in a circuit, then

$$\tan\phi = \frac{X}{R} = \frac{(\omega L - 1/\omega C)}{R} \text{ and ;}$$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X)^2}}$$

Power factor

$$\cos\phi = \frac{R}{Z}$$

(h) Power factor is a unit-less quantity.

(i) If the circuit simply has an inductance coil, there will be no power loss and energy will be stored in the magnetic field.

(j) If a capacitor is just connected in the circuit, no power is lost and energy is stored in the electrostatic field.

(k) An inductor and a capacitor do have some resistance. As a result, there is always some loss of power.

(l) The power factor is one in the resonance condition.

## WATTLESS CURRENT

(a) Wattless current is the component of current whose contribution to average power is zero.

(b) The average level of power is zero because the average of the second component of instantaneous power over the course of a whole cycle is zero.

$$E_0 \sin\omega t (I_0 \sin\phi) \sin(\omega t - \pi/2) = 0$$

(c) The current component connected with this section is known as Wattless current. As a result, the current  $(I_0 \sin\phi) \sin(\omega t - \pi/2)$  is a wattless current whose amplitude is  $I_0 \sin\phi$ .

(d) If RMS value of current in the circuit is  $I_{rms}$  then the RMS value of a wattless current will be  $I_{rms} \sin\phi$ . A wattless current lags or

leads the e.m.f. by an angle  $\pi/2$  RMS value of wattless current:

$$I_{rms} \sin\phi = \frac{I_0}{\sqrt{2}} \sin\phi ; = \frac{I_0}{\sqrt{2}} \frac{X}{Z}. \text{ Since } \sin\phi = \frac{X}{Z}$$

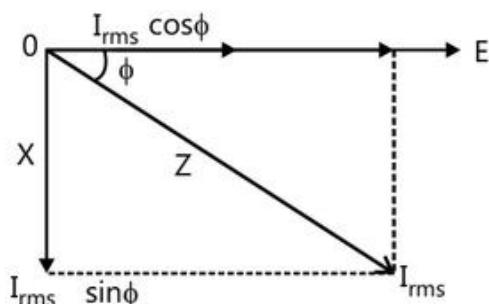


Figure:

Since,  $\sin\phi = \frac{X}{Z}$  where  $Z$  is the resultant reactance of the circuit.

## TRANSFORMERS

- A transformer is a device that converts a low alternating voltage at a higher current to a high alternating voltage at a lower current and vice versa. A transformer, in other words, is an electrical device that increases or decreases alternating voltage.

### Types of Transformers

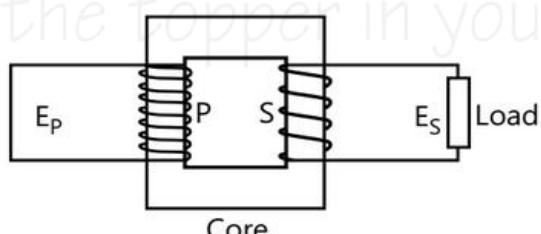


Figure:

(a) **Step-up transformers:** A step-up transformer is a transformer that converts a low alternating voltage at a higher current into a high alternating voltage at a lower current.

(a) **Step-down transformers:** A step-down transformer is a transformer that converts a high alternating voltage at a lower current to a low alternating voltage at a higher current.

**Principle:** A transformer operates on the mutual induction principle. When a changing current runs through a neighbouring coil, an e.m.f. is induced.

**Construction:** It is made up of two coils of insulated wire twisted on the same iron core. The primary (A) winding or coil is the one connected to the alternating current input, while the secondary (S) winding or coil is the one providing output.

**Theory:** An alternating current passes through the primary coil when an alternating source of e.m.f.  $E_p$  is attached to it. An alternating magnetic flux causes an alternating e.m.f. in the secondary coil  $E_s$  due to the flow of alternating current in the main coil. Let  $N_p$  and  $N_s$  denote the number of turns in the primary and secondary coils. The iron core is capable of linking the entire magnetic flux produced by the primary coil's turns with the secondary coil.

According to Faraday's law of electromagnetic induction, the induced e.m.f. in the primary coil,

$$E_p = -N_p \frac{d\phi}{dt} \quad \dots(i)$$

The induced e.m.f in the secondary coil.

$$E_s = -N_s \frac{d\phi}{dt} \quad \dots(ii)$$

Dividing (ii) by (i), we get ;

$$\frac{E_s}{E_p} = \frac{N_s}{N_p};$$

Where

$$\frac{N_s}{N_p} = K$$

the transformation ratio or ratio.

Then,

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = K$$

$K < 1$  for step down transformer. In this case,  $N_s < N_p$  and  $E_s < E_p$  i.e.  $E_p$ , and output alternating voltage < input alternating voltage.

$K > 1$  for step up transformer. In this case,  $N_s > N_p$  and  $E_s > E_p$  i.e., output alternating voltage is greater than the input alternating voltage.

For an ideal transformer (in which no energy losses),

$$\text{output power} = \text{input power} \quad \dots(iii)$$

Let  $I_s$  and  $I_p$  be the current in the primary and secondary coil respectively.

$$\text{Then output power} = E_s I_s$$

$$\text{input power} = E_p I_p$$

from equation (iii)

$$E_p = E_s \text{ or } \frac{E_s}{E_p} = \frac{I_p}{I_s};$$

or ; n general,

$$E \propto \frac{1}{I}$$

Voltage rises as current falls for the same power transfer and vice versa. As a result, whatever gains in the voltage ratio are lost in the current ratio, and vice versa. As a result, a step-up transformer increases alternating voltage by decreasing alternating current, whereas a step-down transformer decreases alternating voltage by raising alternating current.

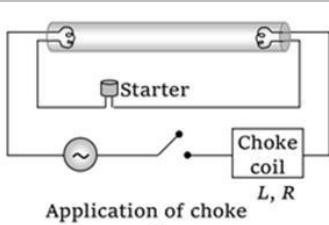
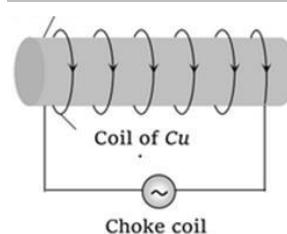
For a transformer, efficiency,

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}} = \frac{E_s I_s}{E_p I_p}$$

For an ideal transformer, efficiency,  $\eta$  is 100%. But in a real transformer, the efficiency varies from 90-99%. This indicates that there are some energy losses in the transformer.

Step-up transformer	Step-down transformer
1. $E_s > E_p$	1. $E_s < E_p$
2. $N_s > N_p$	2. $N_s < N_p$
3. $I_s > I_p$	3. $I_s > I_p$
4. $Z_s < Z_p$	4. $Z_s < Z_p$
5. $k > 1$	5. $k < 1$

### CHOKING COIL



- Let us consider a choke coil of large inductance  $L$  and low resistance  $R$ . Then, the power factor of the given circuit will be given by (as  $R \ll \omega L$ )

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{\omega L}$$

- Now, because we know that  $R \ll \omega L$ , the power factor is minimal, the power absorbed will be very small. Furthermore, due to the coil's high impedance (high inductance), the current flowing through it is quite modest. As a result, such a coil is favoured in electrical circuits for changing current to any desired value while minimising energy waste.

### FORMULAE SHEET

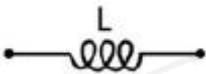
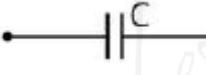
(a) In an AC circuit, sinusoidal voltage source of amplitude  $V_0$  is represented as:

$$V(t) = V_0 \sin \omega t$$

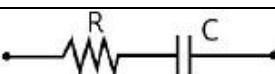
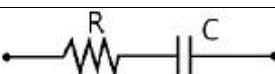
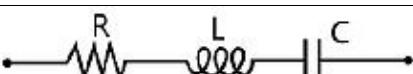
The current in the circuit has amplitude  $I_0$  and lags the applied voltage by phase angle  $\phi$ . Current is represented as:

$$I(t) = I_0 \sin(\omega t - \phi)$$

(b) For a single-element circuit (a resistor, a capacitor or an inductor) connected to the AC voltage source, we summarise the results in the below table:

Circuit elements	Resistance/Reactance	Current Amplitude	Phase angle $\phi$
	$R$	$I_{R_0} = \frac{V_0}{R}$	0
	Inductive Reactance $X_L = \omega L$	$I_{L_0} = \frac{V_0}{X_L}$	$(\pi/2)$ i.e. current lags voltage by $90^\circ$
	Capacitive Reactance $X_C = \frac{1}{\omega C}$	$I_{C_0} = \frac{V_0}{X_C}$	$(-\pi/2)$ i.e., current leads voltage by $90^\circ$

(c) For a circuit having more than one circuit element connected in a series, we summarise the results in the below table:

Circuit elements	Impedance $Z$	Current amplitude	Phase angle $\phi$
	$\sqrt{R^2 + X_L^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$	$0 < \phi < \left(\frac{\pi}{2}\right)$
	$\sqrt{R^2 + X_C^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$	$\left(-\frac{\pi}{2}\right) < \phi < 0$
	$\sqrt{R^2 + (X_L - X_C)^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$	$\phi > 0$ if $X_L > X_C$ $\phi < 0$ if $X_L < X_C$

(d) For series LCR circuit,

(i) The impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(ii) The current lags the voltage by phase

$$\text{angle } \phi = \tan^{-1} \frac{(X_L - X_C)}{R}$$

(iii) The resonant frequency is  $\omega_0 = \sqrt{\frac{1}{LC}}$

At resonance, the current in the series LCR circuit is maximum, while that in parallel LCR circuit is minimum.

(e) Impedance for parallel LCR circuit, is given by

$$\frac{I}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \frac{1}{\omega C}\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

The phase angle by which the current lags the voltage is

$$\phi = \tan^{-1} R \left( \frac{1}{X_L} - \frac{1}{X_C} \right) = \tan^{-1} R \left( \frac{1}{\omega L} - \frac{1}{\omega C} \right)$$

(f) The RMS (root mean square) value of voltage and current in an AC circuit are given as

$$V_{rms} = \frac{V_0}{\sqrt{2}}, \text{ and } I_{rms} = \frac{I_0}{\sqrt{2}}$$

(g) Average power of an AC circuit is

$$\langle P(t) \rangle = I_{rms} V_{rms} \cos \phi \text{ where } \cos \phi = \frac{R}{Z} \text{ is the power factor of the circuit.}$$

(h) Quality factor Q of LCR circuit is

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

(i) For a transformer, the ratio of secondary coil voltage to that of primary coil voltage is

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \text{ where } N_1 \text{ is number of turns in primary coil, and } N_2 \text{ is number of turns in secondary coil.}$$

For the step-up transformer,  $N_2 > N_1$ ; for step down transformer,  $N_2 < N_1$ .

## SOLVED PROBLEM

### Example 1:

An electric heater draws 2.5 A current from a 220-V, 60-Hz power supply. Find

- The average current
- The average of the square of the current
- The current amplitude
- The supply voltage amplitude

### Solution:

In AC circuit, the average value of current over a long time interval is zero but  $I^2$  is not zero. The r.m.s. value of current and voltage is given by

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_{max}}{\sqrt{2}}.$$

(a) The average of sinusoidal AC values over any whole number of cycles is zero.

(b) RMS value of current =  $I_{rms} = 2.5 \text{ A}$  so,  $(I^2)_{av} = (I_{rms})^2 = 6.25 \text{ A}^2$

(c)  $I_{rms} = \frac{I_m}{\sqrt{2}}$  ; So, current amplitude  $I_m = \sqrt{2} I_{rms} = \sqrt{2}(2.5 \text{ A}) = 3.5 \text{ A}$

(d)  $V_{rms} = 220 \text{ V} = \frac{V_m}{\sqrt{2}}$  . So, supply voltage amplitude  $V_m = \sqrt{2}(V_{rms}) = \sqrt{2}(220 \text{ V}) = 311 \text{ V}.$

### Example 2:

Find the RMS value of current  $I = I_m \sin \omega t$  from

(i)  $t = 0$  to  $t = \frac{\pi}{2\omega}$  (ii)  $t = \frac{\pi}{2\omega}$  to  $t = \frac{3\pi}{2\omega}$

### Solution:

In AC circuit over time interval  $0 \leq t \leq T$  the RMS value of current is given by

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T I_m^2 \sin^2(\omega t) dt}{\int_0^T dt}} = \frac{I_m}{\sqrt{2}}$$

$$\text{where } T = \frac{2\pi}{\omega}$$

$$(i) I_{rms} = \sqrt{\frac{\int_0^{\frac{\pi}{\omega}} I_m^2 \sin^2(\omega t) dt}{\frac{\pi}{\omega}}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$(ii) I_{rms} = \sqrt{\frac{\int_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} I_m^2 \sin^2(\omega t) dt}{\frac{\pi}{\omega}}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} \text{ A}$$

### Example 3:

When a voltage  $V_s = 200\sqrt{2} \sin(\omega t + 15^\circ)$  is applied to an AC circuit, the current in the circuit is found to be  $I = 2 \sin(\omega t + \pi/4)$  then average power consumed in the circuit is

(A) 200 W (B)  $400\sqrt{2} \text{ W}$   
 (C)  $100\sqrt{2} \text{ W}$  (D)  $200\sqrt{2} \text{ W}$

**Solution:**

Power in any AC circuit is calculated as  $P_{av} = V_{rms} I_{rms} \cos \phi$  where  $\phi$  is phase angle between  $V$  and  $I$ .

$$P_{av} = V_{rms} I_{rms} \cos \phi = \frac{200\sqrt{2}}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \cos(30^\circ) = 100\sqrt{6} \text{ W}$$

**Example 4:**

An alternating voltage of 220V RMS at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of  $6\Omega$  in series. Calculate (a) The current, (b) Potential difference across the resistance, (c) Potential different across inductance, (d) The time lag.

**Solution:**

The impedance of LR circuit is  $Z = \sqrt{R^2 + (\omega L)^2}$ .

The RMS value of the current is  $I_{rms} = \frac{V_{rms}}{Z}$ . In LR circuit, the current lags the applied voltage by phase angle  $\phi$  obtained as  $\phi = \tan^{-1} \left( \frac{\omega L}{R} \right)$ .

The impedance of the L-R series circuit is given by:

$$Z^2 = [R^2 + (\omega L)^2]^{1/2} = [(R)^2 + (2\pi f L)^2]^{1/2} \\ = [62 + (2 \times 3.14 \times 40 \times 0.01)^2]^{1/2} = 6.504 \Omega$$

(a) RMS value of the current:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{6.504} = 33.83 \text{ A}$$

(b) The potential difference across the resistance is given by:  $V_R = I_{rms} \times R = 33.83 \times 6 = 202.83 \text{ V}$ .

(c) Potential difference across the inductance is given by:

$$V_L = I_{rms} \times (\omega L) = 33.83 \times (2 \times 3.14 \times 0.01) = 96.83 \text{ V}$$

(d) Phase angle  $\phi = \tan^{-1} \left( \frac{\omega L}{R} \right)$ ; So,  $\phi = \tan^{-1} (0.4189) = 22.46^\circ$

$$\text{Now time lag} = \frac{\phi}{360^\circ} = \frac{22.46}{360} = 0.0623 \text{ s}$$

**Example 5:**

A  $\frac{9}{100\pi}$  H inductor and a  $12 \Omega$  resistance are connected in a series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

**Solution:**

The impedance of LR circuit is  $Z = \sqrt{R^2 + (\omega L)^2}$ .

The RMS value of the current is  $I_{rms} = \frac{V_{rms}}{Z}$ . In LR circuit, the current lags the applied voltage by phase angle  $\phi$  obtained as  $\phi = \tan^{-1} \left( \frac{\omega L}{R} \right)$ .

$$\text{Here, } X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{9}{100\pi} = 9 \Omega$$

$$\text{So, } Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15 \Omega$$

$$(a) I = \frac{V}{Z} = \frac{225}{15} = 15 \text{ A and}$$

$$(b) \phi = \tan^{-1} \left( \frac{X_L}{R} \right) = \tan^{-1} \left( \frac{9}{12} \right) = \tan^{-1} 3/4 = 37^\circ$$

i.e., the current will lag the applied voltage by  $37^\circ$  in phase.

**Example 6:**

A choke coil is needed to operate an arc lamp at 160 V (RMS) and 50 Hz. The arc lamp has an effective resistance of 50 $\Omega$  when running of 10 A (RMS). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160V (dc), what additional resistance is required? Compare the power losses in both cases.

**Solution:**

The choke coil is a LR circuit having large inductance and small resistance. The potential difference across the resistor and inductor is added vectorially:  $V^2 = V_R^2 + V_L^2$ .